INSTRUCTIONS: Answer question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) With respect to accumulation factors, explain the principle of consistency. 

(b) Define a nominal rate of interest

(c) What is the “Force of interest per unit time”? 

(d) The rate of interest is

\[ \delta(t) = \begin{cases} 
0.08t & \text{for } 0 \leq t < 5 \\
0.1 - 0.01t & \text{for } 5 \leq t < 10 
\end{cases} \]

Find an accumulation factor from time 0 to time \( t \).

(e) Given that \( \ddot{a}_n = 7.029584 \) and \( \ddot{a}_2n = 10.934563 \), find the rate of interest \( i \) and \( n \).

(f) An individual wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of £12,000 per annum. The payments increase at each anniversary by 3% per annum. The individual would like to buy the annuity with a single premium 10 years from now. Calculate the single premium required in 10 years’ time to purchase the annuity assuming an interest rate of 6% per annum effective.

(g) Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semi-annually. At the same time, Peter deposits 100 into a separate account. Peter’s account is credited interest at a force of interest of \( \delta \). After 7.25 years, the value of each account is the same. Calculate \( \delta \).

(h) An amount \( X \) is deposited in an account that grows interest at an annual effective rate of interest 6%. Another amount \( \frac{X}{2} \) is deposited in another account that earns interest at an annual effective rate of discount \( d \). After 10 years, the total interest earned by both accounts is equal. Find \( d \).