We need to find values $a_1$ and $a_2$ such that
\[
\begin{bmatrix}
-1 \\
1
\end{bmatrix} = a_1 \begin{bmatrix}
2 \\
3
\end{bmatrix} + a_2 \begin{bmatrix}
1 \\
2
\end{bmatrix} = \begin{bmatrix}
2a_1 + a_2 \\
3a_1 + 2a_2
\end{bmatrix}.
\]

To be true, that vector based equation has to match at both coordinates. The first coordinate value gives us the equation $-1 = 2a_1 + a_2$, which can be easily converted into $a_2 = -1 - 2a_1$. The second is
\[
1 = 3a_1 + 2a_2 \quad \implies \quad 1 = 3a_1 - 2 - 4a_1 \quad \implies \quad 3 = -a_1
\]

leading to $a_1 = -3$ and $a_2 = 5$. The final answer is, basically, ‘yes’, though it is good to include
\[
-3 \begin{bmatrix}
2 \\
3
\end{bmatrix} + 5 \begin{bmatrix}
1 \\
2
\end{bmatrix} = \begin{bmatrix}
-6 + 5 \\
-9 + 10
\end{bmatrix} = \begin{bmatrix}
-1 \\
1
\end{bmatrix}.
\]

**Dot Products**

A dot product between two vectors $\mathbf{x}$ and $\mathbf{y}$ is written simply as $\mathbf{x} \cdot \mathbf{y}$ and calculated like so:
\[
\mathbf{x} \cdot \mathbf{y} = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} \cdot \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{bmatrix} = x_1y_1 + x_2y_2 + x_3y_3 + \cdots + x_ny_n.
\]

You multiply the equivalent terms in $\mathbf{x}$ and $\mathbf{y}$ then add them all up, resulting in a real number out of two vectors.

If we take the vector $\mathbf{v} = \begin{bmatrix}
3 \\
4
\end{bmatrix}$ in $\mathbb{R}^2$ we can calculate
\[
\mathbf{v} \cdot \mathbf{v} = 4 \times 4 + 3 \times 3 = 16 + 9 = 25.
\]

Why mention this? Well, consider a more geometric interpretation of $\mathbf{v}$. It is, in actual fact, the hypotenuse of a triangle with remaining lengths 4 and 3. By the Pythagorean theorem, that makes the length of $\mathbf{v}$ equal to the square root of
\[
4^2 + 3^2 = 16 + 9 = 25,
\]