Prove that the matrix
\[
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}
\]
is diagonalizable if \(-4bc < (a - d)^2\) and is not diagonalizable if \(-4bc > (a - d)^2\)

**Proof:**

To prove that the matrix \(A\) is diagonalizable, we will use a theorem that if an \(n \times n\) matrix \(A\) has \(n\) distinct eigenvalues, then \(A\) is diagonalizable. Now the characteristic polynomial of \(A\) is given by,

\[
|\lambda I - A| = \begin{vmatrix} \lambda - a & -b \\ -c & \lambda - d \end{vmatrix}
= (\lambda - a)(\lambda - d) - (bc)
= \lambda^2 + (-d - a)\lambda + (ad - bc)
\]

So by applying the quadratic formula, we get the roots of the characteristic polynomial of \(A\). i.e.

\[
\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

If we solve for \(b^2 - 4ac\) we get,

\[
b^2 - 4ac = (-b)^2 - 4(ad - bc)
= a^2 + 2ad + a^2 - 4ad + 4bc
= a^2 - 2ad + a^2 + 4bc
= (a - d)^2 + 4bc
\]

Now if \(b^2 - 4ac > 0\) i.e. \((a - d)^2 + 4bc > 0\) then the characteristic polynomial of \(A\) has 2 distinct real roots which implies that \(A\) has two distinct real eigenvalues. So, \(A\) is diagonalizable if \(-4bc < (a - d)^2\) while if \((a - d)^2 < -4bc\) then the characteristic polynomial has imaginary roots and \(A\) is not diagonalizable.