Example 2

If a sum of money is invested at 6% p.a. with the interest compounded quarterly for 5 years, what is the present value to the nearest dollar if the future value is $18,856?

Answer:

\[ FV = 18,856, \ r = \frac{0.06}{4} = 0.0015, \]
\[ n = 5 \times 4 = 20 \]

Rearranging \( FV = PV (1 + r)^n \) for \( PV \) gives:

\[ PV = \frac{FV}{(1 + r)^n} \]
\[ PV = \frac{18,856}{(1 + 0.0015)^{20}} \]
\[ PV = 14,000.0222... \]
\[ PV = $14,000 \] (to the nearest dollar)

Example 3

Tony has a credit card which comes with an interest-free period of up to 55 days. The interest rate is 16.9% p.a. charged daily. His monthly bill shows a total of $3580 owing.

a) How much interest is due if he pays off the bill in total before the due date?
b) How much interest will accrue for each day he is late in paying the bill?
c) Tony decides to pay only the minimum amount off the bill, which is $10 or 2% of the closing balance (rounded up to the nearest dollar), whichever is greater. What will his bill be the following month (31 days later) if he doesn’t use his credit card during that time?

Credit Cards

- **Credit cards** allow you to borrow money up to a certain limit as long as you make regular minimum repayments. A credit card is a means of borrowing money to pay for goods and services.
- They can be used to pay for goods and services and for **cash advances** (i.e. withdraw cash from an ATM or bank branch)
- The difference from a **debit card** is that a debit card uses money that is in your account whereas a credit card borrows money that you will need to pay back later
- Many credit cards have an **interest free period** (e.g. 55 days). This means no interest will be charged on purchases if the amount owing on the statement is paid by the due date shown (provided there is no previous outstanding amount).

- **Note:** An outstanding amount is an amount of money that was due for payment previously but has not been paid
- Usually interest-free periods apply only to purchases whereas cash advances usually attract interest straight away.
- If a credit card has no interest-free period, then interest is accrued either from the day of the purchase or from the day the monthly statement is issued
- Interest is usually charged daily. Therefore, the annual rate of interest must be divided by 365 to get the daily rate. Credit card interest is generally simple interest, but may be compound interest.
- Note: Even if simple interest is charged, you will be charged interest on interest in the following month if the outstanding amount isn’t paid in full.
Example 10 (contd.)

b) Flat rate loan, therefore:

\[ I = Prn \]

\[ P = $20 000; r = \frac{0.084}{12} = 0.007; \]
\[ n = 12 \times 7 = 84 \]
\[ \therefore I = 20 000 \times 0.085 \times 84 \]
\[ \therefore I = $11 900 \]

c) The hire purchase option will cost her the least over the full term. Her saving would be:

$11 900 − $5494.40 = $6405.60

d) Her repayments for the bank loan option would be:

\[ \frac{\text{Total amount to be repaid}}{\text{Total number of repayments}} \]
\[ = \frac{20 000 + 11 900}{7 \times 12} \]
\[ = 79.76 \text{ (to nearest cent)} \]

As this monthly repayment is cheaper than the hire purchase option, she may decide to choose this option in order to reduce her monthly repayment.

Fixed vs Variable Interest Rate

A fixed interest rate is when the interest rate is set at the outset of the loan and does not change over the term of the loan.

A variable interest rate is when the interest rate can go up or down over the term of a loan. In general, lending institutions such as banks will change the variable interest rates that they charge based on changes to the Official Cash Rate (Interest Rate) as set by the Reserve Bank of Australia (RBA). It is important for borrowers to consider the effect of possible interest rate rises on their loans to ensure that they will still be able to make the increased repayments.

Sometimes, a lending institution may offer a loan that has a fixed interest rate for a certain period of time before it changes to a variable interest rate. This is common for home loans where the fixed period is often for the first 1 to 5 years of the loan term. This gives the borrower certainty about the repayment amount during this period before the loan changes to the variable interest rate.
ANNUITIES AND LOAN REPAYMENTS

Annuities

Generally speaking, there are two types of annuities - immediate annuities and deferred annuities.

With an immediate annuity, you contribute a lump sum to the annuity account and immediately begin receiving regular payments, which usually last for the rest of your life. Immediate annuities convert a cash pool into a lifelong income stream, providing you with a guaranteed monthly allowance as you grow old.

A deferred annuity is an investment where you contribute regular equal amounts to an account over your working life to build a sizable income stream for your retirement. In this course, you will deal only with deferred annuities.

For this course, a definition of an annuity can therefore be:

An annuity is a fixed sum of money that is paid out to someone at regular intervals. It is a form of insurance or investment which entitles the holder to a guaranteed income stream for a certain number of years or even the rest of their life.

- The contribution per period is the sum of money that is being invested at regular time intervals.
- The future value of an annuity is the total value of the investment at the end of the last contribution period.
- There are tables that we can use to help us find the future values. These usually give the amount for an investment of $1 (see below for an example table). This amount should then be multiplied by the number of dollars invested to give the required value.
- The present value of an annuity is the single sum of money that can be invested now under the same terms (interest rate and length of time) as an annuity with regular contributions, M, and will produce the same financial outcome.

The present value can be used to compare investment options. The one with the greater present value will produce the greater financial outcome over time.

There are tables that we can use to help us find the present value of an annuity (see below for an example table). These tables usually give the present values of an annuity of $1 for different interest rates and periods of time.

The following tables are used for Examples 11 - 15:

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<table>
<thead>
<tr>
<th>Period</th>
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<tr>
<td>12</td>
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</table>

Example 11

If Josh were to invest $8500 per year into an account at 6% for 16 years, what would the future value be?

Answer:
Example 22

Paul and Liz each take out a home loan of $800 000 and pay the same monthly repayments. However, Liz shops around and manages to secure a lower interest rate than Paul.

Using the graph above, find:

a) How many years earlier Liz will have paid off her loan compared to Paul.

b) The remaining balance on Paul’s loan when Liz has paid hers off (to the nearest $10 000).

Answer:

From the graph, we get:

a) Liz’s loan will be paid off 10 years earlier than Paul’s.

b) After 20 years, when Liz’s loan is fully paid off, the remaining balance on Paul’s loan is about $450 000 (to the nearest $10 000).

Example 23

Sophie takes out a home loan. The graph below shows how the principle owing and the total owing vary over the term of the loan.

Answer:

From the graph, we get:

b) After 20 years, when Liz’s loan is fully paid off, the remaining balance on Paul’s loan is about $450 000 (to the nearest $10 000).