Chapter 6: Factoring

Chapter 7: Rational Expressions

Chapter 8: Radicals

Chapter 9: Quadratics

Chapter 10: Functions

Example 5.

$4+(-3)$	Different signs, subtract $4-3$, use sign from bigger number, positive
1	Our Solution

Example 6.

$7+(-10)$	Different signs, subtract $10-7$, use sign from bigger number, negative
-3	Our Solution

For subtraction of negatives we will change the problem to an addition problem which we can then solve using the above methods. The way we change a subtraction to an addition is to add the opposite of the number after the subtraction sign. Often this method is refered to as "add the opposite." This is illustrated in the following examples.

Example 7.

$8-3$	Add the opposite of 3
$8+(-3)$	Different signs, subtract $8-3$, use sign from bigger number, positive
5	Our Solution

Example 8.

$-4-6$	Add the opposite of 6
$-4+(-6)$	Same sign, add $4+6$, keep the negative
-10	Our Solution

Example 9.

$9-(-4)$	Add the opposite of -4
$9+4$	Same sign, add $9+4$, keep the positive
13	Our Solution

Example 10.

$-6-(-2)$	Add the opposite of -2
$-6+2$	Different sign, subtract $6-2$, use sign from bigger number, negative
-4	Our Solution

Example 21.

$\frac{13}{6} - \frac{9}{6}$	Same denominator, subtract numerators $13 - 9$
$\frac{4}{6}$	Reduce answer, dividing by 2
$\frac{2}{3}$	Our Solution

If the denominators do not match we will first have to identify the LCD and build up each fraction by multiplying the numerators and denominators by the same number so the denominator is built up to the LCD.

Example 22.

$\frac{5}{6} + \frac{4}{9}$	LCD is 18.
$\frac{\mathbf{3} \cdot 5}{\mathbf{3} \cdot 6} + \frac{4 \cdot \mathbf{2}}{9 \cdot \mathbf{2}}$	Multiply first fraction by 3 and the second by 2
$\frac{15}{18} + \frac{8}{18}$	Same denominator, add numerators, $15 + 8$
$\frac{23}{18}$	Our Solution

Example 23.

$\frac{2}{3} - \frac{1}{6}$	LCD is 6
$\frac{2 \cdot 2}{2 \cdot 3} - \frac{1}{6}$	Multiply first fraction by 2, the second already has a denominator of 6
$\frac{4}{6} - \frac{1}{6}$	Same denominator, subtract numerators, $4 - 1$
$\frac{3}{6}$	Reduce answer, dividing by 3
$\frac{1}{2}$	Our Solution

Distribute

37) $-8(x-4)$

38) $3(8v+9)$

39) $8n(n+9)$

40) $-(-5+9a)$

41) $7k(-k+6)$

42) $10x(1+2x)$

43) $-6(1+6x)$

44) $-2(n+1)$

45) $8m(5-m)$

46) $-2p(9p-1)$

47) $-9x(4-x)$

48) $4(8n-2)$

49) $-9b(b-10)$

50) $-4(1+7r)$

51) $-8n(5+10n)$

52) $2x(8x-10)$

Simplify.

53) $9(b+10)+5b$

54) $4v-7(1-8v)$

55) $-3x(1-4x)-4x^2$

56) $-8x+9(-9x+9)$

57) $-4k^2-8k(8k+1)$

58) $-9-10(1+9a)$

59) $1-7(5+7p)$

60) $-10(x-2)-3$

61) $-10-4(n-5)$

62) $-$[illegible]$(5-m)+3m$

63) $4(x+7)+8(x+4)$

64) $-2r(1+4r)+8r(-r+4)$

65) $-8(n+6)-8n(n+$[illegible]$)$

66) $9(6b+5)-4b(b+3)$

67) $7(7+3v)+10(3-10v)$

68) $-7(4x-6)+2(10x-10)$

69) $2n(-10n+5)-7(6-10n)$

70) $-3(4+a)+6a(9a+10)$

71) $5(1-6k)+10(k-8)$

72) $-7(4x+3)-10(10x+10)$

73) $(8n^2-3n)-(5+4n^2)$

74) $(7x^2-3)-(5x^2+6x)$

75) $(5p-6)+(1-p)$

76) $(3x^2-x)-(7-8x)$

77) $(2-4v^2)+(3v^2+2v)$

78) $(2b-8)+(b-7b^2)$

79) $(4-2k^2)+(8-2k^2)$

80) $(7a^2+7a)-(6a^2+4a)$

81) $(x^2-8)+(2x^2-7)$

82) $(3-7n^2)+(6n^2+3)$

Example 47.

$\begin{array}{r} -6+x=-2 \\ \underline{\mathbf{+6}\qquad\mathbf{+6}} \\ x=4 \end{array}$	$\begin{array}{r} -10=x-7 \\ \underline{\mathbf{+7}\quad\mathbf{+7}} \\ -3=x \end{array}$	$\begin{array}{r} 5=-8+x \\ \underline{\mathbf{+8}\ \mathbf{+8}} \\ 13=x \end{array}$

Table 2. Subtraction Examples

Multiplication Problems

With a multiplication problem, we get rid of the number by dividing on both sides. For example consider the following example.

Example 48.

$$\begin{array}{rl} 4x=20 & \text{Variable is multiplied by } 4 \\ \overline{\mathbf{4}}\quad\overline{\mathbf{4}} & \text{Divide both sides by } 4 \\ x=5 & \text{Our solution!} \end{array}$$

Then we get our solution $x=5$

With multiplication problems it is very important that care is taken with signs. If x is multiplied by a negative then we will divide by a negative. This is shown in example 9.

Example 49.

$$\begin{array}{rl} -5x = 30 & \text{Variable is multiplied by } -5 \\ \overline{\mathbf{-5}}\quad\overline{\mathbf{-5}} & \text{Divide both sides by } -5 \\ x=-6 & \text{Our Solution!} \end{array}$$

The same process is used in each of the following examples. Notice how negative and positive numbers are handled as each problem is solved.

Example 50.

sure. If the pressure of a certain gas is 40 newtons per square meter when the volume is 600 cubic meters what will the pressure be when the volume is reduced by 240 cubic meters?

31. The time required to empty a tank varies inversely as the rate of pumping. If a pump can empty a tank in 45 min at the rate of 600 kL/min, how long will it take the pump to empty the same tank at the rate of 1000 kL/min?

32. The weight of an object varies inversely as the square of the distance from the center of the earth. At sea level (6400 km from the center of the earth), an astronaut weighs 100 lb. How far above the earth must the astronaut be in order to weigh 64 lb?

33. The stopping distance of a car after the brakes have been applied varies directly as the square of the speed r. If a car, traveling 60 mph can stop in 200 ft, how fast can a car go and still stop in 72 ft?

34. The drag force on a boat varies jointly as the wetted surface area and the square of the velocity of a boat. If a boat going 6.5 mph experiences a drag force of 86 N when the wetted surface area is 41.2 ft^2, how fast must a boat with 28.5 ft^2 of wetted surface area go in order to experience a drag force of 94N?

35. The intensity of a light from a light bulb varies inversely as the square of the distance from the bulb. Suppose intensity is 90 W/m^2 (watts per square meter) when the distance is 5 m. How much further would it be to a point where the intesity is 40 W/m^2?

36. The volume of a cone varies jointly as its height, and the square of its radius. If a cone with a height of 8 centimeters and a radius of 2 centimeters has a volume of 33.5 cm^3, what is the volume of a cone with a height of 6 centimeters and a radius of 4 centimeters?

37. The intensity of a television signal varies inversely as the square of the distance from the transmitter. If the intensity is 25 W/m^2 at a distance of 2 km, how far from the trasmitter are you when the intensity is 2.56 W/m^2?

38. The intensity of illumination falling on a surface from a given source of light is inversely proportional to the square of the distance from the source of light. The unit for measuring the intesity of illumination is usually the footcandle. If a given source of light gives an illumination of 1 foot-candle at a distance of 10 feet, what would the illumination be from the same source at a distance of 20 feet?

1.8

Linear Equations - Number and Geometry

Objective: Solve number and geometry problems by creating and solving a linear equation.

Word problems can be tricky. Often it takes a bit of practice to convert the English sentence into a mathematical sentence. This is what we will focus on here with some basic number problems, geometry problems, and parts problems.

A few important phrases are described below that can give us clues for how to set up a problem.

- **A number** (or unknown, a value, etc) often becomes our variable
- **Is** (or other forms of is: was, will be, are, etc) often represents equals (=)

  x is 5 becomes $x = 5$
- **More than** often represents addition and is usually built backwards, writing the second part plus the first

  Three more than a number becomes $x + 3$
- **Less than** often represents subtraction and is usually built backwards as well, writing the second part minus the first

  Four less than a number becomes $x - 4$

Using these key phrases we can take a number problem and set up and equation and solve.

Example 102.

If 28 less than five times a certain number is 232. What is the number?

$5x - 28$	Subtraction is built backwards, multiply the unknown by 5
$5x - 28 = 232$	Is translates to equals
$\underline{+\mathbf{28} + \mathbf{28}}$	Add 28 to both sides
$5x = 260$	The variable is multiplied by 5
$\overline{\mathbf{5}}\quad\overline{\mathbf{5}}$	Divide both sides by 5
$x = 52$	The number is 52.

This same idea can be extended to a more involved problem as shown in the next example.

Example 103.

Fifteen more than three times a number is the same as ten less than six times the number. What is the number

$$
\begin{array}{rl}
3x+15 & \text{First, addition is built backwards} \\
6x-10 & \text{Then, subtraction is also built backwards} \\
3x+15=6x-10 & \text{Is between the parts tells us they must be equal} \\
\underline{\mathbf{-3x \qquad -3x}} & \text{Subtract } 3x \text{ so variable is all on one side} \\
15=3x-10 & \text{Now we have } a \text{ two} - \text{step equation} \\
\underline{\mathbf{+10 \qquad +10}} & \text{Add 10 to both sides} \\
25=3x & \text{The variable is multiplied by 3} \\
\overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \text{Divide both sides by 3} \\
\frac{25}{3}=x & \text{Our number is } \frac{25}{3}
\end{array}
$$

Another type of number problem involves consecutive numbers. **Consecutive numbers** are numbers that come one after the other, such as 3, 4, 5. If we are looking for several consecutive numbers it is important to first identify what they look like with variables before we set up the equation. This is shown in the following example.

Example 104.

The sum of three consecutive integers is 93. What are the integers?

$$
\begin{array}{rl}
\text{First } x & \text{Make the first number } x \\
\text{Second } x+1 & \text{To get the next number we go up one or } +1 \\
\text{Third } x+2 & \text{Add another 1 (2 total) to get the third} \\
F+S+T=93 & \text{First } (F) \text{ plus Second } (S) \text{ plus Third } (T) \text{ equals 93} \\
(x)+(x+1)+(x+2)=93 & \text{Replace } F \text{ with } x, S \text{ with } x+1, \text{ and } T \text{ with } x+2 \\
x+x+1+x+2=93 & \text{Here the parenthesis aren}'t \text{ needed.} \\
3x+3=93 & \text{Combine like terms } x+x+x \text{ and } 2+1 \\
\underline{\mathbf{-3 \quad -3}} & \text{Add 3 to both sides} \\
3x=90 & \text{The variable is multiplied by 3} \\
\overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \text{Divide both sides by 3} \\
x=30 & \text{Our solution for } x \\
\text{First } 30 & \text{Replace } x \text{ in our origional list with 30} \\
\text{Second } (30)+1=31 & \text{The numbers are 30, 31, and 32} \\
\text{Third } (30)+2=32 &
\end{array}
$$

Sometimes we will work consective even or odd integers, rather than just consecutive integers. When we had consecutive integers, we only had to add 1 to get the next number so we had x, $x+1$, and $x+2$ for our first, second, and third number respectively. With even or odd numbers they are spaced apart by two. So if we want three consecutive even numbers, if the first is x, the next number would be $x+2$, then finally add two more to get the third, $x+4$. The same is

$4x + 2 = 102$ Solve the two − step equation

$\underline{-2 \quad -2}$ Subtract 2 from both sides

$4x = 100$ The variable is multiplied by 4

$\overline{4} \quad \overline{4}$ Divide both sides by 4

$x = 25$ Our solution for x

	Age Now
Nicole	25
Kristen	$32 - 25 = 7$

Plug 25 in for x in the now column
Nicole is 25 and Kristin is 7

A slight variation on age problems is to ask not how old the people are, but rather ask how long until we have some relationship about their ages. In this case we alter our table slightly. In the change column because we don't know the time to add or subtract we will use a variable, t, and add or subtract this from the now column. This is shown in the next example.

Example 113.

Louis is 26 years old. Her daughter is 4 years old. In how many years will Louis be double her daughter's age?

	Age Now	$+t$
Louis	26	
Daughter	4	

As we are given their ages now, these numbers go into the table. The change is unknown, so we write $+t$ for the change

	Age Now	$+t$
Louis	26	$26+t$
Daughter	4	$4+t$

Fill in the change column by adding t to each person's age. Our table is now complete.

$L = 2D$ Louis will be double her daughter

$(26 + t) = 2(4 + t)$ Replace variables with information in change cells

$26 + t = 8 + 2t$ Distribute through parenthesis

$\underline{-t \quad -t}$ Subtract t from both sides

$26 = 8 + t$ Now we have an 8 added to the t

$\underline{-8 - 8}$ Subtract 8 from both sides

$18 = t$ In 18 years she will be double her daughter's age

Age problems have several steps to them. However, if we take the time to work through each of the steps carefully, keeping the information organized, the problems can be solved quite nicely.

World View Note: The oldest man in the world was Shigechiyo Izumi from Japan who lived to be 120 years, 237 days. However, his exact age has been disputed.

years ago, the bronze plaque was one-half the age of the wood plaque. Find the present age of each plaque.

13. A is now 34 years old, and B is 4 years old. In how many years will A be twice as old as B?

14. A man's age is 36 and that of his daughter is 3 years. In how many years will the man be 4 times as old as his daughter?

15. An Oriental rug is 52 years old and a Persian rug is 16 years old. How many years ago was the Oriental rug four times as old as the Persian Rug?

16. A log cabin quilt is 24 years old and a friendship quilt is 6 years old. In how may years will the log cabin quilt be three times as old as the friendship quilt?

17. The age of the older of two boys is twice that of the younger; 5 years ago it was three times that of the younger. Find the age of each.

18. A pitcher is 30 years old, and a vase is 22 years old. How many years ago was the pitcher twice as old as the vase?

19. Marge is twice as old as Consuelo. The sum of their ages seven years ago was 13. How old are they now?

20. The sum of Jason and Mandy's age is 35. Ten years ago Jason was double Mandy's age. How old are they now?

21. A silver coin is 28 years older than a bronze coin. In 6 years, the silver coin will be twice as old as the bronze coin. Find the present age of each coin.

22. A sofa is 12 years old and a table is 36 years old. In how many years will the table be twice as old as the sofa?

23. A limestone statue is 56 years older than a marble statue. In 12 years, the limestone will be three times as old as the marble statue. Find the present age of the statues.

24. A pewter bowl is 8 years old, and a silver bowl is 22 years old. In how many years will the silver bowl be twice the age of the pewter bowl?

25. Brandon is 9 years older than Ronda. In four years the sum of their ages will be 91. How old are they now?

26. A kerosene lamp is 95 years old, and an electric lamp is 55 years old. How many years ago was the kerosene lamp twice the age of the electric lamp?

27. A father is three times as old as his son, and his daughter is 3 years younger

than the son. If the sum of their ages 3 years ago was 63 years, find the present age of the father.

28. The sum of Clyde and Wendy's age is 64. In four years, Wendy will be three times as old as Clyde. How old are they now?

29. The sum of the ages of two ships is 12 years. Two years ago, the age of the older ship was three times the age of the newer ship. Find the present age of each ship.

30. Chelsea's age is double Daniel's age. Eight years ago the sum of their ages was 32. How old are they now?

31. Ann is eighteen years older than her son. One year ago, she was three times as old as her son. How old are they now?

32. The sum of the ages of Kristen and Ben is 32. Four years ago Kristen was twice as old as Ben. How old are they both now?

33. A mosaic is 74 years older than the engraving. Thirty years ago, the mosaic was three times as old as the engraving. Find the present age of each.

34. The sum of the ages of Elli and Dan is 56. Four years ago Elli was 3 times as old as Dan. How old are they now?

35. A wool tapestry is 32 years older than a linen tapestry. Twenty years ago, the wool tapestry was twice as old as the linen tapestry. Find the present age of each.

36. Carolyn's age is triple her daughter's age. In eight years the sum of their ages will be 72. How old are they now?

37. Nicole is 26 years old. Emma is 2 years old. In how many years will Nicole be triple Emma's age?

38. The sum of the ages of two children is 16 years. Four years ago, the age of the older child was three times the age of the younger child. Find the present age of each child.

39. Mike is 4 years older than Ron. In two years, the sum of their ages will be 84. How old are they now?

40. A marble bust is 25 years old, and a terra-cotta bust is 85 years old. In how many years will the terra-cotta bust be three times as old as the marble bust?

Example 114.

Two joggers start from opposite ends of an 8 mile course running towards each other. One jogger is running at a rate of 4 mph, and the other is running at a rate of 6 mph. After how long will the joggers meet?

	Rate	Time	Distance
Jogger 1			
Jogger 2			

The basic table for the joggers, one and two

	Rate	Time	Distance
Jogger 1	**4**		
Jogger 2	**6**		

We are given the rates for each jogger. These are added to the table

	Rate	Time	Distance
Jogger 1	4	$\boldsymbol{t}$	
Jogger 2	6	$\boldsymbol{t}$	

We only know they both start and end at the same time. We use the variable t for both times

	Rate	Time	Distance
Jogger 1	4	t	$\boldsymbol{4t}$
Jogger 2	6	t	$\boldsymbol{6t}$
			8

The distance column is filled in by multiplying rate by time

We have **total distance**, 8 miles, under distance

$4t + 6t = 8$ The distance column gives equation by adding

$10t = 8$ Combine like terms, $4t + 6t$

$\frac{10t}{\mathbf{10}} = \frac{8}{\mathbf{10}}$ Divide both sides by 10

$t = \frac{4}{5}$ Our solution for t, $\frac{4}{5}$ hour (48 minutes)

As the example illustrates, once the table is filled in, the equation to solve is very easy to find. This same process can be seen in the following example

Example 115.

Bob and Fred start from the same point and walk in opposite directions. Bob walks 2 miles per hour faster than Fred. After 3 hours they are 30 miles apart. How fast did each walk?

	Rate	Time	Distance
Bob		**3**	
Fred		**3**	

The basic table with given times filled in
Both traveled 3 hours

$2 = t$ Our solution for t, she catches him after 2 hours

World View Note: The 10,000 race is the longest standard track event. 10,000 meters is approximately 6.2 miles. The current (at the time of printing) world record for this race is held by Ethiopian Kenenisa Bekele with a time of 26 minutes, 17.53 second. That is a rate of 12.7 miles per hour!

As these example have shown, using the table can help keep all the given information organized, help fill in the cells, and help find the equation we will solve. The final example clearly illustrates this.

Example 118.

On a 130 mile trip a car travled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took 2.5 hours. For how long did the car travel 40 mph?

	Rate	Time	Distance
Fast	**55**		
Slow	**40**		

Basic table for fast and slow speeds
The given rates are filled in

2.5

	Rate	Time	Distance
Fast	55	$\boldsymbol{t}$	
Slow	40	$\boldsymbol{2.5 - t}$	

Total time is put above the time column
As we have total time the first time we have t
The second time is the subtraction problem
$2.5 - t$

2.5

	Rate	Time	Distance
Fast	55	t	$\boldsymbol{55t}$
Slow	40	$2.5 - t$	$\boldsymbol{100 - 40t}$
			130

Distance column is found by multiplying rate by time. Be sure to distribute $40(2.5 - t)$ for slow

Total distance is put under distance

$55t + 100 - 40t = 130$ The distance column gives our equation by adding

$15t + 100 = 130$ Combine like terms $55t - 40t$

$\underline{\mathbf{-100 \quad -100}}$ Subtract 100 from both sides

$15t = 30$ The variable is multiplied by 30

$\overline{\mathbf{15}} \quad \overline{\mathbf{15}}$ Divide both sides by 15

$t = 2$ Our solution for t.

	Time
Fast	2
Slow	$2.5 - 2 = 0.5$

To answer the question we plug 2 in for t
The car traveled 40 mph for 0.5 hours (30 minutes)

The first plan is flying 25 mph slower than the second plane. In two hours the planes are 430 miles apart. Find the rate of each plane.

29. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1 h head start, how far from the starting point does the bus overtake the car?

30. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.

31. A truck leaves a depot at 11 A.M. and travels at a speed of 45 mph. At noon, a van leaves the same place and travels the same route at a speed of 65 mph. At what time does the van overtake the truck?

32. A family drove to a resort at an average speed of 25 mph and later returned over the same road at an average speed of 40 mph. Find the distance to the resort if the total driving time was 13 h.

33. Three campers left their campsite by canoe and paddled downstream at an average rate of 10 mph. They then turned around and paddled back upstream at an average rate of 5 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 h?

34. A motorcycle breaks down and the rider has to walk the rest of the way to work. The motorcycle was being driven at 45 mph, and the rider walks at a speed of 6 mph. The distance from home to work is 25 miles, and the total time for the trip was 2 hours. How far did the motorcycle go before if broke down?

35. A student walks and jogs to college each day. The student averages 5 km/hr walking and 9 km/hr jogging. The distance from home to college is 8 km, and the student makes the trip in one hour. How far does the student jog?

36. On a 130 mi trip, a car traveled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took a total of 2.5 h. For how long did the car travel at 40 mph?

37. On a 220 mi trip, a car traveled at an average speed of 50 mph and then reduced its average speed to 35 mph for the remainder of the trip. The trip took a total of 5 h. How long did the car travel at each speed?

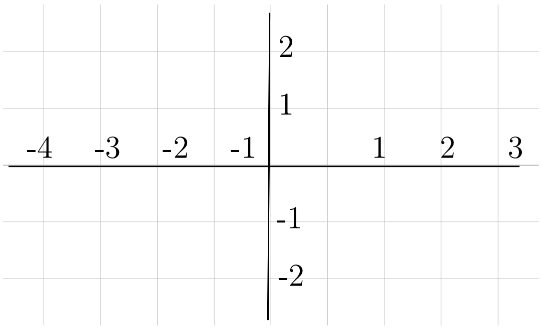
38. An executive drove from home at an average speed of 40 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at and average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

2.1

Graphing - Points and Lines

Objective: Graph points and lines using xy coordinates.

Often, to get an idea of the behavior of an equation we will make a picture that represents the solutions to the equations. A **graph** is simply a picture of the solutions to an equation. Before we spend much time on making a visual representation of an equation, we first have to understand the basis of graphing. Following is an example of what is called the coordinate plane.



The plane is divided into four sections by a horizontal number line (x-axis) and a vertical number line (y-axis). Where the two lines meet in the center is called the origin. This center origin is where $x = 0$ and $y = 0$. As we move to the right the numbers count up from zero, representing $x = 1, 2, 3$.

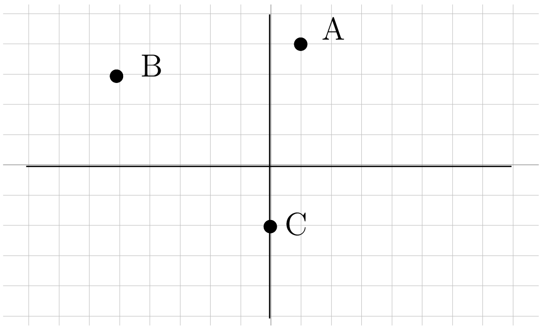
To the left the numbers count down from zero, representing $x = -1, -2, -3$. Similarly, as we move up the number count up from zero, $y = 1, 2, 3$, and as we move down count down from zero, $y = -1, -2, -3$. We can put dots on the graph which we will call points. Each point has an "address" that defines its location. The first number will be the value on the $x-$axis or horizontal number line. This is the distance the point moves left/right from the origin. The second number will represent the value on the $y-$axis or vertical number line. This is the distance the point moves up/down from the origin. The points are given as an ordered pair (x, y).

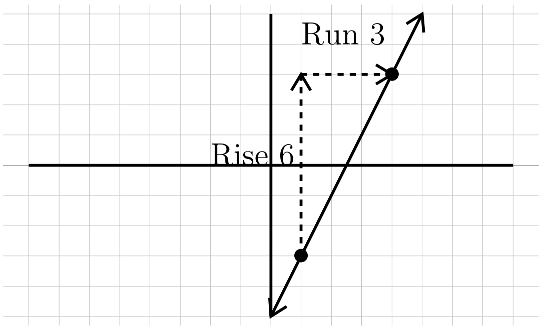
World View Note: Locations on the globe are given in the same manner, each number is a distance from a central point, the origin which is where the prime meridian and the equator. This "origin is just off the western coast of Africa.

The following example finds the address or coordinate pair for each of several points on the coordinate plane.

Example 119.

Give the coordinates of each point.



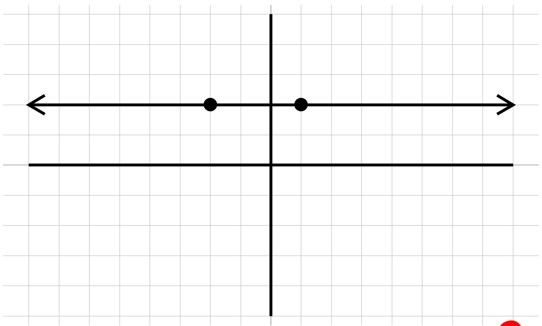


To find the slope of this line, the rise is up 6, the run is right 3. Our slope is then written as a fraction, $\frac{\text{rise}}{\text{run}}$ or $\frac{6}{3}$. This fraction reduces to 2. This will be our slope.

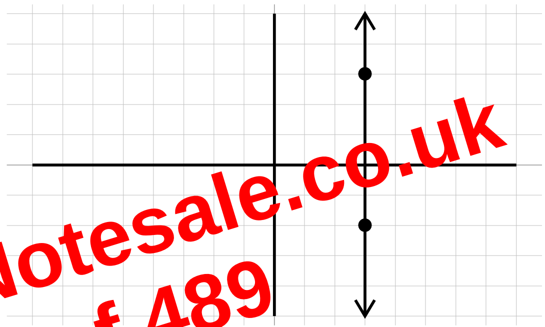
2 Our Solution

There are two special lines that have unique slopes that we need to be aware of. They are illustrated in the following example.

Example 125.



In this graph there is no rise, but the run is 3 units. This slope becomes $\frac{0}{3} = 0$. This line, and all horizontal lines have a zero slope.



This line has a rise of 5, but no run. The slope becomes $\frac{5}{0}$ = undefined. This line, and all vertical lines, have no slope.

As you can see there is a big difference between having a zero slope and having no slope or undefined slope. Remember, slope is a measure of steepness. The first slope is not steep at all, in fact it is flat. Therefore it has a zero slope. The second slope can't get any steeper. It is so steep that there is no number large enough to express how steep it is. This is an undefined slope.

We can find the slope of a line through two points without seeing the points on a graph. We can do this using a slope formula. If the rise is the change in y values, we can calculate this by subtracting the y values of a point. Similarly, if run is a change in the x values, we can calculate this by subtracting the x values of a point. In this way we get the following equation for slope.

$$\textbf{The slope of a line through } (x_1, y_1) \textbf{ and } (x_2, y_2) \textbf{ is } \frac{y_2 - y_1}{x_2 - x_1}$$

2.3

Graphing - Slope-Intercept Form

Objective: Give the equation of a line with a known slope and y-intercept.

When graphing a line we found one method we could use is to make a table of values. However, if we can identify some properties of the line, we may be able to make a graph much quicker and easier. One such method is finding the slope and the y-intercept of the equation. The slope can be represented by m and the y-intercept, where it crosses the axis and $x=0$, can be represented by $(0,b)$ where b is the value where the graph crosses the vertical y-axis. Any other point on the line can be represented by (x,y). Using this information we will look at the slope formula and solve the formula for y.

Example 132.

$$\begin{array}{rl}
m,(0,b),(x,y) & \text{Using the slope formula gives:}\\
\dfrac{y-b}{x-0}=m & \text{Simplify}\\
\dfrac{y-b}{x}=m & \text{Multiply both sides by } x\\
y-b=mx & \text{Add } b \text{ to both sides}\\
\underline{+b\quad +b} & \\
y=mx+b & \text{Our Solution}
\end{array}$$

This equation, $y=mx+b$ can be thought of as the equation of any line that as a slope of m and a y-intercept of b. This formula is known as the slope-intercept equation.

$$\textbf{Slope} - \textbf{Intercept Equation: } \boldsymbol{y=mx+b}$$

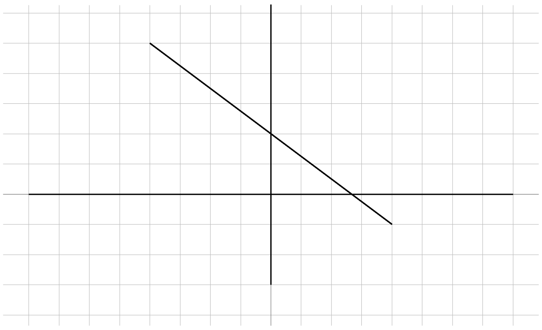
If we know the slope and the y-intercept we can easily find the equation that represents the line.

Example 133.

$$\begin{array}{rl}
\text{Slope}=\dfrac{3}{4},\, y-\text{intercept}=-3 & \text{Use the slope} - \text{intercept equation}\\
y=mx+b & m \text{ is the slope, } b \text{ is the } y-\text{intercept}\\
y=\dfrac{3}{4}x-3 & \text{Our Solution}
\end{array}$$

We can also find the equation by looking at a graph and finding the slope and y-intercept.

Example 134.



15) $x+10y=-37$

16) $x-10y=3$

17) $2x+y=-1$

18) $6x-11y=-70$

19) $7x-3y=24$

20) $4x+7y=28$

21) $x=-8$

22) $x-7y=-42$

23) $y-4=-(x+5)$

24) $y-5=\frac{5}{2}(x-2)$

25) $y-4=4(x-1)$

26) $y-3=-\frac{2}{3}(x+3)$

27) $y+5=-4(x-2)$

28) $0=x-4$

29) $y+1=-\frac{1}{2}(x-4)$

30) $y+2=\frac{6}{5}(x+5)$

Sketch the graph of each line.

31) $y=\frac{1}{3}x+4$

32) $y=-\frac{1}{5}x-4$

33) $y=\frac{6}{5}x-5$

34) $y=-\frac{3}{2}x-1$

35) $y=\frac{3}{2}x$

36) $y=-\frac{3}{4}x+1$

37) $x-y+3=0$

38) $4x+5=5y$

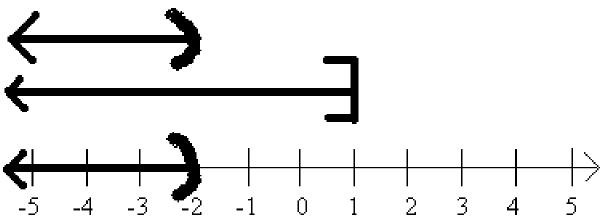
39) $-y-4+3x=0$

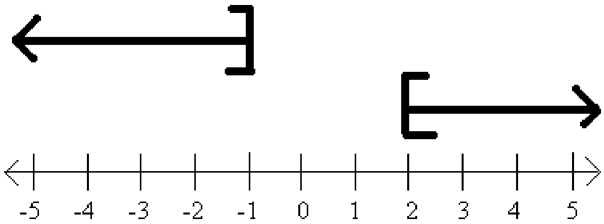
40) $-8=6x-2y$

41) $-3y=-5x+9$

42) $-3y=3-\frac{3}{2}x$

example. The second is if the arrows both point the same way, this is shown below on the left. The third is if the arrows point opposite ways but don't overlap, this is shown below on the right. Notice how interval notation is expressed in each case.





In this graph, the overlap is only the smaller graph, so this is what makes it to the final number line.

Interval Notation: $(-\infty, -2)$

In this graph there is no overlap of the parts. Because their is no overlap, no values make it to the final number line.

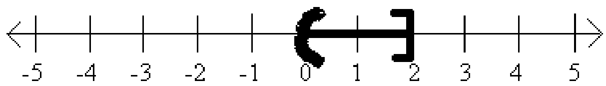
Interval Notation: No Solution or $\varnothing$

The third type of compound inequality is a special type of AND inequality. When our variable (or expression containing the variable) is between two numbers, we can write it as a single math sentence with three parts, such as $5 < x \leqslant 8$, to show x is between 5 and 8 (or equal to 8). When solving these type of inequalities, because there are three parts to work with, to stay balanced we will do the same thing to all three parts (rather than just both sides) to isolate the variable in the middle. The graph then is simply the values between the numbers with appropriate brackets on the ends.

Example 159.

Solve the inequality, graph the solution, and give interval notation.

$$\begin{array}{rl}
-6 \leqslant -4x + 2 < 2 & \text{Subtract 2 from all three parts} \\
\underline{-2 \qquad\quad -2 - 2} & \\
-8 \leqslant -4x < 0 & \text{Divide all three parts by } -4 \\
\overline{-4} \quad \overline{-4} \; \overline{-4} & \text{Dividing by } a \text{ negative flips the symbols} \\
2 \geqslant x > 0 & \text{Flip entire statement so values get larger left to right} \\
0 < x \leqslant 2 & \text{Graph } x \text{ between 0 and 2}
\end{array}$$



$(0, 2]$ Interval Notation

3.2 Practice - Compound Inequalities

Solve each compound inequality, graph its solution, and give interval notation.

1) $\frac{n}{3} \leqslant -3$ or $-5n \leqslant -10$

2) $6m \geqslant -24$ or $m - 7 < -12$

3) $x + 7 \geqslant 12$ or $9x < -45$

4) $10r > 0$ or $r - 5 < -12$

5) $x - 6 < -13$ or $6x \leqslant -60$

6) $9 + n < 2$ or $5n > 40$

7) $\frac{v}{8} > -1$ and $v - 2 < 1$

8) $-9x < 63$ and $\frac{x}{4} < 1$

9) $-8 + b < -3$ and $4b < 20$

10) $-6n \leqslant 12$ and $\frac{n}{3} \leqslant 2$

11) $a + 10 \geqslant 3$ and $8a \leqslant 48$

12) $-6 + v \geqslant 0$ and $2v > 4$

13) $3 \leqslant 9 + x \leqslant 7$

14) $0 \geqslant \frac{x}{9} \geqslant -1$

15) $11 < 8 + k \leqslant 12$

16) $-11 \leqslant n - 9 \leqslant -5$

17) $-3 < x - 1 < 1$

18) $1 \leqslant \frac{p}{8} \leqslant 0$

19) $-4 < 8 - 3m \leqslant 11$

20) $3 + 7r > 59$ or $-6r - 3 > 33$

21) $-16 \leqslant 2n - 10 \leqslant -22$

22) $-6 - 8x \geqslant -6$ or $2 + 10x > 82$

23) $-5b + 10 \leqslant 30$ and $7b + 2 \leqslant -40$

24) $n + 10 \geqslant 15$ or $4n - 5 < -1$

25) $3x - 9 < 2x + 10$ and $5 + 7x \leqslant 10x - 10$

26) $4n + 8 < 3n - 6$ or $10n - 8 \geqslant 9 + 9n$

27) $-8 - 6v \leqslant 8 - 8v$ and $7v + 9 \leqslant 6 + 10v$

28) $5 - 2a \geqslant 2a + 1$ or $10a - 10 \geqslant 9a + 9$

29) $1 + 5k \leqslant 7k - 3$ or $k - 10 > 2k + 10$

30) $8 - 10r \leqslant 8 + 4r$ or $-6 + 8r < 2 + 8r$

31) $2x + 9 \geqslant 10x + 1$ and $3x - 2 < 7x + 2$

32) $-9m + 2 < -10 - 6m$ or $-m + 5 \geqslant 10 + 4m$

4.1 Practice - Graphing

Solve each equation by graphing.

1) $y = -x + 1$
$y = -5x - 3$

2) $y = -\frac{5}{4}x - 2$
$y = -\frac{1}{4}x + 2$

3) $y = -3$
$y = -x - 4$

4) $y = -x - 2$
$y = \frac{2}{3}x + 3$

5) $y = -\frac{3}{4}x + 1$
$y = -\frac{3}{4}x + 2$

6) $y = 2x + 2$
$y = -x - 4$

7) $y = \frac{1}{3}x + 2$
$y = -\frac{5}{3}x - 4$

8) $y = 2x - 4$
$y = \frac{1}{2}x + 2$

9) $y = \frac{5}{3}x + 4$
$y = -\frac{2}{3}x - 3$

10) $y = \frac{1}{2}x + 4$
$y = \frac{1}{2}x + 1$

11) $x + 3y = -9$
$5x + 3y = 3$

12) $x + 4y = -12$
$2x + y = 4$

13) $x - y = 4$
$2x + y = -1$

14) $6x + y = -3$
$x + y = 2$

15) $2x + 3y = -6$
$2x + y = 2$

16) $3x + 2y = 2$
$3x + 2y = -6$

17) $2x + y = 2$
$x - y = 4$

18) $x + 2y = 6$
$5x - 4y = 16$

19) $2x + y = -2$
$x + 3y = 9$

20) $x - y = 3$
$5x + 2y = 8$

21) $0 = -6x - 9y + 36$
$12 = 6x - 3y$

22) $-2y + x = 4$
$2 = -x + \frac{1}{2}y$

23) $2x - y = -1$
$0 = -2x - y - 3$

24) $-2y = -4 - x$
$-2y = -5x + 4$

25) $3 + y = -x$
$-4 - 6x = -y$

26) $16 = -x - 4y$
$-2x = -4 - 4y$

27) $-y + 7x = 4$
$-y - 3 + 7x = 0$

28) $-4 + y = x$
$x + 2 = -y$

29) $-12 + x = 4y$
$12 - 5x = 4y$

30) $-5x + 1 = -y$
$-y + x = -3$

4.2

Systems of Equations - Substitution

Objective: Solve systems of equations using substitution.

When solving a system by graphing has several limitations. First, it requires the graph to be perfectly drawn, if the lines are not straight we may arrive at the wrong answer. Second, graphing is not a great method to use if the answer is really large, over 100 for example, or if the answer is a decimal the that graph will not help us find, 3.2134 for example. For these reasons we will rarely use graphing to solve our systems. Instead, an algebraic approach will be used.

The first algebraic approach is called substitution. We will build the concepts of substitution through several example, then end with a five-step process to solve problems using this method.

Example 170.

$$\begin{array}{rl} \begin{array}{l} x=5 \\ y=2x-3 \end{array} & \text{We already know } x=5\text{, substitute this into the other equation} \\ y=2(\mathbf{5})-3 & \text{Evaluate, multiply first} \\ y=10-3 & \text{Subtract} \\ y=7 & \text{We now also have } y \\ (5,7) & \text{Our Solution} \end{array}$$

When we know what one variable equals we can plug that value (or expression) in for the variable in the other equation. It is very important that when we substitute, the substituted value goes in parenthesis. The reason for this is shown in the next example.

Example 171.

$$\begin{array}{rl} \begin{array}{l} 2x-3y=7 \\ y=3x-7 \end{array} & \text{We know } y=3x-7\text{, substitute this into the other equation} \\ 2x-3(\boldsymbol{3x-7})=7 & \text{Solve this equation, distributing } -3 \text{ first} \end{array}$$

$$\begin{array}{rl}
6x+9y=6 & \text{New second equation}\\
\underline{-6x+5y=22} & \text{First equation still the same, add}\\
14y=28 & \text{Divide both sides by 14}\\
\overline{14}\quad\overline{14} & \\
y=2 & \text{We have our } y!\\
2x+3(2)=2 & \text{Plug into one of the original equations, simplify}\\
2x+6=2 & \text{Subtract 6 from both sides}\\
\underline{-6-6} & \\
2x=-2 & \text{Divide both sides by 2}\\
\overline{2}\quad\overline{2} & \\
x=-2 & \text{We also have our } x!\\
(-2,2) & \text{Our Solution}
\end{array}$$

When we looked at the x terms, $-6x$ and $2x$ we decided to multiply the $2x$ by 3 to get the opposites we were looking for. What we are looking for with our opposites is the least common multiple (LCM) of the coefficients. We also could have solved the above problem by looking at the terms with y, $5y$ and $3y$. The LCM of 3 and 5 is 15. So we would want to multiply both equations, the $5y$ by 3, and the $3y$ by -5 to get opposites, $15y$ and $-15y$. This illustrates an important point, some problems we will have to multiply both equations by a constant (on both sides) to get the opposites we want.

Example 179.

$$\begin{array}{rl}
3x+6y=-9 & \text{We can get opposites in front of } x\text{, find LCM of 6 and 9,}\\
2x+9y=-26 & \text{The LCM is 18. We will multiply to get } 18y \text{ and } -18y
\end{array}$$

$$\begin{array}{rl}
3(3x+6y)=(-9)3 & \text{Multiply the first equation by 3, both sides!}\\
9x+18y=-27 &
\end{array}$$

$$\begin{array}{rl}
-2(2x+9y)=(-26)(-2) & \text{Multiply the second equation by } -2\text{, both sides!}\\
-4x-18y=52 &
\end{array}$$

$$\begin{array}{rl}
9x+18y=-27 & \text{Add two new equations together}\\
\underline{-4x-18y=52} & \\
5x\qquad=25 & \text{Divide both sides by 5}\\
\overline{5}\qquad\quad\overline{5} & \\
x=5 & \text{We have our solution for } x\\
3(5)+6y=-9 & \text{Plug into either original equation, simplify}\\
15+6y=-9 & \text{Subtract 15 from both sides}\\
\underline{-15\qquad -15} &
\end{array}$$

are given to use. This means sometimes this row may have some blanks in it. Once the table is filled in we can easily make equations by adding each column, setting it equal to the total at the bottom of the column. This is shown in the following example.

Example 186.

In a child's bank are 11 coins that have a value of \$1.85. The coins are either quarters or dimes. How many coins each does child have?

	Number	Value	Total
Quarter	q	25	
Dime	d	10	
Total			

Using value table, use q for quarters, d for dimes
Each quarter's value is 25 cents, dime's is 10 cents

	Number	Value	Total
Quarter	q	25	$25q$
Dime	d	10	$10d$
Total			

Multiply number by value to get totals

	Number	Value	Total
Quarter	q	25	$25q$
Dime	d	10	$10d$
Total	11		185

We have 11 coins total. This is the number total.
We have 1.85 for the final total,
Write final total in cents (185)
Because 25 and 10 are cents

$q + d = 11$ — First and last columns are our equations by adding
$25q + 10d = 185$ — Solve by either addition or substitution.

$-10(q + d) = (11)(-10)$ — Using addition, multiply first equation by -10
$-10q - 10d = -110$

$$\begin{array}{r} -10q - 10d = -110 \\ \underline{25q + 10d = 185} \\ 15q \quad\quad = 75 \end{array}$$

Add together equations

Divide both sides by 15

$$\frac{15q}{15} = \frac{75}{15}$$

$q = 5$ — We have our q, number of quarters is 5

$(5) + d = 11$ — Plug into one of original equations
$\underline{-5 \quad\quad -5}$ — Subtract 5 from both sides
$d = 6$ — We have our d, number of dimes is 6

4.6

Systems of Equations - Mixture Problems

Objective: Solve mixture problems by setting up a system of equations.

One application of systems of equations are mixture problems. Mixture problems are ones where two different solutions are mixed together resulting in a new final solution. We will use the following table to help us solve mixture problems:

	Amount	Part	Total
Item 1			
Item 2			
Final			

The first column is for the amount of each item we have. The second column is labeled "part". If we mix percentages we will put the rate (written as a decimal) in this column. If we mix prices we will put prices in this column. Then we can multiply the amount by the part to find the total. Then we can get an equation by adding the amount and/or total columns that will help us solve the problem and answer the questions.

These problems can have either one or two variables. We will start with one variable problems.

Example 191.

A chemist has 70 mL of a 50% methane solution. How much of a 80% solution must she add so the final solution is 60% methane?

	Amount	Part	Total
Start	70	0.5	
Add	x	0.8	
Final			

Set up the mixture table. We start with 70, but don′t know how much we add, that is x. The part is the percentages, 0.5 for start, 0.8 for add.

$$
\begin{array}{rl}
4(30-n)+2.5n=105 & \text{Substitute into untouched equation} \\
120-4n+2.5n=105 & \text{Distribute} \\
120-1.5n=105 & \text{Combine like terms} \\
\underline{-120 \qquad\qquad -120} & \text{Subtract 120 from both sides} \\
-1.5n=-15 & \text{Divide both sides by } -1.5 \\
\overline{-1.5} \quad \overline{-1.5} & \\
n=10 & \text{We have our } n\text{, 10 lbs of nuts} \\
c=30-(10) & \text{Plug into } c= \text{ equation to find } c \\
c=20 & \text{We have our } c\text{, 20 lbs of chocolate} \\
\text{10 lbs of nuts and 20 lbs of chocolate} & \text{Our Solution}
\end{array}
$$

With mixture problems we often are mixing with a pure solution or using water which contains none of our chemical we are interested in. For pure solutions, the percentage is 100% (or 1 in the table). For water, the percentage is 0%. This is shown in the following example.

Example 195.

A solution of pure antifreeze is mixed with water to make a 65% antifreeze solution. How much of each should be used to make 70 L?

	Amount	Part	Final
Antifreeze	a	1	
Water	w	0	
Final	70	0.65	

We use a and w for our variables. Antifreeze is pure, 100% or 1 in our table, written as a decimal. Water has no antifreeze, its percentage is 0. We also fill in the final percent

	Amount	Part	Final
Antifreeze	a	1	a
Water	w	0	0
Final	70	0.65	45.5

Multiply to find final amounts

$$
\begin{array}{rl}
a+w=70 & \text{First equation comes from first column} \\
a=45.5 & \text{Second equation comes from second column} \\
(45.5)+w=70 & \text{We have } a\text{, plug into to other equation} \\
-45.5 \qquad -45.5 & \text{Subtract 45.5 from both sides} \\
w=24.5 & \text{We have our } w \\
45.5\,L \text{ of antifreeze and } 24.5\,L \text{ of water} & \text{Our Solution}
\end{array}
$$

16) A certain grade of milk contains 10% butter fat and a certain grade of cream 60% butter fat. How many quarts of each must be taken so as to obtain a mixture of 100 quarts that will be 45% butter fat?

17) A farmer has some cream which is 21% butterfat and some which is 15% butter fat. How many gallons of each must be mixed to produce 60 gallons of cream which is 19% butterfat?

18) A syrup manufacturer has some pure maple syrup and some which is 85% maple syrup. How many liters of each should be mixed to make 150L which is 96% maple syrup?

19) A chemist wants to make 50ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?

20) A hair dye is made by blending 7% hydrogen peroxide solution and a 4% hydrogen peroxide solution. How many mililiters of each are used to make a 300 ml solution that is 5% hydrogen peroxide?

21) A paint that contains 21% green dye is mixed with a paint that contains 15% green dye. How many gallons of each must be used to make 60 gal of paint that is 19% green dye?

22) A candy mix sells for $2.20 per kilogram. It contains chocolates worth $1.80 per kilogram and other candy worth $3.00 per kilogram. How much of each are in 15 kilograms of the mixture?

23) To make a weed and feed mixture, the Green Thumb Garden Shop mixes fertilizer worth $4.00/lb. with a weed killer worth $8.00/lb. The mixture will cost $6.[illegible]0/lb. How much of each should be used to prepare 500 lb. of the mixture?

24) A grocer is mixing 40 cent per lb. coffee with 60 cent per lb. coffee to make a mixture worth 54¢ per lb. How much of each kind of coffee should be used to make 70 lb. of the mixture?

25) A grocer wishes to mix sugar at 9 cents per pound with sugar at 6 cents per pound to make 60 pounds at 7 cents per pound. What quantity of each must he take?

26) A high-protein diet supplement that costs $6.75 per pound is mixed with a vitamin supplement that costs $3.25 per pound. How many pounds of each should be used to make 5 lb of a mixture that costs $4.65 per pound?

27) A goldsmith combined an alloy that costs $4.30 per ounce with an alloy that costs $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing $2.50 per ounce?

28) A grocery store offers a cheese and fruit sampler that combines cheddar cheese that costs $8 per kilogram with kiwis that cost $3 per kilogram. How many kilograms of each were used to make a 5 kg mixture that costs $4.50 per kilogram?

Example 216.

$\frac{a^3}{a^5}$ Using the quotient rule, subtract exponents

a^{-2} Our Solution, but we will also solve this problem another way.

$\frac{a^3}{a^5}$ Rewrite exponents as repeated multiplication

$\frac{aaa}{aaaaa}$ Reduce three $a's$ out of top and bottom

$\frac{1}{aa}$ Simplify to exponents

$\frac{1}{a^2}$ Our Solution, putting these solutions together gives:

$a^{-2} = \frac{1}{a^2}$ Our Final Solution

This example illustrates an important property of exponents. Negative exponents yield the reciprocal of the base. Once we take the reciprical the exponent is now positive. Also, it is important to note a negative exponent does not mean the expression is negative, only that we need the reciprocal of the base. Following are the rules of negative exponents

Rules of Negative Exponets:
$$\boldsymbol{a^{-m} = \frac{1}{m}}$$
$$\boldsymbol{\frac{1}{a^{-m}} = a^m}$$
$$\boldsymbol{\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}}$$

Negative exponents can be combined in several different ways. As a general rule if we think of our expression as a fraction, negative exponents in the numerator must be moved to the denominator, likewise, negative exponents in the denominator need to be moved to the numerator. When the base with exponent moves, the exponent is now positive. This is illustrated in the following example.

Example 217.

$\frac{a^3b^{-2}c}{2d^{-1}e^{-4}f^2}$ Negative exponents on b, d, and e need to flip

$\frac{a^3cde^4}{2b^2f^2}$ Our Solution

Be very careful when we are squaring a binomial to **NOT** distribute the square through the parenthesis. A common error is to do the following: $(x-5)^2 = x^2 - 25$ (or $x^2 + 25$). Notice both of these are missing the middle term, $-10x$. This is why it is important to use the shortcut to help us find the correct solution. Another important observation is that the middle term in the solution always has the same sign as the middle term in the problem. This is illustrated in the next examples.

Example 252.

$$\begin{aligned}
(2x+5)^2 &\quad \text{Recognize perfect square} \\
(2x)^2 = 4x^2 &\quad \text{Square the first} \\
2(2x)(5) = 20x &\quad \text{Twice the product} \\
5^2 = 25 &\quad \text{Square the last} \\
4x^2 + 20x + 25 &\quad \text{Our Solution}
\end{aligned}$$

Example 253.

$$\begin{aligned}
(3x-7y)^2 &\quad \text{Recognize perfect square} \\
9x^2 - 42\text{xy} + 49y^2 &\quad \text{Square the first, twice the product, square the last. Our Solution}
\end{aligned}$$

Example 254.

$$\begin{aligned}
(5a+9b)^2 &\quad \text{Recognize perfect square} \\
25a^2 + 90\text{ab} + 81b^2 &\quad \text{Square the first, twice the product, square the last. Our Solution}
\end{aligned}$$

These two formulas will be important to commit to memory. The more familiar we are with them, the easier factoring, or multiplying in reverse, will be. The final example covers both types of problems (two perfect squares, one positive, one negative), be sure to notice the difference between the examples and how each formula is used

Example 255.

$(4x-7)(4x+7)$	$(4x+7)^2$	$(4x-7)^2$
$16x^2 - 49$	$16x^2 + 56x + 49$	$16x^2 - 56x + 49$

World View Note: There are also formulas for higher powers of binomials as well, such as $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. While French mathematician Blaise Pascal often gets credit for working with these expansions of binomials in the 17th century, Chinese mathematicians had been working with them almost 400 years earlier!

An example is given to review the (general) steps that are used with whole numbers that we will also use with polynomials

Example 258.

$4\mid\overline{631}$	Divide front numbers: $\frac{6}{4}=1$
$\mathbf{1}$	
$4\mid\overline{631}$	Multiply this number by divisor: $1\cdot 4=4$
$\mathbf{-4}$	Change the sign of this number (make it subtract) and combine
$\mathbf{23}$	Bring down next number
$1\mathbf{5}$	Repeat, divide front numbers: $\frac{23}{4}=5$
$4\mid\overline{631}$	
-4	
23	Multiply this number by divisor: $5\cdot 4=20$
$\mathbf{-20}$	Change the sign of this number (make it subtract) and combine
$\mathbf{31}$	Bring down next number
$15\mathbf{7}$	Repeat, divide front numbers: $\frac{31}{4}=7$
$4\mid\overline{631}$	
-4	
23	
-20	
31	Multiply this number by divisor: $7\cdot 4=28$
$\mathbf{-28}$	Change the sign of this number (make it subtract) and combine
$\mathbf{3}$	We will write our remainder as *a* fraction, over the divisor, added to the end
$157\frac{3}{4}$	Our Solution

This same process will be used to multiply polynomials. The only difference is we will replace the word "number" with the word "term"

Dividing Polynomials

1. Divide front terms

2. Multiply this term by the divisor

3. Change the sign of the terms and combine

4. Bring down the next term

5. Repeat

Step number 3 tends to be the one that students skip, not changing the signs of the terms would be equivalent to adding instead of subtracting on long division with whole numbers. Be sure not to miss this step! This process is illustrated in the following two examples.

Example 259.

$$\frac{3x^3-5x^2-32x+7}{x-4}$$ Rewrite problem as long division

$$x-4\,\overline{)\,3x^3-5x^2-32x+7}$$ Divide front terms: $\dfrac{3x^3}{x}=3x^2$

$$\begin{array}{rl} & \mathbf{3x^2} \\ x-4 \,| & \overline{3x^3-5x^2-32x+7} \\ & \underline{-\mathbf{3x^3}+\mathbf{12x^2}} \\ & \mathbf{7x^2-32x} \end{array}$$

Multiply this term by divisor: $3x^2(x-4)=3x^3-12x^2$
Change the signs and combine
Bring down the next term

$$\begin{array}{rl} & 3x^2+\mathbf{7x} \\ x-4 \,| & \overline{3x^3-5x^2-32x+7} \\ & \underline{-3x^3+12x^2} \\ & 7x^2-32x \\ & \underline{-\mathbf{7x^2}+\mathbf{28x}} \\ & -\mathbf{4x}+\mathbf{7} \end{array}$$

Repeat, divide front terms: $\dfrac{7x^2}{x}=7x$
Multiply this term by divisor: $7x(x-4)=7x^2-28x$
Change the signs and combine
Bring down the next term

$$\begin{array}{rl} & 3x^2+7x-\mathbf{4} \\ x-4 \,| & \overline{3x^3-5x^2-32x+7} \\ & \underline{-3x^3+12x^2} \\ & 7x^2-32x \\ & \underline{-7x^2+28x} \\ & -4x+7 \\ & \underline{+\mathbf{4x}-\mathbf{16}} \\ & -\mathbf{9} \end{array}$$

Repeat, divide front terms: $\dfrac{-4x}{x}=-4$
Multiply this term by divisor: $-4(x-4)=-4x+16$
Change the signs and combine
Remainder put over divisor and subtracted (due to negative)

bers using mental math, then we take any variables that are in common with each term, using the lowest exponent. This is shown in the next example

Example 263.

$$\begin{aligned}
&\text{GCF of } 24x^4y^2z, 18x^2y^4, \text{and } 12x^3yz^5 \\
\frac{24}{6}=4,\ \frac{18}{6}=3,\ \frac{12}{6}&=2 && \text{Each number can be divided by } 6 \\
x^2y& && x \text{ and } y \text{ are in all 3, using lowest exponets} \\
\text{GCF}=6x^2y& && \text{Our Solution}
\end{aligned}$$

To factor out a GCF from a polynomial we first need to identify the GCF of all the terms, this is the part that goes in front of the parenthesis, then we divide each term by the GCF, the answer is what is left inside the parenthesis. This is shown in the following examples

Example 264.

$$\begin{aligned}
4x^2-20x+16& && \text{GCF is 4, divide each term by 4} \\
\frac{4x^2}{4}=x^2,\ \frac{-20x}{4}=-5x,\ \frac{16}{4}=4& && \text{This is what is left inside the parenthesis} \\
4(x^2-5x+4)& && \text{Our Solution}
\end{aligned}$$

With factoring we can always check our solutions by multiplying (distributing in this case) out the answer and the solution should be the original equation.

Example 265.

$$\begin{aligned}
25x^4-15x^3+20x^2& && \text{GCF is } 5x^2\text{, divide each term by this} \\
\frac{25x^4}{5x^2}=5x^2,\ \frac{-15x^3}{5x^2}=-3x,\ \frac{20x^2}{5x^2}=4& && \text{This is what is left inside the parenthesis} \\
5x^2(5x^2-3x+4)& && \text{Our Solution}
\end{aligned}$$

Example 266.

$$3x^3y^2z+5x^4y^3z^5-4xy^4 \qquad \text{GCF is } xy^2\text{, divide each term by this}$$

6.2

Factoring - Grouping

Objective: Factor polynomials with four terms using grouping.

The first thing we will always do when factoring is try to factor out a GCF. This GCF is often a monomial like in the problem $5xy + 10xz$ the GCF is the monomial $5x$, so we would have $5x(y + 2z)$. However, a GCF does not have to be a monomial, it could be a binomial. To see this, consider the following two example.

Example 270.

$$\begin{array}{rl} 3ax - 7bx & \text{Both have } x \text{ in common, factor it out} \\ x(3a - 7b) & \text{Our Solution} \end{array}$$

Now the same problem, but instead of x we have $(2a + 5b)$.

Example 271.

$$\begin{array}{rl} 3a(2a+5b) - 7b(2a+5b) & \text{Both have } (2a+5b) \text{ in common, factor it out} \\ (2a+5b)(3a-7b) & \text{Our Solution} \end{array}$$

In the same way we factored out a GCF of x we can factor out a GCF which is a binomial, $(2a + 5b)$. This process can be extended to factor problems where there is no GCF to factor out, or after the GCF is factored out, there is more factoring that can be done. Here we will have to use another strategy to factor. We will use a process known as grouping. Grouping is how we will factor if there are four terms in the problem. Remember, factoring is like multiplying in reverse, so first we will look at a multiplication problem and then try to reverse the process.

Example 272.

$$\begin{array}{rl} (2a+3)(5b+2) & \text{Distribute } (2a+3) \text{ into second parenthesis} \\ 5b(2a+3) + 2(2a+3) & \text{Distribute each monomial} \\ 10\text{ab} + 15b + 4a + 6 & \text{Our Solution} \end{array}$$

The solution has four terms in it. We arrived at the solution by looking at the two parts, $5b(2a + 3)$ and $2(2a + 3)$. When we are factoring by grouping we will always divide the problem into two parts, the first two terms and the last two terms. Then we can factor the GCF out of both the left and right sides. When we do this our hope is what is left in the parenthesis will match on both the left and right. If they match we can pull this matching GCF out front, putting the rest in parenthesis and we will be factored. The next example is the same problem worked backwards, factoring instead of multiplying.

example the binomials are $(a+b)$ and $(b+a)$, we don't have to do any extra work. This is because addition is the same in either order $(5+3=3+5=8)$.

Example 279.

$7+y-3xy-21x$	Split the problem into two groups
$\boxed{7+y}\boxed{-3xy-21x}$	GCF on left is 1, on the right is $-3x$
$\boxed{1(7+y)}\boxed{-3x(y+7)}$	$y+7$ and $7+y$ are the same, use either one
$(y+7)(1-3x)$	Our Solution

However, if the binomial has subtraction, then we need to be a bit more careful. For example, if the binomials are $(a-b)$ and $(b-a)$, we will factor out the opposite of the GCF on one part, usually the second. Notice what happens when we factor out -1.

Example 280.

$(b-a)$	Factor out -1
$-1(-b+a)$	Addition can be in either order, switch order
$-1(a-b)$	The order of the subtraction has been switched!

Generally we won't show all the above steps, we will simply factor out the opposite of the GCF and switch the order of the subtraction to make it match the other binomial.

Example 281.

$8xy-12y+15-10x$	Split the problem into two groups
$\boxed{8xy-12y}\boxed{15-10x}$	GCF on left is $4y$, on right, 5
$\boxed{4y(2x-3)}\boxed{+5(3-2x)}$	Need to switch subtraction order, use -5 in middle
$\boxed{4y(2y-3)}\boxed{-5(2x-3)}$	Now $2x-3$ match on both! Factor out this GCF
$(2x-3)(4y-5)$	Our Solution

World View Note: Sofia Kovalevskaya of Russia was the first woman on the editorial staff of a mathematical journal in the late 19th century. She also did research on how the rings of Saturn rotated.

As the past few examples illustrate, it is very important to be aware of negatives as we find the pair of numbers we will use to split the middle term. Consier the following example, done incorrectly, ignoring negative signs

Warning 288.

$$\begin{aligned}
x^2+5x-6 &\quad \text{Want to multiply to } 6, \text{ add } 5\\
x^2+2x+3x-6 &\quad 2 \text{ and } 3, \text{ split the middle term}\\
x(x+2)+3(x-2) &\quad \text{Factor by grouping}\\
??? &\quad \text{Binomials do not match!}
\end{aligned}$$

Because we did not use the negative sign with the six to find our pair of numbers, the binomials did not match and grouping was not able to work at the end. Now the problem will be done correctly.

Example 289.

$$\begin{aligned}
x^2+5x-6 &\quad \text{Want to multiply to } -6, \text{ add to } 5\\
x^2+6x-x-6 &\quad 6 \text{ and } -1, \text{ split the middle term}\\
x(x+6)-1(x+6) &\quad \text{Factor by grouping}\\
(x+6)(x-1) &\quad \text{Our Solution}
\end{aligned}$$

You may have noticed a shortcut for factoring these problems. Once we identify the two numbers that are used to split the middle term, these are the two numbers in our factors! In the previous example, the numbers used to split the middle term were 6 and -1, our factors turned out to be $(x+6)(x-1)$. This pattern does not always work, so be careful getting in the habit of using it. We can use it however, when we have no number (technically we have a 1) in front of x^2. In all the problems we have factored in this lesson there is no number in front of x^2. If this is the case then we can use this shortcut. This is shown in the next few examples.

Example 290.

$$\begin{aligned}
x^2-7x-18 &\quad \text{Want to multiply to } -18, \text{ add to } -7\\
&\quad -9 \text{ and } 2, \text{ write the factors}\\
(x-9)(x+2) &\quad \text{Our Solution}
\end{aligned}$$

$$
\begin{array}{rl}
4(x+7y)^2 & \text{Our Solution}
\end{array}
$$

Example 313.

$$
\begin{array}{rl}
5x^2y+15xy-35x^2-105x & \text{GCF first, } 5x \\
5x(xy+3y-7x-21) & \text{Four terms, try grouping} \\
5x[y(x+3)-7(x+3)] & (x+3) \text{ match!} \\
5x(x+3)(y-7) & \text{Our Solution}
\end{array}
$$

Example 314.

$$
\begin{array}{rl}
100x^2-400 & \text{GCF first, } 100 \\
100(x^2-4) & \text{Two terms, difference of squares} \\
100(x+4)(x-4) & \text{Our Solution}
\end{array}
$$

Example 315.

$$
\begin{array}{rl}
108x^3y^2-39x^2y^2+3xy^2 & \text{GCF first, } 3xy^2 \\
3xy^2(36x^2-13x+1) & \text{Three terms, ac method, multiply to 36, add to } -13 \\
3xy^2(36x^2-9x-4x+1) & -9 \text{ and } -4\text{, split middle term} \\
3xy^2[9x(4x-1)-1(4x-1)] & \text{Factor by grouping} \\
3xy^2(4x-1)(9x-1) & \text{Our Solution}
\end{array}
$$

World View Note: Variables originated in ancient Greece where Aristotle would use a single capital letter to represent a number.

Example 316.

$$
\begin{array}{rl}
5+625y^3 & \text{GCF first, } 5 \\
5(1+125y^3) & \text{Two terms, sum of cubes} \\
5(1+5y)(1-5y+25y^2) & \text{Our Solution}
\end{array}
$$

It is important to be comfortable and confident not just with using all the factoring methods, but decided on which method to use. This is why practice is very important!

Factoring - Solve by Factoring

Objective: Solve quadratic equation by factoring and using the zero product rule.

When solving linear equations such as $2x-5=21$ we can solve for the variable directly by adding 5 and dividing by 2 to get 13. However, when we have x^2 (or a higher power of x) we cannot just isolate the variable as we did with the linear equations. One method that we can use to solve for the varaible is known as the zero product rule

Zero Product Rule: If $ab=0$ then either $a=0$ or $b=0$

The zero product rule tells us that if two factors are multiplied together and the answer is zero, then one of the factors must be zero. We can use this to help us solve factored polynomials as in the following example.

Example 317.

$$\begin{array}{rl}
(2x-3)(5x+1)=0 & \text{One factor must be zero}\\
2x-3=0 \quad \text{or} \quad 5x+1=0 & \text{Set each factor equal to zero}\\
\underline{+3\;+3} \qquad\quad \underline{-1\;-1} & \text{Solve each equation}\\
2x=3 \quad \text{or} \quad 5x=-1 & \\
\overline{2}\;\;\overline{2} \qquad\quad \overline{5}\;\;\;\overline{5} & \\
x=\dfrac{3}{2} \quad \text{or} \quad \dfrac{-1}{5} & \text{Our Solution}
\end{array}$$

For the zero product rule to work we must have factors to set equal to zero. This means if the problem is not already factored we will factor it first.

Example 318.

$$\begin{array}{rl}
4x^2+x-3=0 & \text{Factor using the ac method, multiply to } -12\text{, add to } 1\\
4x^2-3x+4x-3=0 & \text{The numbers are } -3 \text{ and } 4\text{, split the middle term}\\
x(4x-3)+1(4x-3)=0 & \text{Factor by grouping}\\
(4x-3)(x+1)=0 & \text{One factor must be zero}\\
4x-3=0 \quad \text{or} \quad x+1=0 & \text{Set each factor equal to zero}
\end{array}$$

Chapter 7 : Rational Expressions

As the above example illustrates, we will multiply out our numerators, but keep our denominators factored. The reason for this is to add and subtract fractions we will want to be able to combine like terms in the numerator, then when we reduce at the end we will want our denominators factored.

Once we know how to find the LCD and how to build up fractions to a desired denominator we can combine them together by finding a common denominator and building up those fractions.

Example 344.

Build up each fraction so they have a common denominator

$$\frac{5a}{4b^3c} \text{ and } \frac{3c}{6a^2b} \quad \text{First identify LCD}$$

$$12a^2b^3c \quad \text{Determine what factors each fraction is missing}$$

$$\text{First: } 3a^2 \quad \text{Second: } 2b^2c \quad \text{Multiply each fraction by missing factors}$$

$$\frac{5a}{4b^3c}\left(\frac{3a^2}{3a^2}\right) \text{ and } \frac{3c}{6a^2b}\left(\frac{2b^2c}{2b^2c}\right)$$

$$\frac{15a^3}{12a^2b^3c} \text{ and } \frac{6b^2c^2}{12a^2b^3c} \quad \text{Our Solution}$$

Example 345.

Build up each fraction so they have a common denominator

$$\frac{5x}{x^2-5x-6} \text{ and } \frac{x-2}{x^2+4x+3} \quad \text{Factor to find LCD}$$

$$(x-6)(x+1) \quad (x+1)(x+3) \quad \text{Use factors to find LCD}$$

$$\text{LCD: } (x-6)(x+1)(x+3) \quad \text{Identify which factors are missing}$$

$$\text{First: } (x+3) \quad \text{Second: } (x-6) \quad \text{Multiply fractions by missing factors}$$

$$\frac{5x}{(x-6)(x+1)}\left(\frac{x+3}{x+3}\right) \text{ and } \frac{x-2}{(x+1)(x+3)}\left(\frac{x-6}{x-6}\right) \quad \text{Multiply numerators}$$

$$\frac{5x^2+15x}{(x-6)(x+1)(x+3)} \text{ and } \frac{x^2-8x+12}{(x-6)(x+1)(x+3)} \quad \text{Our Solution}$$

World View Note: When the Egyptians began working with fractions, they expressed all fractions as a sum of unit fraction. Rather than $\frac{4}{5}$, they would write the fraction as the sum, $\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$. An interesting problem with this system is this is not a unique solution, $\frac{4}{5}$ is also equal to the sum $\frac{1}{3}+\frac{1}{5}+\frac{1}{6}+\frac{1}{10}$.

7.5

Rational Expressions - Complex Fractions

Objective: Simplify complex fractions by multiplying each term by the least common denominator.

Complex fractions have fractions in either the numerator, or denominator, or usually both. These fractions can be simplified in one of two ways. This will be illustrated first with integers, then we will consider how the process can be expanded to include expressions with variables.

The first method uses order of operations to simplify the numerator and denominator first, then divide the two resulting fractions by multiplying by the reciprocal.

Example 353.

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{5}{6}+\frac{1}{2}}\quad \text{Get common denominator in top and bottom fractions}$$

$$\frac{\frac{8}{12}-\frac{3}{12}}{\frac{5}{6}+\frac{3}{6}}\quad \text{Add and subtract fractions, reducing solutions}$$

$$\frac{\frac{5}{12}}{\frac{4}{3}}\quad \text{To divide fractions we multiply by the reciprocal}$$

$$\left(\frac{5}{12}\right)\left(\frac{3}{4}\right)\quad \text{Reduce}$$

$$\left(\frac{5}{4}\right)\left(\frac{1}{4}\right)\quad \text{Multiply}$$

$$\frac{5}{16}\quad \text{Our Solution}$$

The process above works just fine to simplify, but between getting common denominators, taking reciprocals, and reducing, it can be a very involved process. Generally we prefer a different method, to multiply the numerator and denominator of the large fraction (in effect each term in the complex fraction) by the least common denominator (LCD). This will allow us to reduce and clear the small fractions. We will simplify the same problem using this second method.

Example 354.

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{5}{6}+\frac{1}{2}}\quad \text{LCD is 12, multiply each term}$$

7.8

Rational Expressions - Dimensional Analysis

Objective: Use dimensional analysis to preform single unit, dual unit, square unit, and cubed unit conversions.

One application of rational expressions deals with converting units. When we convert units of measure we can do so by multiplying several fractions together in a process known as dimensional analysis. The trick will be to decide what fractions to multiply. When multiplying, if we multiply by 1, the value of the expression does not change. One written as a fraction can look like many different things as long as the numerator and denominator are identical in value. Notice the numerator and denominator are not identical in appearance, but rather identical in value. Below are several fractions, each equal to one where numerator and denominator are identical in value.

$$\frac{1}{1}=\frac{4}{4}=\frac{\frac{1}{2}}{\frac{2}{4}}=\frac{100\,\mathrm{cm}}{1\,m}=\frac{1\,\mathrm{lb}}{16\,\mathrm{oz}}=\frac{1\,\mathrm{hr}}{60\,\mathrm{min}}=\frac{60\,\mathrm{min}}{1\,\mathrm{hr}}$$

The last few fractions that include units are called conversion factors. We can make a conversion factor out of any two measurements that represent the same distance. For example, 1 mile = 5280 feet. We could then make a conversion factor $\frac{1\,\mathrm{mi}}{5280\,\mathrm{ft}}$ because both values are the same, the fraction is still equal to one. Similarly we could make a conversion factor $\frac{5280\,\mathrm{ft}}{1\,\mathrm{mi}}$. The trick for conversions will

Use dimensional analysis to solve the following:

19) On a recent trip, Jan traveled 260 miles using 8 gallons of gas. How many miles per 1-gallon did she travel? How many yards per 1-ounce?

20) A chair lift at the Divide ski resort in Cold Springs, WY is 4806 feet long and takes 9 minutes. What is the average speed in miles per hour? How many feet per second does the lift travel?

21) A certain laser printer can print 12 pages per minute. Determine this printer's output in pages per day, and reams per month. (1 ream = 5000 pages)

22) An average human heart beats 60 times per minute. If an average person lives to the age of 75, how many times does the average heart beat in a lifetime?

23) Blood sugar levels are measured in miligrams of gluclose per deciliter of blood volume. If a person's blood sugar level measured 128 mg/dL, how much is this in grams per liter?

24) You are buying carpet to cover a room that measures 38 ft by 40 ft. The carpet cost $18 per square yard. How much will the carpet cost?

25) A car travels 14 miles in 15 minutes. How fast is it going in miles per hour? in meters per second?

26) A cargo container is 50 ft long, 10 ft wide, and 8 ft tall. Find its volume in cubic yards and cubic meters.

27) A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than $\frac{1}{4}$ of the lot it is built on. Suppose you own a $\frac{1}{3}$ acre lot, what is the maximum allowed footprint for your house in square feet? in square inches? (1 acre = 43560 ft^2)

28) Computer memory is measured in units of bytes, where one byte is enough memory to store one character (a letter in the alphabet or a number). How many typical pages of text can be stored on a 700-megabyte compact disc? Assume that one typical page of text contains 2000 characters. (1 megabyte = 1,000,000 bytes)

29) In April 1996, the Department of the Interior released a "spike flood" from the Glen Canyon Dam on the Colorado River. Its purpose was to restore the river and the habitants along its bank. The release from the dam lasted a week at a rate of 25,800 cubic feet of water per second. About how much water was released during the 1-week flood?

30) The largest single rough diamond ever found, the Cullinan diamond, weighed 3106 carats; how much does the diamond weigh in miligrams? in pounds? (1 carat - 0.2 grams)

Chapter 8 : Radicals

8.3 Practice - Adding Radicals

Simiplify

1) $2\sqrt{5}+2\sqrt{5}+2\sqrt{5}$

2) $-3\sqrt{6}-3\sqrt{3}-2\sqrt{3}$

3) $-3\sqrt{2}+3\sqrt{5}+3\sqrt{5}$

4) $-2\sqrt{6}-\sqrt{3}-3\sqrt{6}$

5) $-2\sqrt{6}-2\sqrt{6}-\sqrt{6}$

6) $-3\sqrt{3}+2\sqrt{3}-2\sqrt{3}$

7) $3\sqrt{6}+3\sqrt{5}+2\sqrt{5}$

8) $-\sqrt{5}+2\sqrt{3}-2\sqrt{3}$

9) $2\sqrt{2}-3\sqrt{18}-\sqrt{2}$

10) $-\sqrt{54}-3\sqrt{6}+3\sqrt{27}$

11) $-3\sqrt{6}-\sqrt{12}+3\sqrt{3}$

12) $-\sqrt{5}-\sqrt{5}-2\sqrt{54}$

13) $3\sqrt{2}+2\sqrt{8}-3\sqrt{18}$

14) $2\sqrt{20}+2\sqrt{20}-\sqrt{3}$

15) $3\sqrt{18}-\sqrt{2}-3\sqrt{2}$

16) $-3\sqrt{27}+2\sqrt{3}-\sqrt{12}$

17) $-3\sqrt{6}-3\sqrt{6}-\sqrt{3}+3\sqrt{6}$

18) $-2\sqrt{2}-\sqrt{2}+3\sqrt{8}+3\sqrt{6}$

19) $-2\sqrt{18}-3\sqrt{8}-\sqrt{20}+2\sqrt{20}$

20) $-3\sqrt{18}-\sqrt{8}+2\sqrt{8}+2\sqrt{8}$

21) $-2\sqrt{24}-2\sqrt{6}+2\sqrt{6}+2\sqrt{20}$

22) $-3\sqrt{8}-\sqrt{5}-3\sqrt{6}+2\sqrt{18}$

23) $3\sqrt{24}-3\sqrt{27}+2\sqrt{6}+2\sqrt{8}$

24) $2\sqrt{6}-\sqrt{54}-3\sqrt{27}-\sqrt{3}$

25) $-2\sqrt[3]{16}+2\sqrt[3]{16}+2\sqrt[3]{2}$

26) $3\sqrt[3]{135}-\sqrt[3]{81}-\sqrt[3]{135}$

27) $2\sqrt[4]{243}-2\sqrt[4]{243}-\sqrt[4]{3}$

28) $-3\sqrt[4]{4}+3\sqrt[4]{324}+2\sqrt[4]{64}$

29) $3\sqrt[4]{2}-2\sqrt[4]{2}-\sqrt[4]{243}$

30) $2\sqrt[4]{6}+2\sqrt[4]{4}+3\sqrt[4]{6}$

31) $-\sqrt[4]{324}+3\sqrt[4]{324}-3\sqrt[4]{4}$

32) $-2\sqrt[4]{243}-\sqrt[4]{96}+2\sqrt[4]{96}$

33) $2\sqrt[4]{2}+2\sqrt[4]{3}+3\sqrt[4]{64}-\sqrt[4]{3}$

34) $2\sqrt[4]{48}-3\sqrt[4]{405}-3\sqrt[4]{48}-\sqrt[4]{162}$

35) $-3\sqrt[5]{6}-\sqrt[5]{64}+2\sqrt[5]{192}-2\sqrt[5]{64}$

36) $-3\sqrt[7]{3}-3\sqrt[7]{768}+2\sqrt[7]{384}+3\sqrt[7]{5}$

37) $2\sqrt[5]{160}-2\sqrt[5]{192}-\sqrt[5]{160}-\sqrt[5]{-160}$

38) $-2\sqrt[7]{256}-2\sqrt[7]{256}-3\sqrt[7]{2}-\sqrt[7]{640}$

39) $-\sqrt[6]{256}-2\sqrt[6]{4}-3\sqrt[6]{320}-2\sqrt[6]{128}$

We do this by multiplying the numerator and denominator by the same thing. The problems we will consider here will all have a monomial in the denominator. The way we clear a monomial radical in the denominator is to focus on the index. The index tells us how many of each factor we will need to clear the radical. For example, if the index is 4, we will need 4 of each factor to clear the radical. This is shown in the following examples.

Example 398.

$$\frac{\sqrt{6}}{\sqrt{5}} \qquad \text{Index is 2, we need two fives in denominator, need 1 more}$$

$$\frac{\sqrt{6}}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) \qquad \text{Multiply numerator and denominator by } \sqrt{5}$$

$$\frac{\sqrt{30}}{5} \qquad \text{Our Solution}$$

Example 399.

$$\frac{3\sqrt[4]{11}}{\sqrt[4]{2}} \qquad \text{Index is 4, we need four twos in denominator, need 3 more}$$

$$\frac{3\sqrt[4]{11}}{\sqrt[4]{2}}\left(\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}}\right) \qquad \text{Multiply numerator and denominator by } \sqrt[4]{2^3}$$

$$\frac{3\sqrt[4]{88}}{2} \qquad \text{Our Solution}$$

Example 400.

$$\frac{4\sqrt[3]{2}}{7\sqrt[3]{25}} \qquad \text{The 25 can be written as } 5^2\text{. This will help us keep the numbers small}$$

$$\frac{4\sqrt[3]{2}}{7\sqrt[3]{5^2}} \qquad \text{Index is 3, we need three fives in denominator, need 1 more}$$

$$\frac{4\sqrt[3]{2}}{7\sqrt[3]{5^2}}\left(\frac{\sqrt[3]{5}}{\sqrt[3]{5}}\right) \qquad \text{Multiply numerator and denominator by } \sqrt[3]{5}$$

$$\frac{4\sqrt[3]{10}}{7\cdot 5} \qquad \text{Multiply out denominator}$$

$$\frac{4\sqrt[3]{10}}{35} \qquad \text{Our Solution}$$

37) $\frac{5\sqrt{2}+\sqrt{3}}{5+5\sqrt{2}}$

38) $\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$

8.8

Radicals - Complex Numbers

Objective: Add, subtract, multiply, rationalize, and simplify expressions using complex numbers.

World View Note: When mathematics was first used, the primary purpose was for counting. Thus they did not originally use negatives, zero, fractions or irrational numbers. However, the ancient Egyptians quickly developed the need for "a part" and so they made up a new type of number, the ratio or fraction. The Ancient Greeks did not believe in irrational numbers (people were killed for believing otherwise). The Mayans of Central America later made up the number zero when they found use for it as a placeholder. Ancient Chinese Mathematicians made up negative numbers when they found use for them.

In mathematics, when the current number system does not provide the tools to solve the problems the culture is working with, we tend to make up new ways for dealing with the problem that can solve the problem. Throughout history this has been the case with the need for a number that is nothing (0), smaller than zero (negatives), between integers (fractions), and between fractions (irrational numbers). This is also the case for the square roots of negative numbers. To work with the square root of negative numbers mathematicians have defined what are called imaginary and complex numbers.

Definition of Imaginary Numbers: $i^2 = -1$ (thus $i = \sqrt{-1}$)

Examples of imaginary numbers include $3i$, $-6i$, $\frac{3}{5}i$ and $3i\sqrt{5}$. A **complex number** is one that contains both a real and imaginary part, such as $2+5i$.

With this definition, the square root of a negative number is no longer undefined. We now are allowed to do basic operations with the square root of negatives. First we will consider exponents on imaginary numbers. We will do this by manipulating our definition of $i^2 = -1$. If we multiply both sides of the definition by i, the equation becomes $i^3 = -i$. Then if we multiply both sides of the equation again by i, the equation becomes $i^4 = -i^2 = -(-1) = 1$, or simply $i^4 = 1$. Multiplying again by i gives $i^5 = i$. One more time gives $i^6 = i^2 = -1$. And if this pattern continues we see a cycle forming, the exponents on i change we cycle through simplified answers of i, -1, $-i$, 1. As there are 4 different possible answers in this cycle, if we divide the exponent by 4 and consider the remainder, we can simplify any exponent on i by learning just the following four values:

Cyclic Property of Powers of i

$$\begin{aligned} i^0 &= 1 \\ i &= i \\ i^2 &= -1 \\ i^3 &= -i \end{aligned}$$

Example 431.

$$\begin{aligned} (3i)(7i) &\quad \text{Multilpy coefficients and } i's \\ 21i^2 &\quad \text{Simplify } i^2 = -1 \\ 21(-1) &\quad \text{Multiply} \\ -21 &\quad \text{Our Solution} \end{aligned}$$

Example 432.

$$\begin{aligned} 5i(3i - 7) &\quad \text{Distribute} \\ 15i^2 - 35i &\quad \text{Simplify } i^2 = -1 \\ 15(-1) - 35i &\quad \text{Multiply} \\ -15 - 35i &\quad \text{Our Solution} \end{aligned}$$

Example 433.

$$\begin{aligned} (2 - 4i)(3 + 5i) &\quad \text{FOIL} \\ 6 + 10i - 12i - 20i^2 &\quad \text{Simplify } i^2 = -1 \\ 6 + 10i - 12i - 20(-1) &\quad \text{Multiply} \\ 6 + 10i - 12i + 20 &\quad \text{Combine like terms } 6 + 20 \text{ and } 10i - 12i \\ 26 - 2i &\quad \text{Our Solution} \end{aligned}$$

Example 434.

$$\begin{aligned} (3i)(6i)(2 - 3i) &\quad \text{Multiply first two monomials} \\ 18i^2(2 - 3i) &\quad \text{Distribute} \\ 36i^2 - 54i^3 &\quad \text{Simplify } i^2 = -1 \text{ and } i^3 = -i \\ 36(-1) - 54(-i) &\quad \text{Multiply} \\ -36 + 54i &\quad \text{Our Solution} \end{aligned}$$

Remember when squaring a binomial we either have to FOIL or use our shortcut to square the first, twice the product and square the last. The next example uses the shortcut

Example 435.

$$\begin{aligned} (4 - 5i)^2 &\quad \text{Use perfect square shortcut} \\ 4^2 = 16 &\quad \text{Square the first} \\ 2(4)(-5i) = -40i &\quad \text{Twice the product} \\ (5i)^2 = 25i^2 = 25(-1) = -25 &\quad \text{Square the last, simplify } i^2 = -1 \\ 16 - 40i - 25 &\quad \text{Combine like terms} \\ -9 - 40i &\quad \text{Our Solution} \end{aligned}$$

Using i we can simplify radicals with negatives under the root. We will use the product rule and simplify the negative as a factor of negative one. This is shown in the following examples.

Example 438.

$$\begin{array}{rl} \sqrt{-16} & \text{Consider the negative as } a \text{ factor of } -1 \\ \sqrt{-1\cdot 16} & \text{Take each root, square root of } -1 \text{ is } i \\ 4i & \text{Our Solution} \end{array}$$

Example 439.

$$\begin{array}{rl} \sqrt{-24} & \text{Find perfect square factors, including } -1 \\ \sqrt{-1\cdot 4\cdot 6} & \text{Square root of } -1 \text{ is } i\text{, square root of 4 is 2} \\ 2i\sqrt{6} & \text{Our Solution} \end{array}$$

When simplifying complex radicals it is important that we take the -1 out of the radical (as an i) before we combine radicals.

Example 440.

$$\begin{array}{rl} \sqrt{-6}\sqrt{-3} & \text{Simplify the negatives, bringing } i \text{ out of radicals} \\ (i\sqrt{6})(i\sqrt{3}) & \text{Multiply } i \text{ by } i \text{ is } i^2=-1\text{, also multiply radicals} \\ -\sqrt{18} & \text{Simplify the radical} \\ -\sqrt{9\cdot 2} & \text{Take square root of 9} \\ -3\sqrt{2} & \text{Our Solution} \end{array}$$

If there are fractions, we need to make sure to reduce each term by the same number. This is shown in the following example.

Example 441.

$$\begin{array}{rl} \dfrac{-15-\sqrt{-200}}{20} & \text{Simplify the radical first} \\ \sqrt{-200} & \text{Find perfect square factors, including } -1 \\ \sqrt{-1\cdot 100\cdot 2} & \text{Take square root of } -1 \text{ and } 100 \\ 10i\sqrt{2} & \text{Put this back into the expression} \\ \dfrac{-15-10i\sqrt{2}}{20} & \text{All the factors are divisible by 5} \\ \dfrac{-3-2i\sqrt{2}}{4} & \text{Our Solution} \end{array}$$

By using $i=\sqrt{-1}$ we will be able to simplify and solve problems that we could not simplify and solve before. This will be explored in more detail in a later section.

9.1

Quadratics - Solving with Radicals

Objective: Solve equations with radicals and check for extraneous solutions.

Here we look at equations that have roots in the problem. As you might expect, to clear a root we can raise both sides to an exponent. So to clear a square root we can rise both sides to the second power. To clear a cubed root we can raise both sides to a third power. There is one catch to solving a problem with roots in it, sometimes we end up with solutions that do not actually work in the equation. This will only happen if the index on the root is even, and it will not happen all the time. So for these problems it will be required that we check our answer in the original problem. If a value does not work it is called an extraneous solution and not included in the final solution.

When solving a radical problem with an even index: check answers!

Example 442.

$$\begin{aligned}
\sqrt{7x+2} &= 4 && \text{Even index! We will have to check answers}\\
(\sqrt{7x+2})^2 &= 4^2 && \text{Square both sides, simplify exponents}\\
7x+2 &= 16 && \text{Solve}\\
\underline{-2\;\;} &\underline{\;-2} && \text{Subtract 2 from both sides}\\
7x &= 14 && \text{Divide both sides by 7}\\
\overline{\;7\;} &\;\;\overline{\;7\;} && \\
x &= 2 && \text{Need to check answer in original problem}\\
\sqrt{7(2)+2} &= 4 && \text{Multiply}\\
\sqrt{14+2} &= 4 && \text{Add}\\
\sqrt{16} &= 4 && \text{Square root}\\
4 &= 4 && \text{True! It works!}\\
x &= 2 && \text{Our Solution}
\end{aligned}$$

Example 443.

$$\begin{aligned}
\sqrt[3]{x-1} &= -4 && \text{Odd index, we don't need to check answer}\\
(\sqrt[3]{x-1})^3 &= (-4)^3 && \text{Cube both sides, simplify exponents}\\
x-1 &= -64 && \text{Solve}
\end{aligned}$$

$$
\begin{aligned}
2-2&=0 && \text{Subtract}\\
0&=0 && \text{True! It works}\\
x&=4 && \text{Our Solution}
\end{aligned}
$$

When there is more than one square root in the problem, after isolating one root and squaring both sides we may still have a root remaining in the problem. In this case we will again isolate the term with the second root and square both sides. When isolating, we will isolate the *term* with the square root. This means the square root can be multiplied by a number after isolating.

Example 447.

$$
\begin{aligned}
\sqrt{2x+1}-\sqrt{x}&=1 && \text{Even index! We will have to check answers}\\
\underline{+\sqrt{x}+\sqrt{x}} & && \text{Isolate first root by adding } \sqrt{x} \text{ to both sides}\\
\sqrt{2x+1}&=\sqrt{x}+1 && \text{Square both sides}\\
(\sqrt{2x+1})^2&=(\sqrt{x}+1)^2 && \text{Evaluate exponents, recall } (a+b)^2=a^2+2ab+b^2\\
2x+1&=x+2\sqrt{x}+1 && \text{Isolate the term with the root}\\
\underline{-x-1-x\qquad -1} & && \text{Subtract } x \text{ and 1 from both sides}\\
x&=2\sqrt{x} && \text{Square both sides}\\
(x)^2&=(2\sqrt{x})^2 && \text{Evaluate exponents}\\
x^2&=4x && \text{Make equation equal zero}\\
\underline{-4x\ \ -4x} & && \text{Subtract } x \text{ from both sides}\\
x^2-4x&=0 && \text{Factor}\\
x(x-4)&=0 && \text{Set each factor equal to zero}\\
x=0 \quad \text{or} \quad x-4&=0 && \text{Solve}\\
\underline{+4+4} & && \text{Add 4 to both sides of second equation}\\
x=0 \quad \text{or} \quad x&=4 && \text{Need to check answers in original}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{2(0)+1}-\sqrt{(0)}&=1 && \text{Check } x=0 \text{ first}\\
\sqrt{1}-\sqrt{0}&=1 && \text{Take roots}\\
1-0&=1 && \text{Subtract}\\
1&=1 && \text{True! It works}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{2(4)+1}-\sqrt{(4)}&=1 && \text{Check } x=4\\
\sqrt{8+1}-\sqrt{4}&=1 && \text{Add}\\
\sqrt{9}-\sqrt{4}&=1 && \text{Take roots}\\
3-2&=1 && \text{Subtract}\\
1&=1 && \text{True! It works}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt[4]{x^4} &= \pm\sqrt[4]{16} && \text{Simplify roots}\\
x &= \pm 2 && \text{Our Solution}
\end{aligned}
$$

World View Note: In 1545, French Mathematicain Gerolamo Cardano published his book *The Great Art, or the Rules of Algebra* which included the solution of an equation with a fourth power, but it was considered absurd by many to take a quantity to the fourth power because there are only three dimensions!

Example 451.

$$
\begin{array}{rl}
(2x+4)^2 = 36 & \text{Use even root property } (\pm)\\
\sqrt{(2x+4)^2} = \pm\sqrt{36} & \text{Simplify roots}\\
2x+4 = \pm 6 & \text{To avoid sign errors we need two equations}\\
2x+4=6 \quad\text{or}\quad 2x+4=-6 & \text{One equation for } +\text{, one equation for } -\\
\underline{-4\;-4} \qquad\qquad \underline{-4\quad -4} & \text{Subtract 4 from both sides}\\
2x=2 \quad\text{or}\quad 2x=-10 & \text{Divide both sides by 2}\\
\overline{2}\;\;\overline{2} \qquad\quad \overline{2}\qquad\;\overline{2} & \\
x=1 \quad\text{or}\quad x=-5 & \text{Our Solutions}
\end{array}
$$

In the previous example we needed two equations to simplify because when we took the root, our solutions were two rational numbers, 6 and -6. If the roots did not simplify to rational numbers we can keep the $\pm$ in the equation.

Example 452.

$$
\begin{array}{rl}
(6x-9)^2 = 45 & \text{Use even root property } (\pm)\\
\sqrt{(6x-9)^2} = \pm\sqrt{45} & \text{Simplify roots}\\
6x-9 = \pm 3\sqrt{5} & \text{Use one equation because root did not simplify to rational}\\
\underline{+9\;+9} \qquad & \text{Add 9 to both sides}\\
6x = 9 \pm 3\sqrt{5} & \text{Divide both sides by 6}\\
\overline{6}\qquad\overline{\quad 6\quad} & \\
x = \dfrac{9\pm 3\sqrt{5}}{6} & \text{Simplify, divide each term by 3}\\
x = \dfrac{3\pm\sqrt{5}}{2} & \text{Our Solution}
\end{array}
$$

9.3

Quadratics - Complete the Square

Objective: Solve quadratic equations by completing the square.

When solving quadratic equations in the past we have used factoring to solve for our variable. This is exactly what is done in the next example.

Example 457.

$$\begin{array}{rcrl}
 & & x^2+5x+6=0 & \text{Factor} \\
 & & (x+3)(x+2)=0 & \text{Set each factor equal to zero} \\
x+3=0 & \text{or} & x+2=0 & \text{Solve each equation} \\
\underline{-3-3} & & \underline{-2-2} & \\
x=-3 & \text{or} & x=-2 & \text{Our Solutions}
\end{array}$$

However, the problem with factoring is all equations cannot be factored. Consider the following equation: $x^2-2x-7=0$. The equation cannot be factored, however there are two solutions to this equation, $1+2\sqrt{2}$ and $1-2\sqrt{2}$. To find these two solutions we will use a method known as completing the square. When completing the square we will change the quadratic into a perfect square which can easily be solved with the square root property. The next example reviews the square root property.

Example 458.

$$\begin{array}{rl}
(x+5)^2=18 & \text{Square root of both sides} \\
\sqrt{(x+5)^2}=\pm\sqrt{18} & \text{Simplify each radical} \\
x+5=\pm 3\sqrt{2} & \text{Subtract 5 from both sides} \\
\underline{-5\quad -5\quad} & \\
x=-5\pm 3\sqrt{2} & \text{Our Solution}
\end{array}$$

$$
\begin{array}{rl}
\underline{-48-48} & \text{Subtract 24} \\
2x^2+20x \quad = -48 & \text{Divide by } a \text{ or } 2 \\
\overline{\;2\;} \quad \overline{\;2\;} \quad\quad \overline{\;2\;} & \\
x^2+10x \quad = -24 & \text{Find number to complete the square: } \left(\frac{1}{2}\cdot b\right)^2 \\
\left(\frac{1}{2}\cdot 10\right)^2 = 5^2 = 25 & \text{Add 25 to both sides of the equation} \\
x^2+10x \quad = -24 & \\
\underline{+25 \quad +25} & \\
x^2+10x+25 = 1 & \text{Factor} \\
(x+5)^2 = 1 & \text{Solve with even root property} \\
\sqrt{(x+5)^2} = \pm\sqrt{1} & \text{Simplify roots} \\
x+5 = \pm 1 & \text{Subtract 5 from both sides} \\
\underline{-5-5} & \\
x = -5\pm 1 & \text{Evaluate} \\
x = -4 \;\text{ or }\; -6 & \text{Our Solution}
\end{array}
$$

Example 463.

$$
\begin{array}{rl}
x^2-3x-2=0 & \text{Separate constant from variables} \\
\underline{+2+2} & \text{Add 2 to both sides} \\
x^2-3x \quad = 2 & \text{Now, find number to complete the square } \left(\frac{1}{2}\cdot b\right)^2 \\
\left(\frac{1}{2}\cdot 3\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} & \text{Add } \frac{9}{4} \text{ to both sides,} \\
\frac{2}{1}\left(\frac{4}{4}\right)+\frac{9}{4} = \frac{8}{4}+\frac{9}{4} = \frac{17}{4} & \text{Need common denominator (4) on right} \\
x^2-3x+\frac{9}{4} = \frac{8}{4}+\frac{9}{4} = \frac{17}{4} & \text{Factor} \\
\left(x-\frac{3}{2}\right)^2 = \frac{17}{4} & \text{Solve using the even root property} \\
\sqrt{\left(x-\frac{3}{2}\right)^2} = \pm\sqrt{\frac{17}{4}} & \text{Simplify roots} \\
x-\frac{3}{2} = \frac{\pm\sqrt{17}}{2} & \text{Add } \frac{3}{2} \text{ to both sides,}
\end{array}
$$

$$x=\frac{30\pm\sqrt{900+1100}}{50} \quad \text{Evaluate addition inside root}$$
$$x=\frac{30\pm\sqrt{2000}}{50} \quad \text{Simplify root}$$
$$x=\frac{30\pm 20\sqrt{5}}{50} \quad \text{Reduce fraction by dividing each term by 10}$$
$$x=\frac{3\pm 2\sqrt{5}}{5} \quad \text{Our Solution}$$

Example 468.

$$\begin{array}{rl}
3x^2+4x+8=2x^2+6x-5 & \text{First set equation equal to zero}\\
\underline{-2x^2-6x+5 \quad -2x^2-6x+5} & \text{Subtract } 2x^2 \text{ and } 6x \text{ and add } 5\\
x^2-2x+13=0 & a=1, b=-2, c=13, \text{ use quadratic formula}\\
x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(13)}}{2(1)} & \text{Evaluate exponent and multiplication}\\
x=\dfrac{2\pm\sqrt{4-52}}{2} & \text{Evaluate subtraction inside root}\\
x=\dfrac{2\pm\sqrt{-48}}{2} & \text{Simplify root}\\
x=\dfrac{2\pm 4i\sqrt{3}}{2} & \text{Reduce fraction by dividing each term by 2}\\
x=1\pm 2i\sqrt{3} & \text{Our Solution}
\end{array}$$

When we use the quadratic formula we don't necessarily get two unique answers. We can end up with only one solution if the square root simplifies to zero.

Example 469.

$$\begin{array}{rl}
4x^2-12x+9=0 & a=4, b=-12, c=9, \text{ use quadratic formula}\\
x=\dfrac{12\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)} & \text{Evaluate exponents and multiplication}\\
x=\dfrac{12\pm\sqrt{144-144}}{8} & \text{Evaluate subtraction inside root}\\
x=\dfrac{12\pm\sqrt{0}}{8} & \text{Evaluate root}\\
x=\dfrac{12\pm 0}{8} & \text{Evaluate } \pm\\
x=\dfrac{12}{8} & \text{Reduce fraction}\\
x=\dfrac{3}{2} & \text{Our Solution}
\end{array}$$

9.5

Quadratics - Build Quadratics From Roots

Objective: Find a quadratic equation that has given roots using reverse factoring and reverse completing the square.

Up to this point we have found the solutions to quadratics by a method such as factoring or completing the square. Here we will take our solutions and work backwards to find what quadratic goes with the solutions.

We will start with rational solutions. If we have rational solutions we can use factoring in reverse, we will set each solution equal to x and then make the equation equal to zero by adding or subtracting. Once we have done this our expressions will become the factors of the quadratic.

Example 471.

$$\begin{array}{rl}
\text{The solutions are } 4 \text{ and } -2 & \text{Set each solution equal to } x \\
x = 4 \quad \text{or} \quad x = -2 & \text{Make each equation equal zero} \\
\underline{-4 \; -4} \quad \underline{+2 \quad +2} & \text{Subtract 4 from first, add 2 to second} \\
x - 4 = 0 \quad \text{or} \quad x + 2 = 0 & \text{These expressions are the factors} \\
(x-4)(x+2) = 0 & \text{FOIL} \\
x^2 + 2x - 4x - 8 = 0 & \text{Combine like terms} \\
x^2 - 2x - 8 = 0 & \text{Our Solution}
\end{array}$$

If one or both of the solutions are fractions we will clear the fractions by multiplying by the denominators.

Example 472.

$$\begin{array}{rl}
\text{The solution are } \frac{2}{3} \text{ and } \frac{3}{4} & \text{Set each solution equal to } x \\
x = \frac{2}{3} \quad \text{or} \quad x = \frac{3}{4} & \text{Clear fractions by multiplying by denominators} \\
3x = 2 \quad \text{or} \quad 4x = 3 & \text{Make each equation equal zero} \\
\underline{-2 \; -2} \quad \underline{-3 \; -3} & \text{Subtract 2 from the first, subtract 3 from the second} \\
3x - 2 = 0 \quad \text{or} \quad 4x - 3 = 0 & \text{These expressions are the factors} \\
(3x-2)(4x-3) = 0 & \text{FOIL} \\
12x^2 - 9x - 8x + 6 = 0 & \text{Combine like terms}
\end{array}$$

16) A sink can be filled from the faucet in 5 minutes. It takes only 3 minutes to empty the sink when the drain is open. If the sink is full and both the faucet and the drain are open, how long will it take to empty the sink?

17) It takes 10 hours to fill a pool with the inlet pipe. It can be emptied in 15 hrs with the outlet pipe. If the pool is half full to begin with, how long will it take to fill it from there if both pipes are open?

18) A sink is $\frac{1}{4}$ full when both the faucet and the drain are opened. The faucet alone can fill the sink in 6 minutes, while it takes 8 minutes to empty it with the drain. How long will it take to fill the remaining $\frac{3}{4}$ of the sink?

19) A sink has two faucets, one for hot water and one for cold water. The sink can be filled by a cold-water faucet in 3.5 minutes. If both faucets are open, the sink is filled in 2.1 minutes. How long does it take to fill the sink with just the hot-water faucet open?

20) A water tank is being filled by two inlet pipes. Pipe A can fill the tank in $4\frac{1}{2}$ hrs, while both pipes together can fill the tank in 2 hours. How long does it take to fill the tank using only pipe B?

21) A tank can be emptied by any one of three caps. The first can empty the tank in 20 minutes while the second takes 32 minutes. If all three working together could empty the tank in $8\frac{8}{59}$ minutes, how long would the third take to empty the tank?

22) One pipe can fill a cistern in $1\frac{1}{2}$ hours while a second pipe can fill it in $2\frac{1}{3}$ hrs. Three pipes working together fill the cistern in 42 minutes. How long would it take the third pipe alone to fill the tank?

23) Sam takes 6 hours longer than Susan to wax a floor. Working together they can wax the floor in 4 hours. How long will it take each of them working alone to wax the floor?

24) It takes Robert 9 hours longer than Paul to rapair a transmission. If it takes them $2\frac{2}{5}$ hours to do the job if they work together, how long will it take each of them working alone?

25) It takes Sally $10\frac{1}{2}$ minutes longer than Patricia to clean up their dorm room. If they work together they can clean it in 5 minutes. How long will it take each of them if they work alone?

26) A takes $7\frac{1}{2}$ minutes longer than B to do a job. Working together they can do the job in 9 minutes. How long does it take each working alone?

27) Secretary A takes 6 minutes longer than Secretary B to type 10 pages of manuscript. If they divide the job and work together it will take them $8\frac{3}{4}$ minutes to type 10 pages. How long will it take each working alone to type the 10 pages?

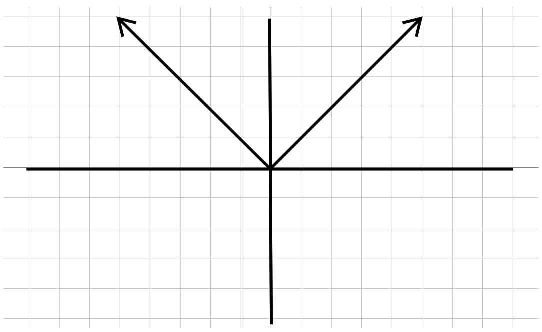
28) It takes John 24 minutes longer than Sally to mow the lawn. If they work together they can mow the lawn in 9 minutes. How long will it take each to mow the lawn if they work alone?

It is important to remember the graph of all quadratics is a parabola with the same U shape (they could be upside-down). If you plot your points and we cannot connect them in the correct U shape then one of your points must be wrong. Go back and check your work to be sure they are correct!

Just as all quadratics (equation with $y = x^2$) all have the same U-shape to them and all linear equations (equations such as $y = x$) have the same line shape when graphed, different equations have different shapes to them. Below are some common equations (some we have yet to cover!) with their graph shape drawn.

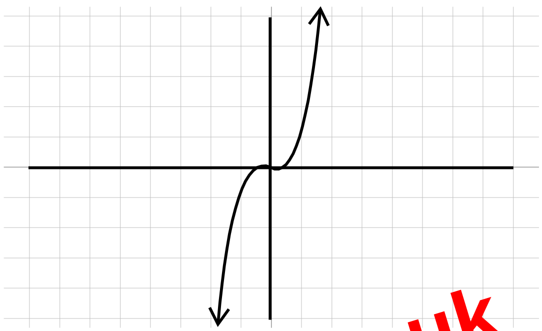
Absolute Value

$y = |x|$



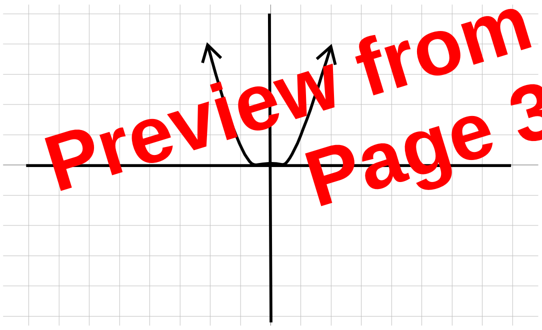
Cubic

$y = x^3$



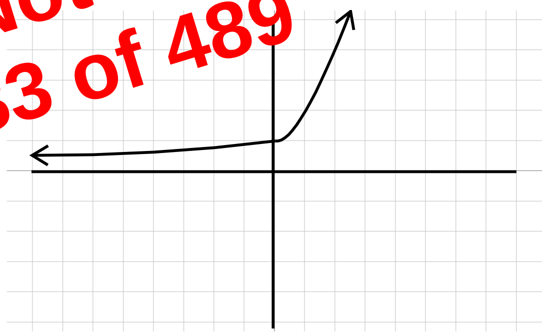
Quadratic

$y = x^2$



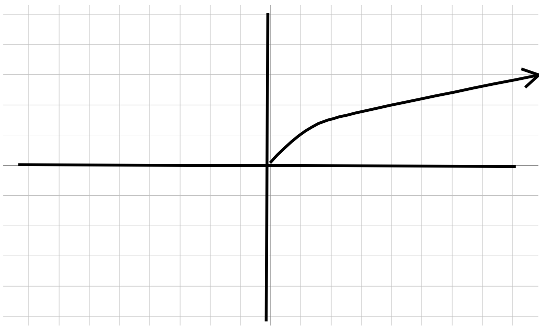
Exponential

$y = a^x$



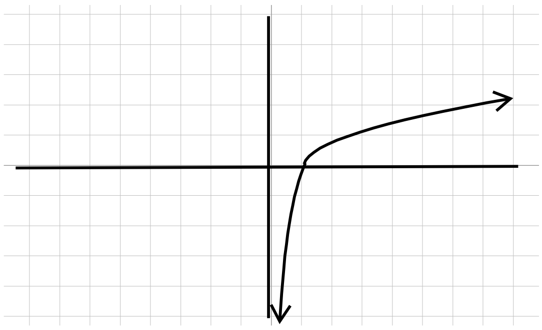
Square Root

$y = \sqrt{x}$



Logarithmic

$y = \log_a x$



10.1

Functions - Function Notation

Objective: Idenfity functions and use correct notation to evaluate functions at numerical and variable values.

There are many different types of equations that we can work with in algebra. An equation gives the relationship between variables and numbers. Examples of several relationships are below:

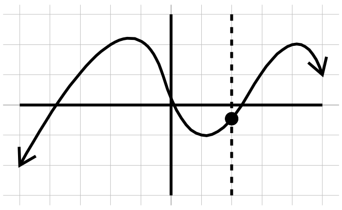
$$\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1 \quad \text{and} \quad y = x^2 - 2x + 7 \quad \text{and} \quad \sqrt{y+x} - 7 = xy$$

There is a speical classification of relationships known as functions. **Functions** have at most one output for any input. Generally x is the variable that we plug into an equation and evaluate to find y. For this reason x is considered an input variable and y is considered an output variable. This means the definition of a function, in terms of equations in x and y could be said, for any x value there is at most one y value that corresponds with it.

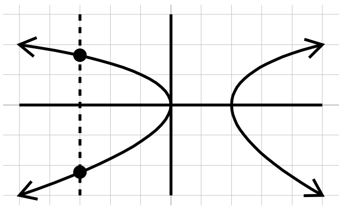
A great way to visualize this definition is to look at the graphs of a few relationships. Because x values are vertical lines we will draw a vertical line through the graph. If the vertical line crosses the graph more than once, that means we have too many possible y values. If the graph crosses the graph only once, then we say the relationship is a function.

Example 504.

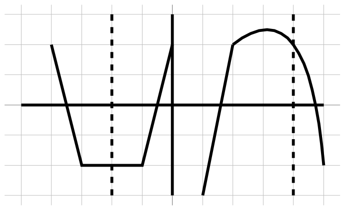
Which of the following graphs are graphs of functions?



Drawing a vertical line through this graph will only cross the graph once, it is a function.



Drawing a vertical line through this graph will cross the graph twice, once at top and once at bottom. This is not a function.



Drawing a vertical line through this graph will cross the graph only once, it is a function.

We can look at the above idea in an algebraic method by taking a relationship and solving it for y. If we have only one solution then it is a function.

Example 505.

$$\begin{array}{rl}
\text{Is } 3x^2 - y = 5 \text{ a function?} & \text{Solve the relation for } y \\
-3x^2 \quad -3x^2 & \text{Subtract } 3x^2 \text{ from both sides} \\
-y = -3x^2 + 5 & \text{Divide each term by } -1 \\
\dfrac{-y}{-1} = \dfrac{-3x^2}{-1} + \dfrac{5}{-1} & \\
y = 3x^2 - 5 & \text{Only one solution for } y. \\
\text{Yes!} & \text{It is } a \text{ function}
\end{array}$$

Example 506.

$$\begin{array}{rl}
\text{Is } y^2 - x = 5 \text{ a function?} & \text{Solve the relation for } y \\
+x \quad +x & \text{Add } x \text{ to both sides} \\
y^2 = x + 5 & \text{Square root of both sdies} \\
\sqrt{y^2} = \pm\sqrt{x+5} & \text{Simplify} \\
y = \pm\sqrt{x+5} & \text{Two solutions for } y \text{ (one } +, \text{ one } -) \\
\text{No!} & \text{Not } a \text{ function}
\end{array}$$

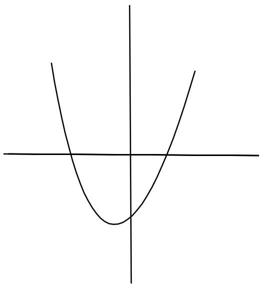
Once we know we have a function, often we will change the notation used to emphasis the fact that it is a function. Instead of writing $y =$, we will use function notation which can be written $f(x) =$. We read this notation "f of x". So for

10.1 Practice - Function Notation

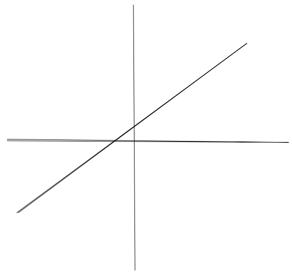
Solve.

1) Which of the following is a function?

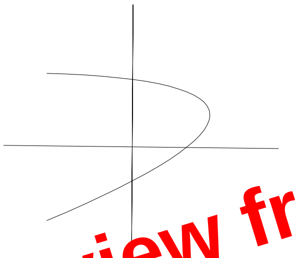
a)



b)



c)



d)

e) $y = 3x - 7$

f) $y^2 - x^2 = 1$

g) $\sqrt{y} + x = 2$

h) $x^2 + y^2 = 1$

Specify the domain of each of the following funcitons.

2) $f(x) = -5x + 1$

3) $f(x) = \sqrt{5 - 4x}$

4) $s(t) = \frac{1}{t^2}$

5) $f(x) = x^2 - 3x - 4$

6) $s(t) = \frac{1}{t^2 + 1}$

7) $f(x) = \sqrt{x - 16}$

8) $f(x) = \frac{-2}{x^2 - 3x - 4}$

9) $h(x) = \frac{\sqrt{3x - 12}}{x^2 - 25}$

10 $y(x) = \frac{x}{x^2 - 25}$

We can also evaluate a composition of functions at a variable. In these problems we will take the inside function and substitute into the outside function.

Example 522.

$$\begin{array}{ll} f(x) = x^2 - x & \\ g(x) = x + 3 & \text{Rewrite as } a \text{ function in function} \\ \text{Find } (f \circ g)(x) & \end{array}$$

$$\begin{array}{rl} f(g(x)) & \text{Replace } g(x) \text{ with } x+3 \\ f(x+3) & \text{Replace the variables in } f \text{ with } (x+3) \\ (x+3)^2 - (x+3) & \text{Evaluate exponent} \\ (x^2+6x+9) - (x+3) & \text{Distribute negative} \\ x^2+6x+9-x-3 & \text{Combine like terms} \\ x^2+5x+6 & \text{Our Solution} \end{array}$$

It is important to note that very rarely is $(f \circ g)(x)$ the same as $(g \circ f)(x)$ as the following example will show, using the same equations, but compositing them in the opposite direction.

Example 523.

$$\begin{array}{ll} f(x) = x^2 - x & \\ g(x) = x + 3 & \text{Rewrite as } a \text{ function in function} \\ \text{Find } (g \circ f)(x) & \end{array}$$

$$\begin{array}{rl} g(f(x)) & \text{Replace } f(x) \text{ with } x^2 - x \\ g(x^2 - x) & \text{Replace the variable in } g \text{ with } (x^2 - x) \\ (x^2 - x) + 3 & \text{Here the parenthesis don}'t \text{ change the expression} \\ x^2 - x + 3 & \text{Our Solution} \end{array}$$

World View Note: The term "function" came from Gottfried Wihelm Leibniz, a German mathematician from the late 17th century.

To investigate what happens to the balance if the compounds happen more often, we will consider the same problem, this time with interest compounded daily.

Example 547.

If \$4000 is invested in an account paying 3% interest compounded daily, what is the balance after 7 years?

$$
\begin{array}{rl}
P=4000, r=0.03, n=365, t=7 & \text{Identify each variable} \\
A=4000\left(1+\frac{0.03}{365}\right)^{365\cdot 7} & \text{Plug each value into formula, evaluate parenthesis} \\
A=4000(1.00008219\quad)^{365\cdot 7} & \text{Multiply exponent} \\
A=4000(1.00008219\quad)^{2555} & \text{Evaluate exponent} \\
A=4000(1.23366741\quad .) & \text{Multiply} \\
A=4934.67 & \\
\$4934.67 & \text{Our Solution}
\end{array}
$$

While this difference is not very large, it is a bit higher. The table below shows the result for the same problem with different compounds.

Compound	Balance
Annually	\$4919.50
Semi-Annually	\$4927.02
Quarterly	\$4930.85
Monthly	\$4933.42
Weekly	\$4934.41
Daily	\$4934.67

As the table illustrates, the more often interest is compounded, the higher the final balance will be. The reason is, because we are calculating compound interest or interest on interest. So once interest is paid into the account it will start earning interest for the next compound and thus giving a higher final balance. The next question one might consider is what is the maximum number of compounds possible? We actually have a way to calculate interest compounded an infinite number of times a year. This is when the interest is compounded continuously. When we see the word "continuously" we will know that we cannot use the first formula. Instead we will use the following formula:

$$
\begin{array}{r}
\textbf{Interest Compounded Continuously: } \boldsymbol{A=Pe^{rt}} \\
A=\text{Final Amount} \\
P=\text{Principle (starting balance)} \\
e=a\text{ constant approximately }2.71828183\quad . \\
r=\text{Interest rate (written as }a\text{ decimal)} \\
t=\text{time (years)}
\end{array}
$$

j. All of the above compounded continuously.

2) What principal will amount to \$2000 if invested at 4% interest compounded semiannually for 5 years?

3) What principal will amount to \$3500 if invested at 4% interest compounded quarterly for 5 years?

4) What principal will amount to \$3000 if invested at 3% interest compounded semiannually for 10 years?

5) What principal will amount to \$2500 if invested at 5% interest compounded semiannually for 7.5 years?

6) What principal will amount to \$1750 if invested at 3% interest compounded quarterly for 5 years?

7) A thousand dollars is left in a bank savings account drawing 7% interest, compounded quarterly for 10 years. What is the balance at the end of that time?

8) A thousand dollars is left in a credit union drawing 7% compounded monthly. What is the balance at the end of 10 years?

9) \$1750 is invested in an account earning 13.5% interest compounded monthly for a 2 year period. What is the balance at the end of 2 years?

10) You lend out \$5500 at 10% compounded monthly. If the debt is repaid in 18 months, what is the total owed at the time of repayment?

11) A \$10,000 Treasury Bill earned 16% compounded monthly. If the bill matured in 2 years, what was it worth at maturity?

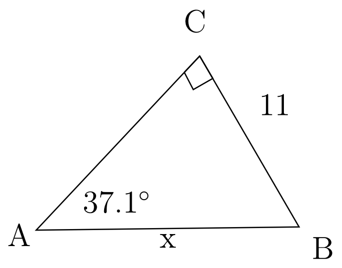
12) You borrow \$25000 at 12.25% interest compounded monthly. If you are unable to make any payments the first year, how much do you owe, excluding penalties?

13) A savings institution advertises 7% annual interest, compounded daily, How much more interest would you earn over the bank savings account or credit union in problems 7 and 8?

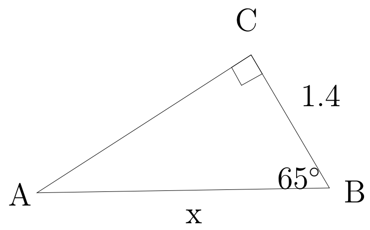
14) An 8.5% account earns continuous interest. If \$2500 is deposited for 5 years, what is the total accumulated?

15) You lend \$100 at 10% continuous interest. If you are repaid 2 months later, what is owed?

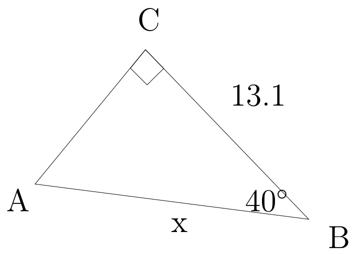
37)



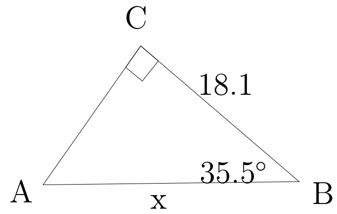
38)



39)



40)

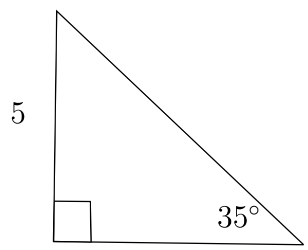


b are the other two sides (legs), then we can use the following formula, $a^2 + b^2 = c^2$ to find a missing side.

Often when solving triangles we use trigonometry to find one part, then use the angle sum and/or the Pythagorean Theorem to find the other two parts.

Example 557.

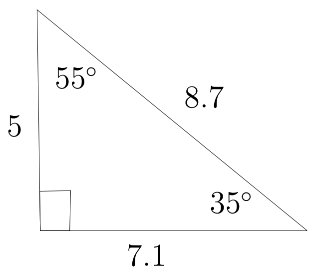
Solve the triangle



We have one angle and one side. We can use these to find either other side. We will find the other leg, the adjacent side to the 35° angle.

The 5 is the opposite side, so we will use the tangent to find the leg.

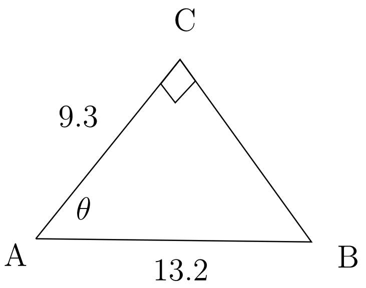
$\tan 35° = \frac{5}{x}$	Tangent is opposite over adjacent
$\frac{0.700}{1} = \frac{5}{x}$	Evaluate tangent, put it over one so we have a proportion
$0.700x = 5$	Find cross product
$\overline{0.700} \quad \overline{0.700}$	Divide both sides by 0.700
$x = 7.1$	The missing leg.
$a^2 + b^2 = c^2$	We can now use pythagorean theorem to find hypotenuse, c
$5^2 + 7.1^2 = c^2$	Evaluate exponents
$25 + 50.41 = c^2$	Add
$75.41 = c^2$	Square root both sides
$8.7 = c$	The hypotenuse
$90° - 35°$	To find the missing angle we subtract from 90°
$55°$	The missing angle



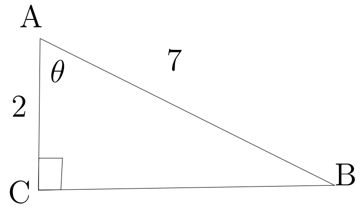
Our Solution

In the previous example, once we found the leg to be 7.1 we could have used the sine function on the 35° angle to get the hypotenuse and then any inverse trig

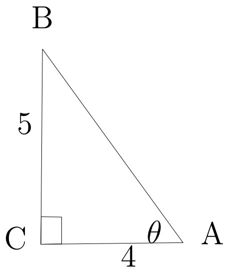
19)



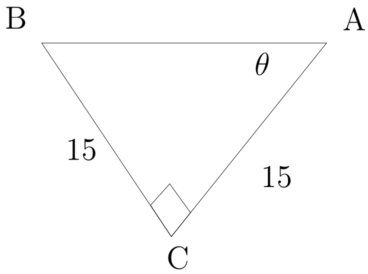
20)



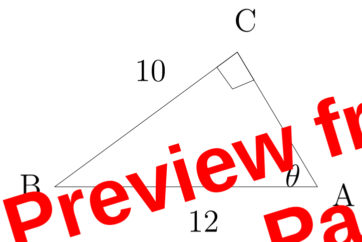
21)



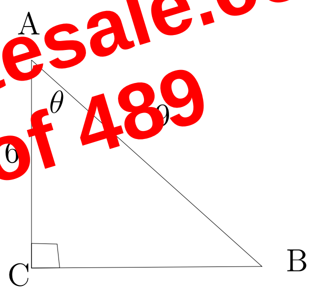
22)



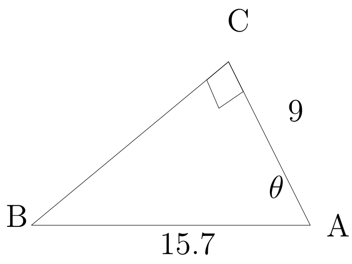
23)



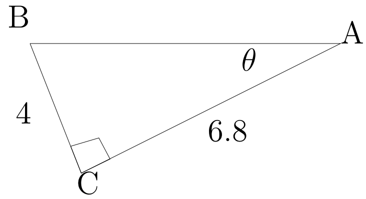
24)



25)



26)



7) $\frac{5}{4}$

8) $\frac{4}{3}$

9) $\frac{3}{2}$

10) $\frac{8}{3}$

11) $\frac{5}{2}$

12) $\frac{8}{7}$

13) $\frac{7}{2}$

14) $\frac{4}{3}$

15) $\frac{4}{3}$

16) $\frac{3}{2}$

17) $\frac{6}{5}$

18) $\frac{7}{6}$

19) $\frac{3}{2}$

20) $\frac{8}{7}$

21) 8

22) $\frac{5}{3}$

23) $-\frac{4}{9}$

24) $-\frac{2}{3}$

25) $-\frac{13}{4}$

26) $\frac{3}{4}$

27) $\frac{33}{20}$

28) $\frac{33}{56}$

29) 4

30) $\frac{18}{7}$

31) $\frac{1}{2}$

32) $-\frac{19}{20}$

33) 3

34) $-\frac{17}{15}$

35) $-\frac{7}{10}$

36) $\frac{5}{14}$

37) $-\frac{8}{7}$

38) $\frac{20}{21}$

39) $\frac{2}{9}$

40) $\frac{4}{3}$

41) $-\frac{21}{26}$

42) $\frac{25}{21}$

43) $-\frac{3}{2}$

44) $-\frac{5}{27}$

45) $\frac{40}{9}$

46) $-\frac{1}{18}$

47) $-\frac{45}{7}$

48) $\frac{13}{15}$

49) $\frac{4}{27}$

50) $\frac{32}{65}$

51) $\frac{1}{15}$

52) 1

53) -1

54) $-\frac{10}{7}$

55) $\frac{2}{7}$

56) 2

57) 3

58) $-\frac{31}{8}$

59) $\frac{37}{20}$

60) $-\frac{5}{3}$

61) $\frac{33}{20}$

62) $\frac{3}{7}$

63) $\frac{47}{56}$

64) $-\frac{7}{6}$

65) $\frac{2}{3}$

66) $-\frac{4}{3}$

67) 1

68) $\frac{7}{8}$

69) $\frac{19}{20}$

70) $-\frac{2}{5}$

71) $-\frac{145}{56}$

72) $-\frac{29}{15}$

73) $\frac{34}{7}$

74) $-\frac{23}{3}$

75) $-\frac{3}{8}$

76) $-\frac{2}{3}$

77) $-\frac{5}{24}$

78) $\frac{39}{14}$

79) $-\frac{5}{6}$

80) $\frac{1}{10}$

81) 2

82) $\frac{62}{21}$

54) $60v - 7$

55) $-3x + 8x^2$

56) $-89x + 81$

57) $-68k^2 - 8k$

58) $-19 - 90a$

59) $-34 - 49p$

60) $-10x + 17$

61) $10 - 4n$

62) $-30 + 9m$

63) $12x + 60$

64) $30r - 16r^2$

65) $-72n - 48 - 8n^2$

66) $-42b - 45 - 4b^2$

67) $79 - 79v$

68) $-8x + 22$

69) $-20n^2 + 80n - 42$

70) $-12 + 57a + 54a^2$

71) $-75 - 20k$

72) $-128x - 121$

73) $4n^2 - 3n - 5$

74) $2x^2 - 6x - 3$

75) $4p - 5$

76) $3x^2 + 7x - 7$

77) $-v^2 + 2v + 2$

78) $-7b^2 + 3b - 8$

79) $-4k^2 + 12$

80) $a^2 + 3a$

81) $3x^2 - 15$

82) $-n^2 + 6$

Answers - Chapter 1

1.1

Answers to One-Step Equations

1) 7

2) 11

3) -5

4) 4

5) 10

6) 6

7) -19

8) -6

9) 18

10) 6

11) -20

12) -7

13) -108

14) 5

15) -8

16) 4

17) 17

18) 4

19) 20

20) -208

21) 3

22) 16

23) -13

24) -9

25) 15

26) 8

27) -10

28) -204

29) 5

30) 2

31) -11

32) -14

33) 14

34) 1

35) -11

36) -15

37) -240

38) -135

39) -16

40) -380

1.2

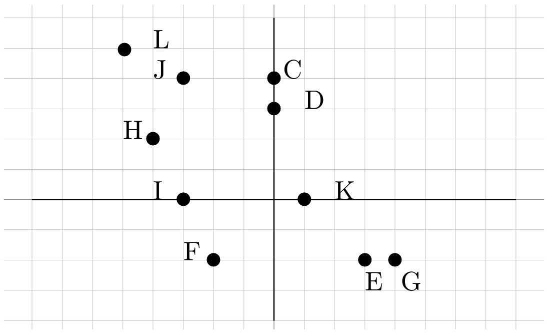
Answers to Two-Step Equations

1) -4

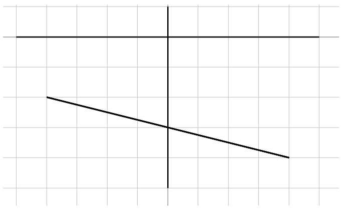
2) 7

1) B(4, -3) C(1, 2) D($-1, 4$) E($-5, 0$) F(2, -3) G(1, 3) H($-1, -4$) I($-2, -1$) J(0, 2) K($-4, 3$)

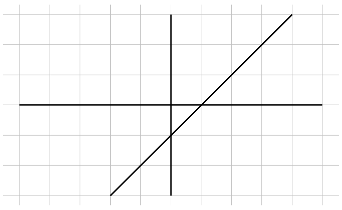
2)



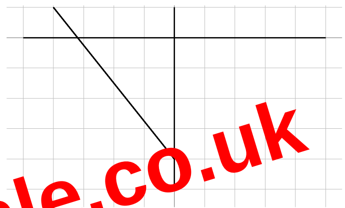
3)



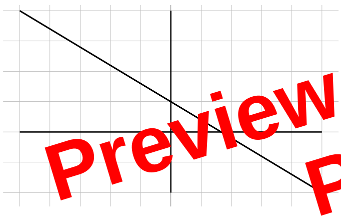
4)



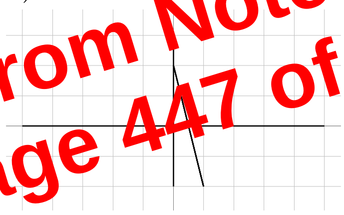
5)



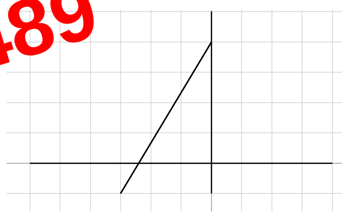
6)



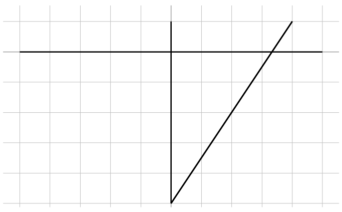
7)



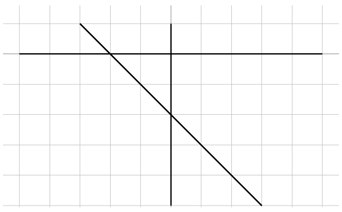
8)



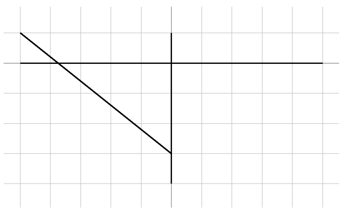
9)



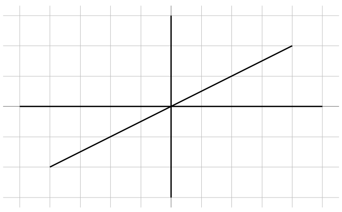
10)



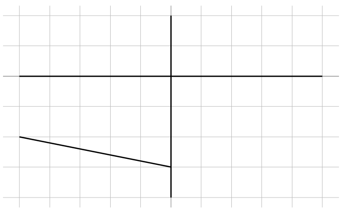
11)



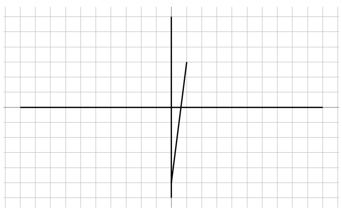
12)

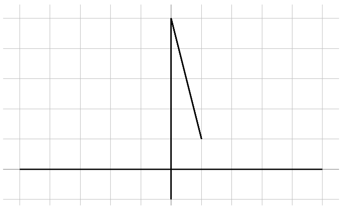


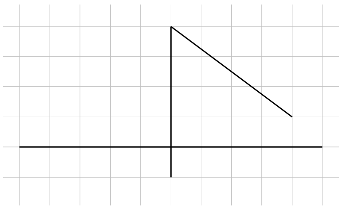
13)

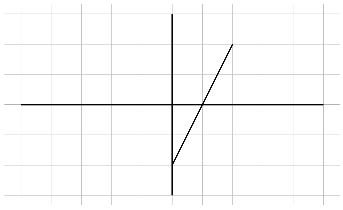


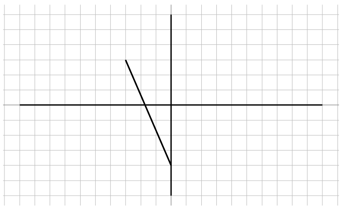
14)

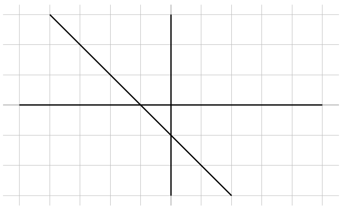


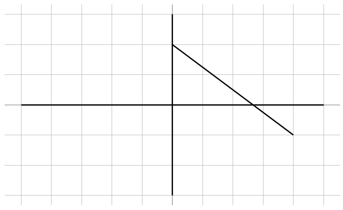
15)


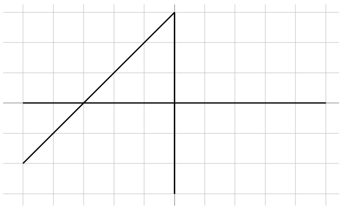
16)


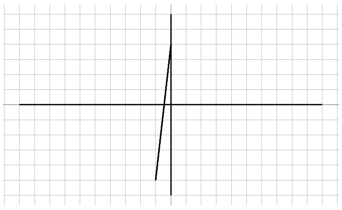
17)


18)


19)


20)


21)


22)


2.2

Answers - Slope

1) $\frac{3}{2}$

2) 5

3) Undefined

4) $-\frac{1}{2}$

5) $\frac{5}{6}$

6) $-\frac{2}{3}$

7) -1

8) $\frac{5}{4}$

9) -1

10) 0

11) Undefined

12) $\frac{16}{7}$

13) $-\frac{17}{31}$

14) $-\frac{3}{2}$

15) $\frac{4}{3}$

16) $-\frac{7}{17}$

17) 0

18) $\frac{5}{11}$

19) $\frac{1}{2}$

20) $\frac{1}{16}$

21) $-\frac{11}{2}$

22) $-\frac{12}{31}$

23) Undefined

24) $\frac{24}{11}$

25) $-\frac{26}{27}$

26) $-\frac{19}{10}$

27) $-\frac{1}{3}$

28) $\frac{1}{16}$

29) $-\frac{7}{13}$

30) $\frac{2}{7}$

31) -5

32) 2

33) -8

34) 3

35) -5

36) 6

37) -4

38) 1

39) 2

40) 1

2.3

Answers - Point-Slope Form

1) $x=2$

2) $x=1$

3) $y-2=\frac{1}{2}(x-2)$

4) $y-1=-\frac{1}{2}(x-2)$

5) $y+5=9(x+1)$

6) $y+2=-2(x-2)$

7) $y-1=\frac{3}{4}(x+4)$

8) $y+3=-2(x-4)$

9) $y+2=-3x$

10) $y-1=4(x+1)$

11) $y+5=-\frac{1}{4}x$

12) $y-2=-\frac{5}{4}x$

13) $y+3=\frac{1}{5}(x+5)$

14) $y+4=-\frac{2}{3}(x+1)$

15) $y-4=-\frac{5}{4}(x+1)$

16) $y+4=-\frac{3}{2}(x-1)$

17) $y=2x-3$

18) $y=-2x+2$

19) $y=-\frac{3}{5}x+2$

20) $y=-\frac{2}{3}x-\frac{10}{3}$

21) $y=\frac{1}{2}x+3$

22) $y=-\frac{7}{4}x+4$

23) $y=-\frac{3}{2}x+4$

24) $y=-\frac{5}{2}x-5$

25) $y=-\frac{2}{5}x-5$

26) $y=\frac{7}{3}x-4$

27) $y=x-4$

28) $y=-3$

29) $x=-3$

30) $y=2x-1$

31) $y=-\frac{1}{2}x$

32) $y=\frac{6}{5}x-3$

33) $y-3=-2(x+4)$

34) $y=3$

35) $y-1=\frac{1}{8}(x-5)$

36) $y-5=-\frac{1}{8}(x+4)$

37) $y+2=\frac{3}{2}(x+4)$

38) $y-1=\frac{3}{8}(x+4)$

39) $y-5=\frac{1}{4}(x-3)$

40) $y+4=-(x+1)$

41) $y+3=-\frac{8}{7}(x-3)$

42) $y+5=-\frac{1}{4}(x+1)$

43) $y=-\frac{3}{4}x-\frac{11}{4}$

44) $y=-\frac{1}{10}x-\frac{3}{2}$

45) $y=-\frac{8}{7}x-\frac{5}{7}$

46) $y=\frac{1}{2}x-\frac{3}{2}$

47) $y=-x+5$

48) $y=\frac{1}{3}x+1$

49) $y=-x+2$

50) $y=x+2$

51) $y=4x+3$

52) $y=\frac{3}{7}x+\frac{6}{7}$

2.5

Answers - Parallel and Perpendicular Lines

1) 2

2) $-\frac{2}{3}$

3) 4

4) $-\frac{10}{3}$

5) 1

6) $\frac{6}{5}$

7) -7

8) $-\frac{3}{4}$

9) 0

10) 2

11) 3

12) $-\frac{5}{4}$

13) -3

14) $-\frac{1}{3}$

15) 2

16) $-\frac{3}{8}$

17) $x=2$

18) $y-2=\frac{7}{5}(x-5)$

19) $y-4=\frac{9}{2}(x-3)$

20) $y+1=-\frac{3}{4}(x-1)$

21) $y-3=\frac{7}{5}(x-2)$

22) $y-3=-3(x+1)$

23) $x = 4$

24) $y - 4 = \frac{7}{5}(x - 1)$

25) $y + 5 = -(x - 1)$

26) $y + 2 = -2(x - 1)$

27) $y - 2 = \frac{1}{5}(x - 5)$

28) $y - 3 = -(x - 1)$

29) $y - 2 = -\frac{1}{4}(x - 4)$

30) $y + 5 = \frac{7}{3}(x + 3)$

31) $y + 2 = -3(x - 2)$

32) $y - 5 = -\frac{1}{2}(x + 2)$

33) $y = -2x + 5$

34) $y = \frac{3}{5}x + 5$

35) $y = -\frac{4}{3}x - 3$

36) $y = -\frac{5}{4}x - 5$

37) $y = -\frac{1}{2}x - 3$

38) $y = \frac{5}{2}x - 2$

39) $y = -\frac{1}{2}x - 2$

40) $y = \frac{3}{5}x - 1$

41) $y = x - 1$

42) $y = 2x + 1$

43) $y = 2$

44) $y = -\frac{2}{5}x + 1$

45) $y = -x + 3$

46) $y = -\frac{5}{2}x + 2$

47) $y = -2x + 5$

48) $y = \frac{3}{4}x + 4$

Answers - Chapter 3

3.1

Answers - Solve and Graph Inequalities

1) $(-5, \infty)$

2) $(-4, \infty)$

3) $(-\infty, -2]$

4) $(-\infty, 1]$

5) $(-\infty, 2]$

6) $(-5, \infty)$

7) $m < -2$

8) $m \leqslant 1$

9) $x \geqslant 5$

10) $a \leqslant -5$

11) $b > -2$

12) $x > 1$

13) $x \geqslant 110: [110, \infty)$

14) $n \geqslant -26: [-26, \infty)$

15) $r < 1: (-\infty, 1)$

16) $m \leqslant -6: (-\infty, -6]$

17) $n \geqslant -6: [-6, \infty)$

18) $x < 6: (-\infty, 6)$

19) $a < 12: (-\infty, 12)$

20) $v \geqslant 1: [1, \infty)$

21) $x \geqslant 11: [11, \infty)$

22) $x \leqslant -18: (-\infty, -18]$

23) $k > 19: (19, \infty)$

24) $n \leqslant -10: (-\infty, -10]$

25) $p < -1: (-\infty, -1)$

26) $x \leqslant 20: (-\infty, 20]$

27) $m \geqslant 2: [2, \infty)$

28) $n \leqslant 5: (-\infty, 5]$

29) $r > 8: (8, \infty)$

30) $x \leqslant -3: (-\infty, -3]$

31) $b > 1: (1, \infty)$

32) $n \geqslant 0: [0, \infty)$

33) $v < 0: (-\infty, 0)$

34) $x > 2: (2, \infty)$

Answers - Chapter 4

4.1

Answers - Graphing

1) $(-1,2)$
2) $(-4,3)$
3) $(-1,-3)$
4) $(-3,1)$
5) No Solution
6) $(-2,-2)$
7) $(-3,1)$
8) $(4,4)$
9) $(-3,-1)$
10) No Solution
11) $(3,-4)$
12) $(4,-4)$
13) $(1,-3)$
14) $(-1,3)$
15) $(3,-4)$
16) No Solution
17) $(2,-2)$
18) $(4,1)$
19) $(-3,4)$
20) $(2,-1)$
21) $(3,2)$
22) $(-4,-4)$
23) $(-1,-1)$
24) $(2,3)$
25) $(-1,-2)$
26) $(-4,-3)$
27) No Solution
28) $(-3,1)$
29) $(4,-2)$
30) $(1,4)$

4.2

Answers - Substitution

1) $(1,-3)$
2) $(-3,2)$
3) $(-2,-5)$
4) $(0,3)$
5) $(-1,-2)$
6) $(-7,-8)$
7) $(1,5)$
8) $(-4,-1)$
9) $(3,3)$
10) $(4,4)$
11) $(2,6)$
12) $(-3,3)$
13) $(-2,-6)$
14) $(0,2)$
15) ([illegible]$,-5)$
16) $(-1,0)$
17) $(-1,8)$
18) $(3,7)$
19) $(2,3)$
20) $(8,-8)$
21) $(1,7)$
22) $(1,7)$
23) $(-3,-2)$
24) $(1,-3)$
25) $(1,3)$
26) $(2,1)$
27) $(-2,8)$
28) $(-4,3)$
29) $(4,-3)$
30) $(-1,5)$
31) $(0,2)$
32) $(0,-7)$
33) $(0,3)$
34) $(1,-4)$
35) $(4,-2)$
36) $(8,-3)$
37) $(2,0)$
38) $(2,5)$
39) $(-4,8)$
40) $(2,3)$

4.3

5) $(p+2)(p-2)$

6) $(2v+1)(2v-1)$

7) $(3k+2)(3k-2)$

8) $(3a+1)(3a-1)$

9) $3(x+3)(x-3)$

10) $5(n+2)(n-2)$

11) $4(2x+3)(2x-3)$

12) $5(25x^2+9y^2)$

13) $2(3a+5b)(3a-5b)$

14) $4(m^2+16n^2)$

15) $(a-1)^2$

16) $(k+2)^2$

17) $(x+3)^2$

18) $(n-4)^2$

19) $(x-3)^2$

20) $(k-2)^2$

21) $(5p-1)^2$

22) $(x+1)^2$

23) $(5a+3b)^2$

24) $(x+4y)^2$

25) $(2a-5b)^2$

26) $2(3m-2n)^2$

27) $2(2x-3y)^2$

28) $5(2x+y)^2$

29) $(2-m)(4+2m+m^2)$

30) $(x+4)(x^2-4x+16)$

31) $(x-4)(x^2+4x+16)$

32) $(x+2)(x^2-2x+4)$

33) $(6-u)(36+6u+u^2)$

34) $(5x-6)(25x^2+30x+36)$

35) $(5a-4)(25a^2+20a+16)$

36) $(4x-3)(16x^2+12x+9)$

37) $(4x+3y)(16x^2-12xy+9y^2)$

38) $4(2m-3n)(4m^2+6mn+9n^2)$

39) $2(3x+5y)(9x^2-15xy+25y^2)$

40) $3(5m+6n)(25m^2-30mn+36n^2)$

41) $(a^2+9)(a+3)(a-3)$

42) $(x^2+16)(x+4)(x-4)$

43) $(4+z^2)(2+z)(2-z)$

44) $(n^2+1)(n+1)(n-1)$

45) $(x^2+y^2)(x+y)(x-y)$

46) $(4a^2+b^2)(2a+b)(2a-b)$

47) $(m^2+9b^2)(m+3b)(m-3b)$

48) $(9c^2+4d^2)(3c+2d)(3c-2d)$

6.6

Answers - Factoring Strategy

1) $3(2a+5y)(4z-3h)$

2) $(2x-5)(x-3)$

3) $(5u-4v)(u-v)$

4) $4(2x+3y)^2$

5) $2(-x+4y)(x^2+4xy+16y^2)$

6) $5(4u-x)(v-3u^2)$

7) $n(5n-3)(n+2)$

8) $x(2x+3y)(x+y)$

9) $2(3u-2)(9u^2+6u+4)$

10) $2(3-4x)(9+12x+16x^2)$

11) $n(n-1)$

12) $(5x+3)(x-5)$

13) $(x-3y)(x-y)$

14) $5(3u-5v)^2$

15) $(3x+5y)(3x-5y)$

16) $(x-3y)(x^2+3xy+9y^2)$

17) $(m+2n)(m-2n)$

18) $3(2a+n)(2b-3)$

19) $4(3b^2+2x)(3c-2d)$

20) $3m(m+2n)(m-4n)$

21) $2(4+3x)(16-12x+9x^2)$

22) $(4m+3n)(16m^2-12mn+9n^2)$

23) $2x(x+5y)(x-2y)$

24) $(3a+x^2)(c+5d^2)$

25) $n(n+2)(n+5)$

26) $(4m-n)(16m^2+4mn+n^2)$

27) $(3x-4)(9x^2+12x+16)$

28) $(4a+3b)(4a-3b)$

29) $x(5x+2)$

30) $2(x-2)(x-3)$

31) $3k(k-5)(k-4)$

32) $2(4x+3y)(4x-3y)$

33) $(m-4x)(n+3)$

34) $(2k+5)(k-2)$

35) $(4x-y)^2$

36) $v(v+1)$

37) $3(3m+4n)(3m-4n)$

38) $x^2(x+4)$

39) $3x(3x-5y)(x+4y)$

40) $3n^2(3n-1)$

41) $2(m-2n)(m+5n)$

42) $v^2(2u-5v)(u-3v)$

6.7

Answers - Solve by Factoring

1) $7,-2$

2) $-4,3$

3) $1,-4$

4) $-\frac{5}{2},7$

5) $-5,5$

6) $4,-8$

7) $2,-7$

8) $-5,6$

9) $-\frac{5}{7},-3$

10) $-\frac{7}{8},8$

11) $-\frac{1}{5},2$

12) $-\frac{1}{2},2$

13) $4,0$

14) $8,0$

15) $1,4$

16) $4,2$

17) $\frac{3}{7},-8$

18) $-\frac{1}{7},-8$

19) $\frac{4}{7},-3$

20) $\frac{1}{4},3$

21) $-4,-3$

22) $8,-4$

23) $8,-2$

24) $4,0$

25) $\frac{8}{3},-5$

26) $-\frac{1}{2},\frac{5}{3}$

27) $-\frac{3}{7},-3$

28) $-\frac{4}{3},-3$

29) $-4,1$

30) $2,-3$

31) $-7,7$

32) $-4,-6$

33) $-\frac{5}{2},-8$

34) $-\frac{6}{5},-7$

33) $\frac{x^2+y^2}{xy}$

34) $\frac{2x-1}{2x+1}$

35) $\frac{(1-3x)^2}{x^2(x+3)(x-3)}$

36) $\frac{x+y}{xy}$

7.6

Answers - Proportions

1) $\frac{40}{3}=a$

2) $n=\frac{14}{3}$

3) $k=\frac{12}{7}$

4) $x=16$

5) $x=\frac{3}{2}$

6) $n=34$

7) $m=21$

8) $x=\frac{79}{8}$

9) $p=49$

10) $n=25$

11) $b=-\frac{40}{3}$

12) $r=\frac{36}{5}$

13) [illegible] $=\frac{5}{2}$

14) $n=\frac{32}{5}$

15) $a=\frac{6}{7}$

16) $v=-\frac{16}{7}$

17) $v=\frac{69}{5}$

18) $n=\frac{61}{3}$

19) $x=\frac{38}{3}$

20) $k=\frac{73}{3}$

21) $x=-8,5$

22) $x=-7,5$

23) $m=-7,8$

24) $x=-3,9$

25) $p=-7,-2$

26) $n=-$[illegible]$, 9$

27) $n=-1$

28) $n=-4,-1$

29) $x=-7,1$

30) $x=-1,3$

31) \$9.31

32) 16

33) 2.5 in

34) 12.1 ft

35) 39.4 ft

36) [illegible].1 m

37) T: 38, V: 57

38) J: 4 hr, S: 14 hr

39) \$8

40) C: 36 min,
K: 51 min

7.7

Answers - Solving Rational Equations

1) $-\frac{1}{2},\frac{2}{3}$

2) $-3,1$

3) 3

4) $-1,4$

5) 2

6) $\frac{1}{3}$

7) -1

8) $-\frac{1}{3}$

9) -5

10) $-\frac{7}{15}$

11) $-5,0$

12) $5,10$

13) $\frac{16}{3},5$

14) 2, 13

15) -8

16) 2

17) $-\frac{1}{5},5$

18) $-\frac{9}{5},1$

19) $\frac{3}{2}$

20) 10

21) $0, 5$

22) $-2, \frac{5}{3}$

23) $4, 7$

24) -1

25) $\frac{2}{3}$

26) $\frac{1}{2}$

27) $\frac{3}{10}$

28) 1

29) $-\frac{2}{3}$

30) -1

31) $\frac{13}{4}$

32) 1

33) -10

34) $\frac{7}{4}$

7.8

Answers - Dimensional Analysis

1) 12320 yd

2) 0.0073125 T

3) 0.0112 g

4) 135,000 cm

5) 6.1 mi

6) 0.5 yd^2

7) 0.435 km^2

8) 86,067,200 ft^2

9) 6,500,000 m^3

10) 239.58 cm^3

11) 0.0072 yd^3

12) 5.13 ft/sec

13) 6.31 mph

14) 104.32 mi/hr

15) 111 m/s

16) 2,623,269,600 km/yr

17) 11.6 lb/in^2

18) 63,219.51 km/hr^2

19) 32.5 mph; 447 yd/oz

20) 6.608 mi/hr

21)17280 pages/day; 103.4 reams/month

22) 2,365,200,000 beats/lifetime

23) 1.28 g/L

24) $3040

25) 56 mph; 25 m/s

26) 148.15 yd^3; 113 m^3

27) 3630 ft^2, 522,720 in^2

28) 350,000 pages

29) 15,603,840,000 ft^3/week

30) 621,200 mg; 1.42 lb

Answers - Chapter 8

8.1

Answers - Square Roots

1) $7\sqrt{5}$

2) $5\sqrt{5}$

3) 6

4) 14

42) $-18xz\sqrt[4]{4x^3yz^3}$

8.3

Answers - Adding Radicals

1) $6\sqrt{5}$

2) $-3\sqrt{6}-5\sqrt{3}$

3) $-3\sqrt{2}+6\sqrt{5}$

4) $-5\sqrt{6}-\sqrt{3}$

5) $-5\sqrt{6}$

6) $-3\sqrt{3}$

7) $3\sqrt{6}+5\sqrt{5}$

8) $-\sqrt{5}+\sqrt{3}$

9) $-8\sqrt{2}$

10) $-6\sqrt{6}+9\sqrt{3}$

11) $-3\sqrt{6}+\sqrt{3}$

12) $-2\sqrt{5}-6\sqrt{6}$

13) $-2\sqrt{2}$

14) $8\sqrt{5}-\sqrt{3}$

15) $5\sqrt{2}$

16) $-9\sqrt{3}$

17) $-3\sqrt{6}-\sqrt{3}$

18) $3\sqrt{2}+3\sqrt{6}$

19) $-12\sqrt{2}+2\sqrt{5}$

20) $-3\sqrt{2}$

21) $-4\sqrt{6}+4\sqrt{5}$

22) $-\sqrt{5}-3\sqrt{6}$

23) $8\sqrt{6}-9\sqrt{3}+4\sqrt{2}$

24) $-\sqrt{6}-10\sqrt{3}$

25) $2\sqrt[3]{2}$

26) $6\sqrt[3]{5}-3\sqrt[3]{3}$

27) $-\sqrt[4]{3}$

28) $10\sqrt[4]{4}$

29) $\sqrt[4]{2}-3\sqrt[4]{3}$

30) $5\sqrt[4]{6}+2\sqrt[4]{4}$

31) $6\sqrt[4]{3}-3\sqrt[4]{4}$

32) $-6\sqrt[4]{3}+2\sqrt[4]{6}$

33) $2\sqrt[4]{2}+\sqrt[4]{3}+6\sqrt[4]{4}$

34) $-2\sqrt[4]{3}-9\sqrt[4]{5}-3\sqrt[4]{2}$

35) $\sqrt[5]{6}-6\sqrt[5]{2}$

36) $\sqrt[7]{3}-6\sqrt[7]{6}+3\sqrt[7]{5}$

37) $4\sqrt[5]{5}-4\sqrt[5]{6}$

38) $-11\sqrt[7]{2}-2\sqrt[7]{5}$

39) $-4\sqrt[6]{4}-6\sqrt[6]{5}-4\sqrt[6]{2}$

8.4

Answers - Multiply and Divide Radicals

1) $-48\sqrt{5}$

2) $-25\sqrt{6}$

3) $6m\sqrt{5}$

4) $-25r^2\sqrt{2r}$

5) $2x^2\sqrt[3]{x}$

6) $6a^2\sqrt[3]{5a}$

7) $2\sqrt{3}+2\sqrt{6}$

8) $5\sqrt{2}+2\sqrt{5}$

11) 100

12) -74

13) $\frac{1}{5}$

14) 27

15) $-\frac{9}{26}$

16) $n^2 - 2n$

17) $-x^3 - 4x - 2$

18) $-x^3 + 2x^2 - 3$

19) $-x^2 - 8x + 2$

20) $2t^2 - 8t$

21) $4x^3 + 25x^2 + 25x$

22) $-2t^3 - 15t^2 - 25t$

23) $x^2 - 4x + 5$

24) $3x^2 + 4x - 9$

25) $\frac{n^2+5}{3n+5}$

26) $-2x + 9$

27) $\frac{-2a+5}{3a+5}$

28) $t^3 + 3t^2 -$ [illegible] $t + 5$

29) $n^3 + 8n + 5$

30) $\frac{4x+2}{x^2+2x}$

31) $n^6 - 9n^4 + 20n^2$

32) $18n^2 - 15n - 25$

33) $x + 3$

34) $-\frac{2}{3}$

35) $t^4 + 8t^2 + 2$

36) $\frac{3n-6}{-n^2-4n}$

37) $\frac{-x^3-2x}{-3x+4}$

38) $x^4 - 4x^2 - 3$

39) $\frac{-n^2-2n}{3}$

40) $\frac{32+23n-n^3}{8}$

41) -155

42) 5

43) 21

44) 4

45) 103

46) 12

47) -50

48) 112

49) 176

50) 14[illegible]

51) $16x^2 + 12x - 4$

52) $-8a + 14$

53) $-8a + 2$

54) t

55) $4x^3$

56) $-2n^2 - 12n - 16$

57) $-2x + 8$

58) $27t^3 - 108t^2 + 141t - 60$

59) $-16t - 5$

60) $3x^3 + 6x^2 - 4$

10.3

Answers - Inverse Functions

1) Yes

2) No

10.5

Answers - Logarithmic Functions

1) $9^2 = 81$

2) $b^{-16} = a$

3) $7^{-2} = \frac{1}{49}$

4) $16^2 = 256$

5) $13^2 = 169$

6) $11^0 = 1$

7) $\log_8 1 = 0$

8) $\log_{17} \frac{1}{289} = -2$

9) $\log_{15} 225 = 2$

10) $\log_{144} 12 = \frac{1}{2}$

11) $\log_{64} 2 = \frac{1}{6}$

12) $\log_{19} 361 = 2$

13) $\frac{1}{3}$

14) 3

15) $-\frac{1}{3}$

16) 0

17) 2

18) -3

19) 2

20) $\frac{1}{2}$

21) 6

22) 5

23) 5

24) 512

25) $\frac{1}{4}$

26) 1000

27) 121

28) 256

29) 6552

30) $\frac{45}{11}$

31) $-\frac{125}{3}$

32) $-\frac{1}{4}$

33) $-\frac{54}{11}$

34) $-\frac{2401}{3}$

35) $-\frac{1}{2}$

36) $-\frac{1}{11}$

37) $-\frac{621}{10}$

38) $\frac{283}{243}$

39) $\frac{2}{5}$

40) 3

10.6

Answers - Interest Rate Problems

1)

a. 740.12; 745.91

b. 851.11; 859.99

c. 950.08; 953.44

d. 1979.22; 1984.69

e. 1209.52; 1214.87

f. 1528.02; 1535.27

g. 2694.70; 2699.72

h. 3219.23; 3224.99

i. 7152.17; 7190.52

2) 1640.70

3) 2868.41

4) 2227.41

5) 1726.16

6) 1507.08

7) 2001.60

8) 2009.66

9) 2288.98

10) 6386.12

11) 13742.19

12) 28240.43

13) 12.02; 3.96

14) 3823.98

15) 101.68