Around the neighborhood of $x_0$, if $f''(x)$ is continuous at $x_0$, if $f''(x)$ changes to increasing at $x_0$, then $x_0$ is a local minimum.

Around the neighborhood of $x_0$, if $f''(x)$ is continuous at $x_0$, and if $f''(x)$ does not change orientation at $x_0$, then $x_0$ is a local minimum or a local maximum.

### Inflection Points

Inflection points may occur when:

1. $f''(x) = 0$ or $f''(x)$ does not exist.
2. $f''(x) > 0$ on an interval $I$, this indicates that $f(x)$ is concave up on $I$.
3. $f''(x) < 0$ on an interval $I$, this indicates that $f(x)$ is concave down on $I$.

(a) If $f'(x) = 0$ and $f''(x) < 0$, then $f(x)$ has a local maximum at $x_0$.
(b) If $f''(x) > 0$, then $f(x)$ has an inflection point at $x_0$.
(c) If $f''(x) < 0$, then $f(x)$ has an inflection point at $x_0$.
(d) If $f''(x) = 0$ or $f''(x)$ does not exist, the test fails and we cannot conclude if $x_0$ is a local minimum or a local maximum or an inflection point.

Guidelines for curve sketching can be found on page 249 of your textbook.

Example:

Sketch the graph of $f(x) = 3x^3 + 8x^2$.