Chapter No 4

Quadratic Equations

Quadratic Equation

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general:

\[ ax^2 + bx + c = 0 \]

where \( a \neq 0 \) is called Quadratic Equation in variable \( x \).

3 Methods.

To solve Quadratic Equation there are three different methods named as:

1. Factorization method.
2. Completing Square method.
3. Quadratic Formula method.

Example 1

Solve by Factorization \[ x^2 - 7x + 10 = 0 \]

\[ x^2 - 7x + 10 = 0 \]
\[ x(x-2) - 5(x-2) = 0 \]
\[ (x-2)(x-5) = 0 \]
\[ x-2 = 0 \quad \text{or} \quad x-5 = 0 \]
\[ x = 2 \quad \text{or} \quad x = 5 \]
\[ \{ 2, 5 \} \]

Example 2

Solve \[ x^2 + 4x - 437 = 0 \] by Completing Sq.

\[ x^2 + 4x = 437 \]

Adding \( (\frac{4}{2})^2 = 2^2 \) on both sides.

\[ (x + 2)^2 = 437 + 4 \]
\[ (x + 2)^2 = 441 \]
\[ x + 2 = \pm 21 \]
\[ x = 21 - 2 \quad \text{or} \quad x = -21 - 2 \]
\[ x = 19 \quad \text{or} \quad x = -23 \]
\[ \{ 19, -23 \} \]

Example 3

Solve \[ 6x^2 + x - 15 = 0 \] by Quadratic Formula

Comparing \[ 6x^2 + x - 15 = 0 \]

with \[ ax^2 + bx + c = 0 \]

we have \( a = 6, b = 1, c = -15 \)

By using Quadratic Formula,

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-1 \pm \sqrt{1 + 4(6)(-15)}}{2(6)} \]
\[ x = \frac{-1 \pm \sqrt{1 + 360}}{12} \]
\[ x = \frac{-1 \pm \sqrt{361}}{12} \]
\[ x = \frac{-1 \pm 19}{12} \]
\[ x = \frac{-1 + 19}{12} \quad \text{or} \quad x = \frac{-1 - 19}{12} \]
\[ x = \frac{18}{12} \quad \text{or} \quad x = \frac{-20}{12} \]
\[ x = \frac{3}{2} \quad \text{or} \quad x = -\frac{5}{3} \]
\[ \{ \frac{3}{2}, -\frac{5}{3} \} \]

Example 4

Solve \[ 8x^2 - 14x - 15 = 0 \] by Quadratic Formula

Comparing \[ 8x^2 - 14x - 15 = 0 \]

with \[ ax^2 + bx + c = 0 \]

We have \( a = 8, b = -14, c = -15 \)

By using Quadratic Formula,

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{14 \pm \sqrt{196 + 4(8)(-15)}}{16} \]
\[ x = \frac{14 \pm \sqrt{196 + 480}}{16} \]
\[ x = \frac{14 \pm \sqrt{676}}{16} \]
\[ x = \frac{14 \pm 26}{16} \]
\[ x = \frac{14 + 26}{16} \quad \text{or} \quad x = \frac{14 - 26}{16} \]
\[ x = \frac{40}{16} \quad \text{or} \quad x = \frac{-12}{16} \]
\[ x = \frac{5}{2} \quad \text{or} \quad x = -\frac{3}{4} \]
\[ \{ \frac{5}{2}, -\frac{3}{4} \} \]