the common base $c$. Thus,

$$A_3 = \frac{1}{2}c(d_l + d_r) + \frac{1}{2}b(h_l + h_r)$$

Substitution of $H$ for $h_l + h_r$ and $D$ for $d_l + d_r$ reduces the relation to

$$A_3 = \frac{c D}{2} + \frac{b H}{4}$$

(6-4)

Another convenient formula for the area of a regular three-level section is found by extending the side slopes to an intersection at the center line to form a triangle, called the grade triangle, below (or above) the base. The dimensions of the grade triangle are constant until the base or slope changes; its fixed area is $b^2/4$. Consequently, the “grade-triangle formula” is

$$A_3 = \frac{D}{2} \left( c + \frac{b}{2} \right) - \frac{b^2}{4}$$

(6-4)

Formula 6-5 is slightly more convenient than 6-4 for computing a long series of three-level sections in cut or fill, owing to the constant terms $b/2$ and $b^2/4$.

The special three-level section having a grade point at the ground surface (as at $D$ or $G$, Fig. 6-1) is also determined by formula 6-4. One of the four triangles disappears; therefore, $D = d_l$ (or $d_r$) + $\frac{1}{2}b$, and $H = h_l$ or $h_r$. The grade-triangle formula also applies if properly modified, but its use is not recommended at the transition between cut and fill.

At a side-hill section (as at $E$) the end areas for cut and fill are kept separate. Obviously, both are triangles. In the general case, with the grade point not at the center line, each area is

$$A_T = \frac{1}{2}w h$$

(6-6)

where $w$ is the actual base width of the triangle. At section $E$ in Fig. 6-3, $w = h/2$.

The area of a five-level section (as at $B$) is found by combining the indicated triangles having common bases. The final relation is

$$A_5 = \frac{1}{2}(c + f_l)(d_l + f_r)$$

(6-7)

If the section is in cut, $c_l$ and $c_r$ are substituted for $f_l$ and $f_r$.

6-6. End Areas by Coordinates

The area of an irregular section, such as $A$ in Fig. 6-1, is best found by a coordinate method. The procedure takes the origin of coordinates at the
a certain direction than is indicated on the plans. Again, there may be one fill that acts as a bottle-neck. Building it ahead of schedule, possibly by using extra borrow or longer hauls than those theoretically needed, may save time and money. These preferences may be realized by exercising good judgment in altering the theoretical balance lines. The result may be submission of lower bid prices. Even if the bid is not lower, the contractor is better satisfied—a condition that should produce a better job and friendlier relations with the owners.

Even if a grading contract contains no overhaul clause (this practice is becoming more common), the mass diagram is still very useful in the work of grade-line design. Approximate balance points are shown on the final plans to indicate the grading schedule to the contractor. It is then his responsibility to calculate or estimate the hauls and adjust his bid prices for excavation to cover their cost.

Theoretically, the solution in Fig. 6-15 is the more economical one because it has less overhaul, more free haul, and the same amounts of borrow and waste. Yet, in practice, the solution in Fig. 6-14 might be preferred because of the two-way hauling and the shorter haul distances.

Making a distribution analysis by mass diagram is not the purely mechanical process implied in the preceding discussion. Factors other than obtaining theoretical maximum economy enter into the planning of grading operations. For example, on steep grades the contractor prefers loaded hauls to be downhill. Moreover, he may prefer to haul more of a particular cut in