Rectangular Components of a vector.

A number of vectors can be replaced by a single resultant vector. This simplifies the analysis of the effect of vectors on objects. The reverse is true, we can replace a single vector by a number of component vectors. The sum of the component vectors will have a resultant equal to the original vector. This may seem an unnecessary complication, but if the component vectors are chosen to be at right angles, it allows the use of coordinate geometry and does away with the need for inaccurate scale drawing.

Vectors replaced by rectangular components

The process of replacing a vector by its rectangular components is known as resolution. Vectors are resolved into rectangular components. Once the vector has been resolved, the original vector is replaced. We deal only with the components.

Example: An object is projected with a velocity of 60 m.s$^{-1}$ at an angle of 60° to the horizontal. Calculate the vertical and horizontal components of the velocity.

vertical component = $60 \sin 60° = 52$ m.s$^{-1}$

Horizontal component = $60 \cos 60° = 30$ m.s$^{-1}$
**Equations of motion**

**Measurement of acceleration**

**Acceleration (a)**: Acceleration is the change in velocity per unit time. Acceleration is a vector quantity measured in metres per second per second (\( ms^{-2} \)).

**Measurement of acceleration**: To measure the acceleration of an object we require to measure the velocity of the object at two points in its journey and the time taken to travel between the two chosen points.

**Single obstructor and two light gates**

Light gate 1 provides time for initial velocity.
Light gate 2 provides time for final velocity.
Time between light gates is recorded manually or electronically.

\[
\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}
\]

**Double obstructor and single light gate**

The motion computer measures the time taken for the first segment of card to pass the light gate and the time taken for the second segment. It also measures the time for the gap in between.
From this information and the length of the segment, the computer works out the acceleration.
Velocity / acceleration - time graphs

A velocity - time graph shows the velocity of an object during a journey. As we are only dealing with linear motion, direction will be indicated by ‘+’ and ‘−’ signs.

+ = upwards
− = to left
− = downwards

SIGN CONVENTION FOR VELOCITY

Object projected vertically upwards

Bouncing ball

Higher Physics : Mechanics and the Properties of Matter
**Momentum and impulse**

**Momentum**

The momentum of a moving object is the product of its mass and velocity. Momentum is a vector quantity measured in kg m s\(^{-1}\).

\[
p = m \cdot v
\]

\[
\text{momentum} = \text{mass} \times \text{velocity}
\]

\[
20 \text{ kg} \quad 3 \text{ ms}^{-1} \quad 3 \text{ ms}^{-1} \quad 20 \text{ kg}
\]

\[
\text{momentum} = +60 \text{ kg ms}^{-1} \quad \text{momentum} = -60 \text{ kg ms}^{-1}
\]

Any process which changes the speed of an object, will change its momentum. A resultant force will cause a change in velocity \((F = ma)\). The change in momentum caused by a resultant force is called an **Impulse**.

**Collisions**

When two objects collide, the collision generates forces which act on both objects. As a result of this, the momentum of both objects is changed by the collision. If there are no external forces acting on the objects and the only forces are those created by the collision, the vector sum of the momentums before the collision is equal the vector sum of the momentums after the collision. We say that momentum is conserved.

**Conservation of Momentum**. When two objects collide, the vector sum of the momentums before the collision is equal to the vector sum of the momentums after the collision provided there are no external forces involved.

\[
m_A u_A + m_B u_B = m_A v_A + m_B v_B \quad \text{vector addition}
\]

External forces which might be involved include gravity, friction and electric forces.
Newtons’ Third Law

\[ F_A = - F_B \]

During a collision between two objects A and B, the force on A due to B is equal and in the opposite direction to the force on B due to A.

This is a direct result of the conservation of momentum. The change in momentum experienced by A is equal and in the opposite direction to the change in momentum experienced by B.

\[ m_A u_A + m_B u_B = m_A v_A + m_B v_B \]

rewritten

\[ m_A (v_A - u_A) = - m_B (v_B - u_B) \]

change in momentum A = - change in momentum B

If objects are in contact for time t seconds.

\[ \frac{m_A (v_A - u_A)}{t} = - \frac{m_B (v_B - u_B)}{t} \]

\[ m_A a_A = - m_B a_B \]

\[ F_A = - F_B \]

The force acting on A is mirrored by the force acting on B. They are equal and opposite at any time during the collision.
Measurement of Average Force

Mass of ball = 160 g
Distance between lightgates = 0.4 m
Time between lightgates = 0.56 s
Contact time = 25 ms

Momentum gained by ball = mass x velocity
= \frac{0.16 \times 0.4}{0.56} = 0.114 \text{ kg m s}^{-1}

Impulse = change in momentum
\[ F \times t = 0.114 \]
\[ F \times 25 \times 10^{-3} = 0.114 \]
\[ F = 4.6 \text{ N} \]

Average force on ball = 4.6 N
Higher Physics : Electricity and Electronics

Summary Notes

Electric fields and resistors in circuits Page 1
Alternating current and voltage Page 12
Capacitance Page 16
Analogue electronics Page 26
Electric Fields and conductors.
Conductors contain electric charges which are easily moved (electrons). When a conductor is exposed to an electric field, these charges will move.

Conductor: positive and negative charges uniformly distributed

(battery, mains)

Charge

Work is done by a source when electric charges are moved round a circuit. The work done comes from the electrical energy given to the charge as it passes through the source.

The electrical energy given to each coulomb of charge as it passes through the source is termed the e.m.f of the source (e.m.f - electromotive force).

E.m.f is measured in Volts (J C⁻¹).

The work done moving Q coulombs of charge round a circuit is

When dealing with circuits, we usually describe charge in terms of current

\[ W = QV \] where V is the pd across the source.

\[ Q = It \]

I is the current flowing in the circuit

Substituting:

\[ W = ItV \]

This is more familiar as

\[ \frac{W}{t} = P = VI \]

\[ P = VI \] and \[ W = QV \] are equivalent expressions.
The potential difference across the terminals of a battery, or any other source, decreases as the current drawn from the source increases. The behaviour of the source can be predicted if we assume the source consists of a source of constant e.m.f with a small internal resistor in series with it.

\[ V = E - Ir \]

\( Ir \) is often termed the ‘lost volts’.

The maximum current drawn from a source occurs when the terminal p.d. falls to zero. This is termed the short-circuit current and is given by

\[ I_s = \frac{E}{r} \]
Example When a voltmeter is connected across the terminals of a battery it reads 9.2 V. When the battery is connected in series with a 5 ohm resistor, the voltmeter reads 8.5 V. Find 

(a) the e.m.f of the battery.

(b) the internal resistance of the battery;

(c) the maximum current which can be drawn from the battery.

(a) The e.m.f of the battery is 9.2 V. The voltmeter draws virtually no current so the reading on the voltmeter is equal to the e.m.f of the battery.

(b) \[ V = E - Ir \]
\[ 8.5 = 9.2 - 1.7r \]
\[ r = \frac{9.2 - 8.5}{1.7} \]
\[ r = 0.41 \text{ ohms} \]

(c) The maximum current which can be drawn from the battery is the short circuit current when the terminal p.d. falls to zero volts.

\[ 0 = E - Ir \]
\[ I_s = \frac{9.2}{0.41} \]
\[ I_s = 22.4 \text{ A} \]

Internal resistance and power transfer to external resistive loads

The maximum power transferred between a source and an external circuit occurs when the resistance of the external circuit is equal to the internal resistance of the source. The p.d. across the circuit when this happens is \( \frac{1}{2} E \). The maximum voltage transfer occurs when the external resistance is much higher than the internal resistance of the source.
**Alternating current and voltage**

**Measuring frequency using a CRO.**

The speed of the electron beam as it moves horizontally across the face of the screen is controlled by the **time base** control. This is usually scaled in seconds per screen division. Once the signal is stationary on screen, the number of screen divisions per cycle of signal is measured. Multiplied by the time base setting, this measures the period of the signal. Frequency is calculated from \( \frac{1}{\text{period}} \).

\[
\text{frequency} = \frac{1}{\text{period}}
\]

\[
= \frac{1}{3 \times 200 \times 10^{-6}}
\]

\[
= 1667 \text{ Hz}
\]

**AC Supplies**

The voltage of an a.c. supply changes between positive and negative. Over time, the voltage spends the same time as a positive voltage as it does as a negative voltage. The average voltage, over time, is zero.
Power calculations and ac.

*For dc supplies*  \[ P = \frac{V^2}{R} \]

*For ac supplies*  \[ P = \frac{\text{average } V^2}{R} \]

We can find an average value for \( V^2 \).

For sinusoidal a.c. voltage  
\[ V = V_p \sin(\omega t) \]

\[ V^2 = V_p^2 \sin^2(\omega t) \]

The graph of this function is shown above.

The average value over time is \( \frac{V_p^2}{2} \).

This means that for ac  \[ P = \frac{V_p^2}{2R} = \frac{V_{\text{rms}}^2}{R} \]

Where:  
\[ V_{\text{rms}} = \frac{V_p}{\sqrt{2}} \quad V_p \text{ is the peak ac voltage} \]

The r.m.s voltage of an a.c. source is the equivalent d.c. voltage which will produce the same heating effect when applied across a resistive load.

**Example:** Mains electricity is supplied at 230 V r.m.s. at a frequency of 50 Hz. Calculate the peak voltage.

\[ V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}} \]

\[ V_{\text{peak}} = \sqrt{2} \times V_{\text{rms}} \]

\[ = 1.41 \times 230 \]

\[ = 325 \text{ V} \]
Energy stored in a capacitor

Work must be done to charge a capacitor. Once the plates of a capacitor have gained some charge, the stored charge repels more charge coming onto the plate. Work has to be done by the external source to overcome the repulsion and move charge onto the plates. The energy stored in the capacitor is equal to the work done charging the capacitor.

\[
\text{Energy stored in a capacitor} = \frac{1}{2} Q V
\]

Suppose a capacitor is given a charge \( Q \) coulombs and that it has now got a p.d of \( V \) Volts across it. A tiny amount of charge \( \Delta Q \) is now moved from one plate to the other. The work done in moving this tiny amount of charge is \( V \Delta Q \).

\( V \Delta Q \) is the area of the small strip on the graph. If we charge up the capacitor by \( \Delta Q \) coulombs each time, then the total work done will be the sum of the areas of the strips. This is simply the total area under the graph,

\[
\text{work done charging a capacitor} = \frac{1}{2} Q V
\]

Energy stored in a capacitor = \( \frac{1}{2} Q V \)

substituting \( V = \frac{Q}{C} \)

\[
= \frac{1}{2} C V^2
\]

substituting \( V = \frac{Q}{C} \)

\[
= \frac{Q^2}{2C}
\]

The energy is stored in the stretched molecules of the material between the plates of the capacitor. Positive and negative charges in the material are pulled apart by the electric field between the plates.

Example How much energy is stored in a 1000 \( \mu \)F capacitor when there is a p.d of 6 V across it?

\[
E = \frac{1}{2} C V^2
\]

\[
= 0.5 \times 1000 \times 10^{-6} \times 6 \times 6
\]

\[
= \text{1.8} \times 10^{-2} \ \text{Joules}
\]
Effect of RC circuits on square wave signals

Capacitors do not affect the shape of a sine wave signal. However they do affect the shape of other types of signal like the square wave shown below. The shape is affected by how fast the capacitor can charge or discharge.

The change in the p.d. across the resistor R

The signal across the resistor R shows the changes in the current flowing in the circuit.

\[ V_R = I \times R \]

constant
If we investigate the characteristics of a practical inverting mode amplifier, we obtain a graph like the one shown opposite.

\[ \text{gain} = -10 \]

\[ = -\frac{10 \text{ k}\Omega}{1 \text{ k}\Omega} \]

This is an identical result to the one we would expect for the ideal op-amp.

The output voltage of the op-amp is derived from the supply voltage. This means that the output voltage cannot rise above or fall below the supply voltage. In the situation where the output voltage has reached this limit, the op-amp is said to be saturated.

\[ \text{sat} \]

\[ \text{out} \]

\[ \text{in} \]

\[ \text{supply voltage} \]

\[ \frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = -6.0 \]

\[ \frac{\Delta V_{\text{in}}}{\Delta V_{\text{out}}} = 0.6 \]
An optical fibre is made from two types of glass, one surrounded by the other. The inner glass has an absolute refractive index of 1.5, the outer glass has an absolute refractive index of 1.4. Calculate the critical angle for light passing through the inner glass.

\[ n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} \]

\[ \text{speed of light in material} = \frac{\text{speed of light in vacuum}}{n} = \frac{c}{n} \]

for inner glass \[ v = \frac{c}{1.5} \]

for outer glass \[ v = \frac{c}{1.4} \]

refractive index for inner glass relative to outer glass \[ = \frac{\text{speed in outer}}{\text{speed in inner}} \]

\[ = \frac{c / 1.4}{c / 1.5} = \frac{1.5}{1.4} \]

The critical angle \( \theta_c \)

\[ \sin \theta_c = \frac{1}{n} = \frac{1.4}{1.5} = 0.933 \]

\[ \theta_c = 69^\circ \]
A ceiling lamp can be raised or lowered to provide table top illumination at differing intensities as required.

When the lamp was positioned at a distance of 1.5 m above the table a light meter recorded an intensity of 0.35 Wm$^{-2}$.

Calculate the intensity at a distance of 0.8 m.

Assuming lamp acts like a point source.

\[
I = \frac{\text{constant}}{r^2}
\]

\[
0.35 = \frac{\text{constant}}{1.5^2}
\]

constant = 0.35 x 1.5 x 1.5

= 0.79

for a distance of 0.8 m

\[
I = \frac{\text{constant}}{0.8^2}
\]

= \frac{0.79}{0.64}

= 1.2 Wm$^{-2}$

as expected, ½ the distance (approx) 4 times the intensity.

Readings are corrected for background light intensity.
Photodiode is set up in photoconductive mode where the metered output varies directly as the intensity.

Experimental confirmation.
Energy Levels in Solids.

The energy levels in free gas atoms are well defined. When the same atoms are formed into a solid, the atoms are linked together and the electrons can occupy a whole series of energy levels grouped into bands.

The electrons can move easily within the band but find it more difficult to move between bands.

Conductors, Insulators and Semiconductors.

Electrons in a solid can move between energy levels within a band. For electrical conduction to take place there must be electrons in a band and vacant energy levels to move to. The ability to conduct electricity depends on the electron arrangement in the Conduction band and the lower Valence band. Electrons in the valence band are part of the interatomic bonding. The valence band is normally full.

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Insulator</th>
<th>Semiconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduction band</td>
<td>Empty</td>
<td>Electron</td>
</tr>
<tr>
<td>Valence band</td>
<td>Full</td>
<td>Hole</td>
</tr>
</tbody>
</table>

Conduction is possible through the partly filled conduction band. No conduction is possible unless electrons can move to the conduction band.
Error in Calculated Quantities.

Where a quantity is calculated from different measured factors, the error in the calculated quantity is found from the largest percentage error in the factors.

**Example:** In an experiment to measure the specific heat capacity of water, the following results were obtained.

- **mass of water** = 505 ± 2 g
- **energy input** = 6.55 ± 0.05 kJ
- **temperature rise** = 3.1 ± 0.5 °C

\[ c = \frac{E}{m \times \Delta T} \]
\[ = \frac{6550}{0.505 \times 3.1} \]
\[ = 4184 \text{ J kg}^{-1} \text{ °C}^{-1} \]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>0.4</td>
</tr>
<tr>
<td>energy</td>
<td>0.8</td>
</tr>
<tr>
<td>temperature</td>
<td>16</td>
</tr>
</tbody>
</table>

Percentage error in calculated quantity = largest error in factors = 16%

This would give us an answer 4184 ± 670 J kg\(^{-1}\) °C\(^{-1}\)

**Quoted answer** 4.1 ± 0.7 kJ kg\(^{-1}\) °C\(^{-1}\)