\[ p = \frac{dF_\perp}{dA} \]  \hspace{1cm} \text{(definition of pressure)}

If the pressure is the same at all points of a finite plane surface with area \( A \), then

\[ p = \frac{F_\perp}{A}, \]

where \( F_\perp \) is the net normal force on one side of the surface. The SI unit of pressure is the \textit{pascal}, where

\[ 1 \text{ pascal} = 1 \text{ Pa} = N/m^2 \]

Two related units, used principally in meteorology, are the \textit{bar}, equal to \( 10^5 \) Pa, and the \textit{millibar}, equal to \( 10^3 \) Pa. \textbf{Atmospheric pressure} \( P_a \) is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 \textit{atmosphere} (atm), defined to be exactly 101,325 Pa. To four significant figures,

\[ (P_a)_{av} = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \text{ bar} = 1013 \text{ millibar} = 14.70 \text{lb/in.}^2 \]

\textbf{Note:} Don't confuse pressure and force. In everyday language the words "pressure" and "force" mean pretty much the same thing. In fluid mechanics, however, these words describe distinct quantities with different characteristics. Fluid pressure acts perpendicular to any surface in the fluid, no matter how that surface is oriented, as shown in figure.

Hence pressure has no intrinsic direction of its own; it's a scalar. By contrast, force is a vector with a definite direction. Remember, too, that pressure is force per unit area. As shown in figure, a surface with twice the area has twice as much force exerted on it by the fluid, so the pressure is the same.

\subsection*{1.4 Pressure, Depth, and Pascal's Law}

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. We used that approximation in our discussion of bulk stress and strain. But often the fluid's weight is not negligible. Atmospheric pressure is less at high altitude than at sea level, which is why an
The wave propagating in the fluid-filled tube at time intervals of 1/8 of a period, for a total time of one period. The pattern of compressions and rarefactions moves steadily to the right, just like the pattern of crests and troughs in a sinusoidal transverse wave. Each particle in the fluid oscillates in SHM parallel to the direction of wave propagation (that is, left and right) with the same amplitude A and period T as the piston. The particles shown by the two red dots are one wavelength apart, and so oscillate in phase with each other.

Just like the sinusoidal transverse wave, in one period T the longitudinal wave travels one wavelength λ to the right. Hence the fundamental equation \( v = \lambda f \) holds for longitudinal waves as well as for transverse waves, and indeed for all types of periodic waves. Just as for transverse waves, in this chapter and the next we will consider only situations in which the speed of longitudinal waves does not depend on the frequency.

1.4 Wave Function for a sinusoidal wave

Let's see how to determine the form of the wave function for a sinusoidal wave. Suppose a sinusoidal wave travels from left to right (the direction of increasing \( x \)) along the string. Every particle of the string oscillates with simple harmonic motion with the same amplitude and frequency. But the oscillations of particles at different points on the string are not all in step with each other. The particle at point B is at its maximum positive value of \( y \) at \( t = 0 \) and returns to \( y = 0 \) at \( t = 2/8 \) T; these same events occur for a particle at point A or point C at \( t = 4/8 \) T and \( t = 6/8 \) T, exactly one half-period later. For any two particles of the string, the motion of the particle on the right (in terms of the wave, the "downstream" particle) lags behind the motion of the particle on the left by an amount proportional to the distance between the particles. Hence the cyclic motions of various points on the string are out of step with each other by various fractions of a cycle. We call these differences phase differences, and we say that the phase of the motion is different for different points. For example, if one point has its maximum positive displacement at the same time that another has its maximum negative displacement, the two are a half cycle out of phase. (This is the case for points A and B, or points B and C.)

Suppose that the displacement of a particle at the left end of the string (\( x = 0 \)), where the wave originates, is given by –

\[
y(x = 0, t) = A \sin \omega t = A \sin 2\pi ft.
\]

That is, the particle oscillates in simple harmonic motion with amplitude A, frequency \( f \), and angular frequency \( \omega = 2\pi f \). The notation \( y(x = 0, t) \) reminds us that the motion of this particle is a special case of the wave function \( y(x, t) \) that describes the entire wave. At \( t = 0 \) the particle at \( x \)
A simple pendulum is an idealized model consisting of a point mass suspended by a mass less, unstretchable string. When the point mass is pulled to one side of its straight-down equilibrium position and released, it oscillates about the equilibrium position. Familiar situations such as a wrecking ball on a crane’s cable or a person on a swing can be modeled as simple pendulums. The path of the point mass (sometimes called a pendulum bob) is not a straight line but the arc of a circle with radius L equal to the length of the string. We use as our coordinate the distance x measured along the arc. If the motion is simple harmonic, the restoring force must be directly proportional to x or (because $x = L\theta$) to $\theta$ Is it?

We represent the forces on the mass in terms of tangential and radial components. The restoring force $F$ is the tangential component of the net force:

$$F = -mg \sin \theta$$

The restoring force is provided by gravity; the tension T merely acts to make the point mass move in an arc. The restoring force is proportional not to $\theta$ but to sine, so the motion is not simple harmonic. However, if the angle $\theta$ is small, sine is very nearly equal to $\theta$ in radians. For example, when $\theta = 0.1 \text{ rad}$ (about 6°), $\sin \theta = 0.0998$, a difference of only 0.2%. With this approximation,

$$F = -mg\theta = -ma \sin \theta$$

The restoring force is then proportional to the coordinate for small displacements, and the force constant is $k = mg/L$. The angular frequency $\omega$ of a simple pendulum with small amplitude is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}} \quad \text{(Simple pendulum, small amplitude)}$$

The corresponding frequency and period relations are

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad \text{(Simple pendulum, small amplitude)}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}} \quad \text{(Simple pendulum, small amplitude)}$$

Note that these expressions do not involve the mass of the particle. This is because the restoring force, a component of the particle’s weight, is proportional to $m$. Thus the mass appears on both sides of $\sum F = m \ddot{a}$ and cancels out. (This is the same physics that explains why bodies of different masses fall with the same acceleration in a vacuum.) For small oscillations, the period of