1.3 Functions

1. Find these values.

We simple round up or down in each case.

(a) \(\lfloor 1.1 \rfloor = 1\)
(b) \(\lceil 1.1 \rceil = 2\)
(c) \(\lceil 2.99 \rceil = 3\)
(d) \(\lfloor -2.99 \rfloor = -3\)
(e) \(\lceil \frac{1}{2} + \lfloor \frac{1}{2} \rfloor \rfloor = \lceil 1 \rceil = 1\)
(f) \(\lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil = \lfloor 0 \rfloor + \lceil 1 \rceil = 1\)

2. Determine whether \(f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}\) is onto if

(a) \(f(m, n) = 2m - n\)

This is clearly onto since \(f(0, -n) = -n\) for any integer \(n\).

(b) \(f(m, n) = m^2 - n^2\)

This is not onto; for example, 2 is not in the range. To see this, if \(m^2 - n^2 = (m - n)(m + n) = 2\), then \(m + n\) must be even for \(m - n\) and \(m + n\) to have the same parity (both even or both odd). In either case, both \(m - n\) and \(m + n\) are even, so this expression is divisible by 4 and hence cannot equal 2.

(c) \(f(m, n) = |m| - |n|\)

This is onto. To achieve negative values, we set \(m = 0\) and to achieve nonnegative values we set \(n = 0\).

(d) \(f(m, n) = m^2 - 4\)

This is not onto for the same reason as in part (b). In fact, the range here is clearly a subset of the range in that part.

3. Determine whether each of these functions is a bijection from \(\mathbb{R}\) to \(\mathbb{R}\).

If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not one-to-one or not onto.

(a) \(f(x) = -3x + 4\)