Find the derivatives of inverse functions.

The inverse of an ordered pair \((x, y)\) is \((y, x)\).

\[ f(x) = y \iff x = f^{-1}(y) \]

**Theorem:** Let \(f(x)\) represent a differentiable on an interval. If \(f^{-1}(x)\) denotes the inverse of \(f(x)\), then \(f^{-1}(x)\) is differentiable at any \(x\) and

\[ [f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]} \]

where \([f^{-1}(x)]'\) denotes the derivative of inverse of \(f(x)\), \(f'(x)\) is the derivative of \(f(x)\) and \(f'[f^{-1}(x)] \neq 0\).

**Proof:**
If \(f^{-1}(x)\) is the inverse of \(f^{-1}(x)\), then \(f[f^{-1}(x)] = f^{-1}[f(x)] = x\).

Differentiating \(f[f^{-1}(x)] = x\) applying chain rule:

\[
\frac{df[f^{-1}(x)]}{dx} \cdot \frac{df^{-1}(x)}{dx} = 1, \text{ but } \frac{df[f^{-1}(x)]}{df^{-1}(x)} = f'[f^{-1}(x)] \text{ and } \frac{df^{-1}(x)}{dx} = [f^{-1}(x)]' \\
\]

Thus \(f'[f^{-1}(x)] \cdot [f^{-1}(x)]' = 1\) equation (1)

Dividing equation (1) by \(f'[f^{-1}(x)]\),

\[ [f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}, \text{ where } [f^{-1}(x)]' \text{ is the derivative of the inverse of } f(x) \text{ and } \]

\[ f'[f^{-1}(x)] \neq 0. \]

The formula \([f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}\) can be used to find the derivative of an inverse