The Rectangular (or uniform) Distribution

A continuous random variable $X$ having a probability density function $f(x)$ such that

$$f(x) = \frac{1}{b - a} \quad \text{for} \quad a \leq x \leq b$$

where $a$ and $b$ are constants, is said to follow a rectangular (or uniform) distribution.

If $X$ is distributed in this way, we write

$$X \sim R(a, b)$$

The graph of $y = f(x)$ is a straight line parallel to the $x$-axis.

The area of the rectangle must equal 1 and therefore we can write,

$$P(a \leq X \leq b) = 1$$

![Graph of $f(x)$](image)

$$f(x) = \frac{1}{b - a}$$

Area $= \frac{1}{b - a} (b - a) = 1$

Also,

$$\int_{a}^{b} f(x) \, dx = \int_{a}^{b} \frac{1}{b - a} \, dx = \frac{1}{b - a} [x]_{a}^{b} = \frac{1}{b - a} (b - a) = 1$$

Obviously, the graph for $f(x)$ depends on the values of $a$ and $b$.

For example, if $X \sim R(0, 4)$ then $f(x) = \frac{1}{4}$ if $0 \leq x \leq 4$.

Example 1 If $X \sim R(6, 9)$ find $P(7.2 \leq x \leq 8.4)$

$$X \sim R(6, 9), \quad 9.4 - 7.2 = 1.2 \quad \text{Simple area of a Square}$$

$$6 - 9 = \frac{3}{2}, \quad 1.2 \times \frac{3}{2} = 0.4$$

$$\boxed{\text{Area} = \text{Square (1), So } 3 \times x = 1,}$$

$$x = \frac{1}{3}$$

Mean and variance of a continuous uniform distribution

If $X \sim R(a, b)$ then

$$E[X] = \frac{1}{2} (a + b)$$

$$\text{Var}[X] = \frac{1}{12} (b - a)^2$$

Quotable from the formula booklet.

By symmetry, $E[X]$ is half-way between $a$ and $b$ so $E[X] = \frac{1}{2} (a + b)$.

Example 1 Given the graph of the continuous random variable $X$ is

Find,

a) the p.d.f. of $X$

b) $E[X]$

c) $\text{Var}[X]$

d) $P(X > 5)$

Solution

$$f(x)$$

$$\int_{a}^{b} f(x) \, dx = \int_{a}^{b} \frac{1}{b - a} \, dx = \frac{1}{b - a} [x]_{a}^{b} = \frac{1}{b - a} (b - a) = 1$$

$$\text{P.D.F.} = \frac{1}{6 - 3} = \frac{1}{3}, \text{for } 3 \leq x \leq 6$$

$$E(x) = \frac{1}{3} (a + b) = \frac{1}{3} (6 + 3) = \frac{9}{3} = 3$$

$$\text{Var}(x) = \frac{1}{12} (b - a)^2 = \frac{1}{12} (6 - 3)^2 = \frac{3}{4}$$

$$d) P(x > 5) = (6 - 5) \times \frac{1}{3} = \frac{5}{3}$$

PROOF FOR MEAN AND VARIANCE OF RECTANGULAR DISTRIBUTION

To come – may be asked for in exam.