Inverse Functions

Inverse Function
Def. if \( f \) is a 1-1 function with domain \( D \) and range \( R \). Then the inverse function of \( f \) (denoted by \( f^{-1} \)) is the function with domain \( R \) and range \( D \) defined by:

\[
f^{-1}(b) = a \text{ if and only if } f(a) = b\]

where \( a, b \in \mathbb{R} \)
Inverse Functions

Properties of the inverse function by relating it to its original function.

Property 1:

\[ D_f = R_{f^{-1}} \quad \text{and} \quad R_f = D_{f^{-1}} \]

"The range of the function is the domain of the inverse, and the range of the inverse is the domain of the function."

Property 2: Inverse Composition Rule

\[ (f^{-1} \circ f)(x) = f^{-1}(f(x)) = x \]; for every \( x \) in the domain of \( f \)

\[ (f \circ f^{-1})(x) = f(f^{-1}(x)) = x \]; for every \( x \) in the domain of \( f^{-1} \)