The Rectangular (or uniform) Distribution

A continuous random variable X having a probability density function f(x) such that
\[ f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b \]
where a and b are constants, is said to follow a rectangular (or uniform) distribution.

If X is distributed in this way, we write
\[ X \sim R(a, b) \]

The graph of \( y = f(x) \) is a straight line parallel to the x-axis.
The area of the rectangle must equal 1 and therefore we can write,
\[ P(a \leq X \leq b) = 1 \]

\[ \begin{array}{c}
  f(x) = \frac{1}{b-a} \\
  \text{Area} = \left[ \frac{1}{b-a} \right] (b-a) = 1 \\
  \end{array} \]

\[ \int_a^b f(x) \, dx = \frac{1}{b-a} \left[ x \right]_a^b = \frac{1}{b-a} (b-a) = 1 \]

Obviously, the graph for f(x) depends on the values of a and b.
For example, if \( X \sim R(0, 4) \) then f(x) = \( \frac{1}{4} \) if 0 \leq x \leq 4.

Example 1
If \( X \sim R(6, 9) \) find \( P(7.2 \leq x \leq 8.4) \)
\[ \begin{array}{c}
  x \sim R(6, 9) \\
  9 - 7.2 = 1.2 \\
  1.2 \times \frac{1}{3} = 0.4 \\
  \end{array} \]

\[ \begin{array}{c}
  \text{Simple area of a Square} \\
  x = \frac{1}{3} \]

\[ \begin{array}{c}
  \text{Area} = \text{Square (1), So } 3 \times x = 1, \\
  \end{array} \]

Mean and variance of a continuous uniform distribution

If \( X \sim R(a, b) \) then
\[ E[X] = \frac{1}{2} (a + b) \]
\[ \text{Var}[X] = \frac{1}{12} (b - a)^2 \]

By symmetry, E[X] is half-way between a and b so E[X] = \( \frac{1}{2} (a + b) \).

Example 1 Given the graph of the continuous random variable X is
\[ \begin{array}{c}
  f(x) \\
  3 \quad 6 \\
  x \\
  \end{array} \]

Find,
\( a) \) the p.d.f. of X
\( b) \) E[X]
\( c) \) Var[X]
\( d) \) P(X > 5)

Solution
\[ \begin{array}{c}
  f(x) \\
  \frac{1}{6-3} = \frac{1}{3}, \text{ for } 3 \leq x \leq 6 \\
  \end{array} \]

\[ \begin{array}{c}
  a) \text{ P.D.F.} = \frac{1}{b-a} \\
  b) E(x) = \frac{1}{2} (a+b) = \frac{1}{2} (6+3) = 4.5 \\
  c) \text{ Var}(x) = \frac{1}{12} (b-a)^2 = \frac{1}{12} (6-3)^2 = \frac{3}{12} = \frac{1}{4} \\
  d) P(x > 5) = (6 - 5) \times \frac{1}{3} = \frac{1}{3} \]

PROOF FOR MEAN AND VARIANCE OF RECTANGULAR DISTRIBUTION

To come – may be asked for in exam.