# CIRCULAR MOTION #

When a particle moves in a plane such that its distance from a fixed (or moving) point is constant, then its motion is known as Circular motion. The fixed point is called centre, and the distance of particle from it is called radius.

We belie circular motion always in the frame of centre.

Kinematics of Circular Motion

**Variables of Motion:**

1. **Angular Position:** The angle made by the position vector w.r.t. origin, with the reference line is called angular position. Clearly, angular position depends on the choice of the origin.
\[ e_1 = \cos \theta \] 
\[ e_2 = \sin \theta \] 
\[ e_3 = \tan \theta \]

\[ T = \frac{e_1}{e_2} \] 
\[ \tau = \frac{e_3}{e_2} \]

\[ \frac{d}{dt} \left( \frac{e_1}{e_2} \right) = \frac{d}{dt} (\sin \theta) \]

\[ \frac{d}{dt} \left( \frac{e_3}{e_2} \right) = \frac{d}{dt} (\tan \theta) \]

Uniform and instantaneous angular acceleration, \( \Delta \theta = a \Delta t \) and unit vector, \( \mathbf{a} = \alpha \mathbf{T} \)
VERTICAL CIRCULAR MOTION

Let a particle of mass \( m \) is moving in vertical circular path with the help of a string of length \( l \) as shown as

In vertical circular motion a bob with string (similar case) speed, tension decreases as we move from lowest to highest point.

\[ F_c = \frac{mv^2}{l} = T - mg \]
Q2: Prove that a motor car passing over a concave bridge is safer than the same car resting on the same bridge.

If car is moving

\[ N_x - mg = m\frac{v^2}{R} \]
\[ N_y = mg + m\frac{v^2}{R} \]
\[ N_y > N_x \text{ (H.P.)} \]

A car is moving with uniform speed over a circular bridge of radius \( R \) which subtends an angle 90° at its center. Find the minimum possible speed so that the car can cross the bridge without losing its contact with it.

Answer: \( a = 0 \): \( v \) = Constant
Circular Turning on Roads

The necessary frictional force is being provided by banking the roads by following:

1. By friction only
2. By banking of roads only
3. By friction and banking of road both

**Friction Only:**

- Let "c" be a car moving on a horizontal circular road with constant speed "v".

\[
N = mg \quad F = F_f
\]

\[
F_{max} = \mu mg \quad F = \frac{mv^2}{r}
\]

\[\Rightarrow\] Maximum value of \( F \) for which car will not slide.

\[
F = F_{max}
\]

\[
\frac{mv^2}{r} = \mu mg
\]

\[
\Rightarrow v_{max} = \sqrt{\mu gr}
\]

\[
\Rightarrow v_{min} = \frac{v}{\sqrt{\mu}}
\]
(a) \( V_{\text{min}} \) friction is acting upwars (along the circle).

\[ \begin{align*}
\text{Normal} + \text{friction} = mg - \tau \\
\text{friction} = mg z - \tau \\
\text{with } z = \frac{v^2}{Rg} \\
\text{case 2: friction}
\end{align*} \]

\[ \begin{align*}
\tan \theta - \tan \phi = \frac{v^2}{Rg} \\
1 + \tan \theta = \frac{v^2}{Rg}
\end{align*} \]

\[ \theta = \text{angle of repose} \]

\[ \text{if } \theta < \phi \text{ then } V_{\text{min}} = 0 \]

(b) \( V_{\text{max}} \) friction is acting downward (along the incline).