Chapter 4

Applications of Derivatives

4.1

Extreme Values of Functions

DEFINITIONS

Let $f$ be a function with domain $D$. Then $f$ has an absolute
maximum value on $D$ at a point $c$ if

$$f(x) \leq f(c) \quad \text{for all } x \in D$$

and an absolute minimum value on $D$ at $c$ if

$$f(x) \geq f(c) \quad \text{for all } x \in D.$$

FIGURE 4.1 Absolute extrema for
the sine and cosine functions on
$[-\pi/2, \pi/2]$. These values can depend
on the domain of a function.
**Theorem 5**—Second Derivative Test for Local Extrema: Suppose \( f' \) is continuous on an open interval that contains \( x = c \).

1. If \( f'(c) = 0 \) and \( f''(c) < 0 \), then \( f \) has a local maximum at \( x = c \).
2. If \( f'(c) = 0 \) and \( f''(c) > 0 \), then \( f \) has a local minimum at \( x = c \).
3. If \( f'(c) = 0 \) and \( f''(c) = 0 \), then the test fails. The function \( f \) may have a local maximum, a local minimum, or neither.

### Interval 1

<table>
<thead>
<tr>
<th>( x &lt; 0 )</th>
<th>( 0 &lt; x &lt; 3 )</th>
<th>( 3 &lt; x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
<td>( x &lt; 0 )</td>
<td>( 0 &lt; x &lt; 2 )</td>
</tr>
<tr>
<td>Sign of ( f' )</td>
<td>decreasing</td>
<td>decreasing</td>
</tr>
<tr>
<td>Behavior of ( f )</td>
<td>concave up</td>
<td>concave down</td>
</tr>
</tbody>
</table>

### Interval 2

<table>
<thead>
<tr>
<th>( x &lt; 0 )</th>
<th>( 0 &lt; x &lt; 2 )</th>
<th>( 2 &lt; x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of ( f'' )</td>
<td>( 0 &lt; x &lt; 2 )</td>
<td>( 2 &lt; x )</td>
</tr>
<tr>
<td>Behavior of ( f )</td>
<td>concave up</td>
<td>concave down</td>
</tr>
</tbody>
</table>

**Figure 4.29** The graph of \( f(x) = x^4 - 4x^3 + 10 \) (Example 7).
FIGURE 4.34 This one-liter can uses the least material when \( h = 2r \) (Example 2).

FIGURE 4.35 The graph of \( A = 2\pi r^2 + \frac{2000}{r} \) is concave up.

FIGURE 4.36 The rectangle inscribed in the semicircle in Example 3.

FIGURE 4.37 A light ray refracted (deflected from its path) as it passes from one medium to a denser medium (Example 4).