\[ f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} \ldots + \frac{f^n(0)x^n}{n!} \]

First term of \( e^x, \cos x, \ln(1 + x) \) and \( \sin x \) are in the formula book and can be derived using the general formula.

Reduction formula is the general integration expression, by integrating by parts so \( I_n \) is in terms of \( I_{n-1} \) and/or \( I_{n-2} \).

The sum can be approximate by using rectangles area as the terms, a line above and below give the limits.

**Numerical Methods**

An iteration equation, \( x_{n+1} = F(x_n) \) will either converge or diverge to a root of the equation \( x = F(x) \).

Convergence occurs by either staircase or cobweb.

Each iteration there is an error between the root \( \alpha \), when it converges these errors get successively smaller:

- \( e_{n+1} \approx f'(\alpha)e_n \) where \( f'(\alpha) \neq 0 \)
- \( e_{n+1} \propto e_n^2 \) where \( f'(\alpha) = 0 \)

Newton Raphson uses quadratic order convergence, hence quickly tends to the root.

Sums of areas of rectangles above for upper limit and below for lower limit the curve can approximate the integral.

The more rectangles the less deviation from the true value.

The limits of integral or summation depend on the nature of the curve i.e. whether it’s increasing or a decreasing function.

![Converging Staircase](image1)

![Converging Spiral](image2)

![Diverging Staircase](image3)

It takes the tangent to the curve at a point then uses the tangents x intercept as the next value for the iteration.

Start with \( x_n \), go to the curve at this point, construct tangent using \( f'(x_n) \) as gradient and formulate line equation, substitute \( y = 0 \) to find where it cuts the x-axis which becomes the new x value, \( x_{n+1} \):

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]