ARDEN'S THEOREM

Arden's Theorem

In order to find out a regular expression of a Finite Automaton, we use Arden’s Theorem along with the properties of regular expressions.

Statement –

Let $P$ and $Q$ be two regular expressions.

If $P$ does not contain null string, then $R = Q \&plus; RP$ has a unique solution that is $R = QP^*$

Proof –

$R = Q + Q \&plus; RP$ [After putting the value $R = Q \&plus; RP$]

= $Q \&plus; QP &plus; RPP$

When we put the value of $R$ recursively again and again, we get the following equation –

$R = Q \&plus; QP + QP^2 + QP^3$.....

$R = Q (\epsilon + P + P^2 + P^3 + ....)$

$R = QP^*$ [As $P^*$ represents $(\epsilon + P + P^2 + P^3 + ....)$ ]

Hence, proved.

Assumptions for Applying Arden’s Theorem –

- The transition diagram must not have NULL transitions
- It must have only one initial state

Method

Step 1 – Create equations as the following form for all the states of the DFA having $n$ states with initial state $q_1$.

$q_1 = q_1R_{11} + q_2R_{21} + ... + q_nR_{n1} + \epsilon$

$q_2 = q_1R_{12} + q_2R_{22} + ... + q_nR_{n2}$

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$q_n = q_1R_{1n} + q_2R_{2n} + ... + q_nR_{nn}$

$R_{ij}$ represents the set of labels of edges from $q_i$ to $q_j$, if no such edge exists, then $R_{ij} = \emptyset$

Step 2 – Solve these equations to get the equation for the final state in terms of $R_{ij}$

Problem

Construct a regular expression corresponding to the automata given below –