§11.4 The Cross Product

1. Definition of \( \vec{a} \times \vec{b} \).

2. Basic properties:
   (a) \( \vec{a} \times \vec{a} = \vec{0} \)
   (b) \( \vec{a} \times \vec{b} = - (\vec{b} \times \vec{a}) \)
   (c) \( \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \)
   (d) \( (\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a} \)
   (e) \( c(\vec{a} \times \vec{b}) = (c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b}) \)

3. Additional properties:
   (a) \( \vec{a} \times \vec{b} \) is perpendicular to both \( \vec{a} \) and \( \vec{b} \). This is extremely useful.
   (b) \( ||\vec{a} \times \vec{b}|| \) = \( ||\vec{a}|| ||\vec{b}|| \sin \theta \)
   (c) \( \vec{a} \) and \( \vec{b} \) are parallel iff \( \vec{a} \times \vec{b} = \vec{0} \) but this is not a particularly good way to check.

§11.5 Lines in Space

1. Intro: What determines a line? What can we use for an equation? The fundamental way to define a line is to have a point on the line and a vector pointing along (parallel to) the line. If \( \vec{a} \hat{i} + \vec{b} \hat{j} + \vec{c} \hat{k} \) is parallel to the line and if the line contains the point \( P = (x_0, y_0, z_0) \) then:

2. Parametric Equations: \( x = x_0 + at \), \( y = y_0 + bt \) and \( z = z_0 + ct \). Each \( t \) gives a point on the line.

3. Vector Equation: \( \vec{r} = (x_0 + at) \hat{i} + (y_0 + bt) \hat{j} + (z_0 + ct) \hat{k} \). Each \( t \) gives a vector which points from the origin to a point on the line. This is unique since on any given line there are many points and many vectors pointing along the line.

4. Symmetric Equations: Solve for \( t \) in each of the parametric equations and set them all equal. If one doesn't have a \( t \) in it just leave it be. If two do not then leave those alone and don't even write the third because that variable could be anything.

5. If a line has point \( P \) and vector \( \vec{L} \) then the distance from another point \( Q \) to the line is \( \frac{||\vec{L} \times \vec{PQ}||}{||\vec{L}||} \).

§11.6 Planes in Space

1. A plane is determined by a point \( P = (x_0, y_0, z_0) \) and a normal vector \( \vec{N} = a \hat{i} + b \hat{j} + c \hat{k} \) which is perpendicular to the plane. A point \( Q = (x, y, z) \) is on the plane iff the vector from \( P \) to \( Q \) is perpendicular to \( \vec{N} \), meaning \( (a \hat{i} + b \hat{j} + c \hat{k}) \cdot \vec{PQ} = 0 \) which is \( a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \). This is often rearranged to get \( ax + by + cz = d \).

2. If a plane has point \( P \) and normal vector \( \vec{N} \) then the distance from another point \( Q \) to the plane is \( \frac{||\vec{N} \cdot \vec{PQ}||}{||\vec{N}||} \).

3. Sketching planes:
   - Those like \( ax + by + cz = d \), draw a little triangle using the intercepts.
   - Those like \( z = 0 \) or \( x = 2 \) or \( y = -3 \), parallel to the coordinate planes.
   - Those like \( 2x + y = 10 \), draw a line and extend in the direction of the missing variable.