Applications of Integrals

**Net Area**: \( \int_a^b f(x) \, dx \) represents the net area between \( f(x) \) and the \( x \)-axis with area above \( x \)-axis positive and area below \( x \)-axis negative.

**Area Between Curves**: The general formulas for the two main cases for each are,

\[
\text{if } y = f(x) \Rightarrow A = \int_a^b [\text{upper function}] - [\text{lower function}] \, dx \\
\text{and } x = f(y) \Rightarrow A = \int_c^d [\text{right function}] - [\text{left function}] \, dy
\]

If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.

**Volumes of Revolution**: The two main formulas are \( V = \pi \int_a^b [(f(x))^2 - (g(x))^2] \, dx \) and \( V = \int_a^b A(y) \, dy \). Here is some general information about each method of computing and some examples.

### Rings

\[
A = \pi \left( \text{outer radius}^2 - \text{inner radius}^2 \right)
\]

**Limits**: \( x/y \) of right/bot ring to \( x/y \) of left/top ring

**Horz. Axis use** \( f(x), g(x), A(x) \) and \( dx \)

**Vert. Axis use** \( f(y), g(y), A(y) \) and \( dy \)

### Cylinders

\[
A = 2\pi \text{(radius)} \cdot \text{(width/height)}
\]

**Limits**: \( x/y \) of inner cyl. to \( x/y \) of outer cyl.

**Horz. Axis use** \( f(x), g(x), A(x) \) and \( dx \)

**Vert. Axis use** \( f(y), g(y), A(y) \) and \( dy \)

#### Ex. Axis: \( y = a > 0 \)

\[
\text{outer radius : } a - f(x) \\
\text{inner radius : } a - g(x)
\]

#### Ex. Axis: \( y = a \leq 0 \)

\[
\text{outer radius : } |a| + g(x) \\
\text{inner radius : } |a| + f(x)
\]

#### Ex. Axis: \( y = a > 0 \)

\[
\text{radius : } a - y \\
\text{width : } f(y) - g(y)
\]

#### Ex. Axis: \( y = a \leq 0 \)

\[
\text{radius : } |a| + y \\
\text{width : } f(y) - g(y)
\]

These are only a few cases for horizontal axis of rotation. If axis of rotation is the \( x \)-axis use the \( y = a \leq 0 \) case with \( a = 0 \). For vertical axis of rotation \( (x = a > 0 \text{ and } x = a \leq 0) \) interchange \( x \) and \( y \) to get appropriate formulas.