From these diagrams it is clear that \( A - B \neq B - A \), if \( A \neq B \).
If \( A = \{1, 2, 3, 4, 5, 6\} \), \( B = \{2, 4, 6, 8, 10\} \) then \( A - B = \{1, 3, 5\} \),
\( B - A = \{8, 10\} \)

Some properties of difference operation are as under:

1. \( U - A = A' \) \hspace{0.5cm} (quite obvious!)

2. \( A \subseteq B \Rightarrow A - B = \emptyset \)
   
   \[ A - B = \{x \mid x \in A \text{ and } x \notin B\} \]
   
   \[ = \{x \mid x \in A \text{ and } x \in B'\} \]
   
   \[ = \{x \mid x \in B \text{ and } x \notin B'\} \] \hspace{1cm} (\( A \subseteq B \))
   
   \[ = \emptyset \]

3. Symmetric Difference set: For sets \( A, B \in P(U) \), the set consisting of
   all elements which are in the set \( A \) or in the set \( B \), but not in both is called
   the symmetric difference of the sets \( A \) and \( B \). Symmetric difference of two
   sets is denoted by \( A \Delta B \).
   
   Thus \( A \Delta B = (A \cup B) - (A \cap B) \)
   
   We will prove that, \( A \Delta B = A - B \cup (B - A) \)
   
   \[ (A \cup B) - (A \cap B) = (A \cup B) \cap (A' \cap B') \] \hspace{1cm} (\( A - B = A \cap B' \))
   
   \[ = (A \cup B') \cap (A' \cup B') \] \hspace{1cm} (De Morgan’s law)
   
   \[ = (A \cup B) \cap (A \cup B') \cap (A' \cup B') \] \hspace{1cm} (Distributive law)
   
   \[ = [ (A \cap A') \cup (B \cap A') ] \cup [ (A \cap B') \cup (B \cap B') ] \]
   
   \[ = [ \emptyset \cup (B \cap A') ] \cup [ (A \cap B') \cup \emptyset ] \]
   
   \[ = (B \cap A') \cup (A \cap B') \]
   
   \[ = (B - A) \cup (A - B) \]
   
   \[ = (A - B) \cup (B - A) \]

This set is depicted below by the coloured region in the Venn diagram 2.6.

![Venn Diagram](image-url)
Just as we have ordered pairs \((x, y)\), we can have ordered triplet or ordered \(n\)-tuple \((x_1, x_2, x_3, \ldots, x_n)\). If \(A, B\) and \(C\) are non-empty sets, their cartesian product is defined as

\[
A \times B \times C = \{ (x, y, z) \mid x \in A, y \in B, z \in C \}
\]

In analogy with \(A^2 = A \times A\), we will write \(A^3 = A \times A \times A\).

**Example 12**: Find \(A \times B\), if \(A = \{a, b, c\}\) and \(B = \{a, b\}\).

**Solution**: \(A \times B = \{(a, a), (a, b), (b, a), (b, b), (c, a), (c, b)\}\)

**Example 13**: If \(A = \{1, 2, 3\}\), \(B = \{2, 6, 7\}\), \(C = \{2, 7\}\), verify \(A \times (B \setminus C) = (A \times B) \setminus (A \times C)\).

**Solution**: Here \(B \setminus C = \{6\}\)

\[
\therefore A \times (B \setminus C) = \{(1, 6), (2, 6), (3, 6)\}
\]

Now \(A \times B = \{(1, 2), (1, 6), (1, 7), (2, 2), (2, 6), (2, 7), (3, 2), (3, 6), (3, 7)\}\)

\[
A \times C = \{(1, 2), (1, 7), (2, 2), (2, 7), (3, 2), (3, 6), (3, 7)\}
\]

\[
\therefore (A \times B) \setminus (A \times C) = \{(1, 6), (2, 6), (3, 6)\}
\]

Thus, \(A \times (B \setminus C) = (A \times B) \setminus (A \times C)\)

**Example 14**: If \(A \neq \emptyset\) and \(A \times B = A \times C\), prove that \(B = C\).

**Solution**: If \(B = C = \emptyset\), then \(A \times \emptyset = A \times \emptyset = \emptyset\) and \(B = C\).

Obviously, only one of \(A \times B\) and \(A \times C\) being empty is not possible since \(A \neq \emptyset\).

Suppose \(A \neq \emptyset\), \(C \neq \emptyset\).

Since \(A \neq \emptyset\), there exists \((x, y)\) such that \(x \in A\).

\[
\therefore \quad \text{For every } y \in B, (x, y) \in A \times B
\]

\[
\therefore \quad (x, y) \in A \times C
\]

\[
(A \times B = A \times C)
\]

\[
\therefore \quad x \in A, y \in C
\]

Hence, \(\forall y, y \in B \Rightarrow y \in C\)

\[
\therefore \quad B \subseteq C
\]

Similarly, it can be proved that \(C \subseteq B\)

\[
\therefore \quad B = C
\]

**Example 15**: \(A = \{1, 2, 3, 4\}\), \(B = \{(a, b) \mid b \text{ divisible by } a; a, b \in A\}\).

List elements of \(B\).

**Solution**: Here, \(1, 2, 3, 4\) are divisible by 1; \(2, 4\) are divisible by 2; \(3\) is divisible by 3; and \(4\) is divisible by 4.

\[
\therefore \quad B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}
\]

**Example 16**: If \(A \times A = B \times B\), then prove that \(A = B\).

**Solution**: If \(A = \emptyset\), then \(\emptyset = B \times B \Rightarrow B = \emptyset\). This shows that \(A = B\).

Suppose \(A \neq \emptyset\). Let \(x \in A\)

\[
\therefore \quad (x, x) \in A \times A
\]
(4) Given that \( B \subseteq A \)

\[ \therefore \ A \cap B = B \]

Now from the result (3) proved above we have

\[ n(A - B) = n(A) - n(A \cap B) \]

\[ = n(A) - n(B) \quad (A \cap B = B) \]

(5) \( A \cap A' = \emptyset \) and \( A \cup A' = U \)

\[ \therefore \ n(U) = n(A) + n(A') \]

\[ \therefore \ n(A') = n(U) - n(A) \]

Note that if \( A \subseteq B \) then \( n(A) \leq n(B) \).

For finite sets \( A \) and \( B \), \( n(A \times B) = n(A) \cdot n(B) \).

**Example 19**: If \( A = \{1, 2, 3, 4\} \), \( B = \{2, 4\} \), verify \( n(A \times B) = n(A) \cdot n(B) \).

**Solution**: Let \( A = \{1, 2, 3, 4\} \), \( B = \{2, 4\} \)

Here \( A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4), (4, 2), (4, 4)\} \)

\[ \therefore \ n(A \times B) = 8 \]

Also \( n(A) = 4, n(B) = 2, n(A \times B) = 8 \)

\[ \therefore \ n(A \times B) = n(A) \cdot n(B) \]

**Example 20**: \( A \) and \( B \) are non-singleton and \( n(A \times B) = 21 \). Also \( A \subseteq B \). Find \( n(A) \) and \( n(B) \).

**Solution**: \( n(A \times B) = 21 = 3 \times 7 = 1 \times 21 \)

But, \( n(A) \neq 1, n(B) \neq 1 \)

\[ \therefore \ n(A) = 3 \text{ and } n(B) = 7 \text{ or } n(A) = 7 \text{ and } n(B) = 3. \]

But, \( n(A) \leq n(B) \quad (A \subseteq B) \)

\[ \therefore \ n(A) = 3, n(B) = 7 \]

**Example 21**: In a troop of 20 dancers performing Bharatnatyam or Kuchipudi, 12 dancers perform Bharatnatyam and 4 perform both Bharatnatyam and Kuchipudi. Find the number of dancers performing Kuchipudi.

(Note: Each dancer performs Bharatnatyam or Kuchipudi)

**Solution**: Let \( A = \text{Set of dancers performing Bharatnatyam} \)

\( B = \text{Set of dancers performing Kuchipudi} \)

Then given that, \( n(A) = 12, n(A \cap B) = 4, n(A \cup B) = 20 \)

Now, \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)
\[ \because \ 20 = 12 + n(B) - 4 \]
\[ 20 = n(B) + 8 \]
\[ \therefore n(B) = 12 \]
Thus number of dancers performing Kuchipudi is 12.
\[ \because \ \text{Number of Dancers performing only Kuchipudi} = n(B) - n(A \cap B) \]
\[ = 12 - 4 = 8 \]

**Example 22**: In a group of people, 28 like Gujarati movies, 30 like Hindi movies, 42 like English movies; 5 like both Gujarati and Hindi movies, 8 like Hindi and English movies, 8 like Gujarati and English movies and 3 like Gujarati, Hindi and English movies. What is the least number of people in the group?

**Solution**:
Here \( G \) = the set of people who like Gujarati movies
\( H \) = the set of people who like Hindi movies
\( E \) = the set of people who like English movies
then given that
\[ n(G) = 28, \ n(H) = 30, \ n(E) = 42 \]
\[ n(G \cap H) = 5, \ n(E \cap H) = 8, \ n(G \cap E) = 8, \ n(G \cap E \cap H) = 3 \]
Now, \[ n(G \cup E \cup H) = n(G) + n(E) + n(H) - n(G \cap E) - n(G \cap H) - n(E \cap H) + n(G \cap E \cap H) \]
\[ = 8 + 50 + 42 - 5 - 8 - 8 + 3 \]
\[ = 103 - 21 = 82 \]

Some persons may not like to watch a movie. Hence there are at least 82 people in the group.

**Miscellaneous Problems**

**Example 23**: If \( A \cap B = A \cap C \), \( A \cup B = A \cup C \), then prove that \( B = C \) \((B \neq \emptyset, C \neq \emptyset)\)

**Method 1**: Let \( x \in B \)
\[ \therefore x \in A \cup B \]
\[ \therefore x \in A \cup C \]
there are two possibilities
(1) \( x \in A \) or (2) \( x \in C \)
(1) \( x \in A \)
\[ \therefore \quad B \not\subset C \text{ is not true.} \]
\[ \therefore \quad B \subset C. \]
Similarly \( C \subset B \).
Hence \( B = C \).

**Example 25**: Prove that \( P(A) = P(B) \Rightarrow A = B \)

**Solution**: \( A \subset A \Rightarrow A \in P(A) \)
\[ \Rightarrow A \in P(B) \]
\[ \Rightarrow A \subset B \]
(\( P(A) = P(B) \))

Similarly \( B \subset A \).
\[ \therefore \quad A = B \]

**Example 26**: \( n(A \times A) = 9 \). \((a, b) \in A \times A \text{. Also } c \in A \text{. Write the set } A \).

**Solution**: Let \( n(A) = k \)
Now, \( n(A \times A) = k^2 = 9 \)
\[ \therefore \quad k = 3 \]
\((a, b) \in A \times A \)
\[ \therefore \quad a \in A, \ b \in A \]
Further \( c \) is given that \( c \in A \).
Thus \( A \) has 3 elements, namely \( a, b, c \).
\[ \therefore \quad A = \{a, b, c\} \]

**Example 27**: \( A \cap B = \emptyset \) and \( A \cup B = U \), prove that \( A' = B \).

**Solution**: Let \( x \in B \)
\[ \therefore \quad x \notin A \text{ as } A \cap B = \emptyset \]
\[ \therefore \quad x \in A' \quad \text{(i)} \]
Let \( x \in A' \)
\[ \therefore \quad x \notin A \]
but \( x \in U \)
\[ \therefore \quad x \in A \cup B \quad \text{(A \cup B = U)} \]
\[ \therefore \quad x \in A \text{ or } x \in B \]
\[ \therefore \quad x \in B \quad \text{(x \notin A)} \]
\[ \therefore \quad A' \subset B \quad \text{(ii)} \]
From (i) and (ii) \( A' = B \).