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To rationalise a fraction in the form \( \frac{1}{a - \sqrt{b}} \), multiply the numerator and the denominator by \( a + \sqrt{b} \)

1.3 Quadratic functions
1.3.1 What is a quadratic function?

A quadratic function is a function that is in the form

\[ y = ax^2 + bx + c, \]  
where \( a \) is not equal to 0.

This can also be written as

\[ f(x) = ax^2 + bx + c \]

1.3.2 What are the characteristics of a graph of a quadratic function?

The graph is a curve.

The graph has only one turning point.

The graph is symmetrical on either side of the turning point.

1.4 Quadratic equations
1.4.1 Meaning

What is a quadratic equation?

A quadratic equation is an equation of the form

\[ ax^2 + bx + c = 0, \]  
where \( x \) is an unknown variable, and \( a, b \) and \( c \) are constants, known as the quadratic coefficient, the linear coefficient and the constant term respectively.

What are you doing when you are solving a quadratic equation?

You are finding the values of \( x \) for which the equation \( ax^2 + bx + c = 0 \) is true.

How many are there?

A quadratic equation has two solutions.

How can the solutions of quadratic equation be seen visually?

On the graph of a quadratic equation, the values of \( x \) when the curve intercepts the x-axis, that is, when \( y = 0 \), are the solutions to the quadratic equation.
1.9.4 What is the technique for reflecting a graph of $f(x)$ in the axes?

A translation of $y = f(-x)$ is a reflection of the original curve in the $y$-axis.

A translation of $y = -f(x)$ is a reflection of the original curve in the $x$-axis.

2. Coordinate geometry in the $(x, y)$ plane

2.1 Equation of a straight line

2.1.1 What are the identities for the equations for straight lines?

$y = mx + c$, where $m$ and $c$ are integers.

$ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

*How are these identities related?*

They are interchangeable with each other. An equation in the form $y = mx + c$ can be transposed into the form $ax + by + c = 0$.

*What is useful about the form $y = mx + c$?*

The constant $m$ is the gradient of the line, while $c$ is the intercept of the line on the $y$-axis.

2.1.2 What is the formula for calculating the gradient of a line given two points that lie on it?

Given that the first point is at $(x_1, y_1)$ and the second point is $(x_2, y_2)$,

$m = (y_2 - y_1) / (x_2 - x_1)$

2.1.3 What is the formula for finding the equation of a line given the gradient $m$ and one of the points it passes through?

Given that the gradient is $m$ and the point that it passes through is $(x_1, y_1)$,

$y - y_1 = m(x - x_1)$

2.1.4 What is the formula for finding the equation for a straight line given two points that it passes through?

Given that the first point is at $(x_1, y_1)$ and the second point $(x_2, y_2)$,

$(y - y_1) / (y_2 - y_1) = (x - x_1) / (x_2 - x_1)$

2.2 Conditions for parallel and perpendicular lines

2.2.1 What are the conditions for two lines to be parallel?

Two lines are parallel if their gradients are the same.
where a is the first term, and an is the last term in the series.

Why are there two forms for this equation?

The formula for the nth term of an arithmetic series is:

\[ a_n = a_1 + (n-1)d \]

This value of the nth term can be substituted into the formula for the sum of an arithmetic series if the last term in the series is unknown, but the common difference is.

Similarly, if the common difference is unknown, but the last term is, you can use the simpler form of the equation.

Explain how this equation is derived.

Take the general arithmetic series as:

\[ S = a + (a + d) + (a + 2d) + (a + 3d) + \ldots + (a + (n-2)d) + (a + (n-1)d) \]

Reversing the sum:

\[ S = (a + (n-1)d) + (a + (n-2)d) + (a + (n-3)d) + \ldots + (a + d) + a \]

Adding the sums:

\[ 2S = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] + \ldots + [2a + (n-1)d] \]

There are n lots of \([2a + (n-1)d]\). That means we can express this sum as:

\[ 2S = n[2a + (n-1)d] \]

\[ S = \frac{n}{2} \times [2a + (n-1)d] \]

3.2.4 Question Technique 2

What is the technique for answering a question where given an arithmetic series with its first term, common difference and length visible, you are asked to find the sum of the series?

1. Substitute the values into the formula for the sum of an arithmetic series

2. Simplify the equation to find the sum

What is the technique for answering a question where given an arithmetic series with its first term, last term and length, you are asked to find the sum of the series?

1. Substitute the value of n, a1 and an into the second form of the equation - \( S = \frac{n}{2}(a_1 + an) \)

2. Simplify the equation to find the sum.
Is this the only thing that this gradient can represent? Explain your answer.

No. The straight line \( y = \frac{1}{3}x \) is not the only possibility. Any straight line with gradient \( \frac{1}{3} \) may be written as \( y = \frac{1}{3}x + c \), where \( c \) is an arbitrary constant.

This is the most general equation which gives \( dy/dx = 1/3 \).

What does this mean that the equation \( dy/dx = 1/3 \) represents?

It means that equation represents all straight lines with a gradient of \( 1/3 \).

What does it mean to find the anti-derivative with respect to \( x \) of this equation?

From the theorem, we know that antiderifferentiation is equivalent to integration, so by taking the antiderivative with respect to \( x \) of the equation, we are finding the integral with respect to \( x \). Thus, calculating the integral requires that we perform the reverse process of differentiation.

We know that the most general equation for \( y \) which when differentiated gives \( dy/dx = 1/3 \) is

\[ y = \frac{1}{3}x + C, \text{ where } C \text{ is an arbitrary constant.} \]

Thus, for \( dy/dx = 1/3 \), the antiderivative, or integral is calculated as

\[ \int dy/dx \, dx = \int 1/3 \, dx \]

Evaluating:

\[ y = \frac{1}{3}x + C \]

5.1.3 What is the general formula for integration of \( x^n \)?

If \( dy/dx = x^n \)

\[ y = \frac{1}{n+1} \cdot x^{n+1} + c \]

Why is this formula for integration?

As we now know, integration is the reverse process of differentiation. The formula for differentiation is:

\[ \text{if } y = x^n, \, dy/dx = nx^{n-1} \]

The formula for integration has the function of providing the reverse process to this formula.

5.1.4 How is the formula for integration applied to a polynomial?

It is applied in the same way that the formula for differentiation applies to a polynomial. Integrate each term separately.