Find the derivatives of inverse functions.

The inverse of an ordered pair (x, y) is (y, x).

\[ f(x) = y \iff x = f^{-1}(y). \]

**Theorem:** Let \( f(x) \) represent a differentiable on an interval. If \( f^{-1}(x) \) denotes the inverse of \( f(x) \), then \( f^{-1}(x) \) is differentiable at any \( x \) and

\[ [f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]} \]

where \( [f^{-1}(x)]' \) denotes the derivative of inverse of \( f(x) \), \( f'(x) \) is the derivative of \( f(x) \) and \( f'[f^{-1}(x)] \neq 0 \).

**Proof:**
If \( f^{-1}(x) \) is the inverse of \( f^{-1}(x) \), then \( f[f^{-1}(x)] = f^{-1}[f(x)] = x \).

Differentiating \( f[f^{-1}(x)] = x \) applying chain rule:

\[ \frac{df[f^{-1}(x)]}{dx} \cdot \frac{df^{-1}(x)}{dx} = 1, \text{ but } \frac{df[f^{-1}(x)]}{df^{-1}(x)} = f'[f^{-1}(x)] \text{ and } \frac{df^{-1}(x)}{dx} = [f^{-1}(x)]' \]

Thus \( f'[f^{-1}(x)] \cdot [f^{-1}(x)]' = 1 \) equation (1)

Dividing equation (1) by \( f'[f^{-1}(x)] \),

\[ [f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}, \text{ where } [f^{-1}(x)]' \text{ is the derivative of the inverse of } f(x) \text{ and } f'[f^{-1}(x)] \neq 0. \]

The formula \( [f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]} \) can be used to find the derivative of an inverse...