1. Let $ABC$ be an acute-angled triangle; $AD$ be the bisector of $\angle BAC$ with $D$ on $BC$; and $BE$ be the altitude from $B$ on $AC$. Show that $\angle CED > 45^\circ$.

**Solution:**
Draw DL perpendicular to $AB$; $DK$ perpendicular to $AC$; and $DM$ perpendicular to $BE$. Then $EM = DK$. Since $AD$ bisects $\angle A$, we observe that $\angle BAD = \angle KAD$. Thus in triangles $ALD$ and $AKD$, we see that $\angle LAD = \angle KAD$; $\angle AKD = 90^\circ = \angle ALD$; and $AD$ is common. Hence triangles $ALD$ and $AKD$ are congruent, giving $DL = DK$. But $DL > DM$, since $BE$ lies inside the triangle(by acuteness property). Thus $EM > DM$. This implies that $\angle EDM > \angle DEM = 90^\circ - \angle EDM$. We conclude that $\angle EDM > 45^\circ$. Since $\angle CED = \angle EDM$, the result follows.

**Alternate Solution:**
Let $\angle BAC = \theta$. We have $CD = ab/(b + c)$ and $CE = a \cos C$. Using sine rule in triangle $CED$, we have

$$\frac{CD}{\sin \theta} = \frac{CE}{\sin(C + \theta)}.$$  

This reduces to

$$(b + c) \sin \theta \cos C = b \sin C \cos \theta + b \cos C \sin \theta.$$  

Simplification gives $c \sin \theta \cos C = b \sin C \cos \theta$ so that

$$\tan \theta = \frac{b \sin C}{c \cos C} = \frac{\sin B}{\cos C} = \frac{\sin B}{\sin(\pi/2 - C)}.$$  

Since $ABC$ is acute-angled, we have $A < \pi/2$. Hence $B + C > \pi/2$ or $B > (\pi/2) - C$. Therefore $\sin B > \sin(\pi/2 - C)$. This implies that $\tan \theta > 1$ and hence $\theta > \pi/4$.

2. Let $a, b, c$ be three natural numbers such that $a < b < c$ and $\gcd(c - a, c - b) = 1$. Suppose there exists an integer $d$ such that $a + d, b + d, c + d$ form the sides of a right-angled triangle. Prove that there exist integers $l, m$ such that $c + d = l^2 + m^2$.

**Solution:**
We have

$$(c + d)^2 = (a + d)^2 + (b + d)^2.$$  

This reduces to

$$d^2 + 2d(a + b - c) + a^2 + b^2 - c^2 = 0.$$  
