1. Prove that $2(\mathbf{a}) = (\mathbf{a}) \cdot (\mathbf{a} + \mathbf{b})$. Also give its geometrical interpretation. 
Hence find the area of a parallelogram whose diagonals are the vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

2. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be unit vectors, such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = 0$ and the angle between $\mathbf{b}$ and $\mathbf{c}$ is $\pi/6$. Prove that $\mathbf{a} = \pm 2\mathbf{b} \times \mathbf{c}$.

3. A line makes angles $\alpha, \beta, \gamma$ and $\delta$ with the face diagonals of a cube. Prove that 
$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$$

4. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three mutually perpendicular vectors of equal magnitude, find the angle between $\mathbf{a}$ and $(\mathbf{a} + \mathbf{b} + \mathbf{c})$.

5. Let $\mathbf{a} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$. Express $\mathbf{b}$ in the form of $\mathbf{b} = \mathbf{c} + \mathbf{d}$. Where $\mathbf{c}$ is parallel to $\mathbf{a}$ and $\mathbf{d}$ is perpendicular to $\mathbf{a}$. 

(6 Marks)
<table>
<thead>
<tr>
<th>Nutrient</th>
<th>Requirement</th>
<th>Cost</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thiamine</td>
<td>1.00 mg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phosphorous</td>
<td>7.50 mg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron</td>
<td>10.00 mg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The minimum requirement of the nutrients in the diet are 1.00 mg of thiamine, 7.50 mg of phosphorous and 10.00 mg of iron. The cost of $F_1$ is 20 paise per 25 gms while the cost of $F_2$ is 15 paise per 25 gms. Find the minimum cost of diet.

11. A farmer is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them $X$, $Y$ and $Z$), it is necessary to buy two additional products, say, A and B. One unit of products A contains 36 units of $X$, 3 units of $Y$, and 20 units of $Z$. One unit of product B contains 6 units of $X$, 12 units of $Y$ and 10 units of $Z$. The minimum requirements of $X$, $Y$ and $Z$ is 108 units, 36 units and 100 units respectively. Product A costs Rs.20 per and product B costs Rs.40 per unit. Formulate the above as a linear programming problem to minimize the total cost, and solve the problem by using graphical method.

12. If a youngman rides his motor cycle at 25 km per hour he has to spend Rs 2 per km on petrol, if he rides it at a faster speed of 40 km. per hour, the petrol cost increase to Rs. 5 per km. He has Rs. 100 to spend on petrol and wish to find what is the maximum distance he can travel, within one hour. Express this as a linear programming problem and then solve it graphically.

13. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However at least twice as many passengers prefer to travel by economy class to the first class. Determine how many each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit?

14. Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops D, E, F whose requirements are 60, 50, 40 quintals, respectively. The costs of transportation per quintal from godowns to the shops are given in the following table:

<table>
<thead>
<tr>
<th>To</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>2.50</td>
<td>3</td>
</tr>
</tbody>
</table>

How should the supplies be transported in order that the transportation cost is minimum?

15. An oil company has two depots, A and B, with capacities of 7000L and 4000L respectively. The Company is to supply oil to three petrol pumps D, E, F whose
9. Find the probability distribution of the number of face cards when two cards are drawn without replacement from a well shuffled deck of 52 cards.
10. Find the probability distribution of number of doublets in 4 throws of a pair of dice.
11. Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?
12. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of the cases are they likely to contradict each other in the stating the same fact?
13. A factory has three machines I,II and III which produce 30%, 50% and 20% respectively of the total items of the same variety. Out of these 2%, 5% and 3% respectively are found to be defective. An item is picked up at random and found to be defective. Find the probability that it is produced by the III machine.

   Ans.:\text{-6}/37

14. A random variable X has the following probability distribution:

\[
\begin{array}{c|ccccccccc}
X & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
P(X) & 0 & K & 2K & 2K & 3K & K^2 & 2K^2 & 7K^2 & K
\end{array}
\]

(i) Find K (ii) Evaluate \( P(X<3) \), \( P(X\geq6) \) and \( P(0<X<3) \).

15. Find the mean of binomial distribution \( B(4,1/3) \).

16. A die is thrown five times, if getting an odd number is a success, find the probability of getting at least four success.

17. Probabilities of solving a specific problem independently A and B are \( \frac{1}{2} \) and \( \frac{1}{3} \). respectively. If both try to solve the problem independently. Find the probability that

   i. The problem is solved
   ii. Exactly one of them solves the problem.

18. A bag contains \( (2n+1) \) coins. It is known that \( n \) of these coins have a head on both sides where the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is \( \frac{31}{42} \). Determine the value of \( n \).

   Ans. \( n=10 \)

19. A ship is fitted with three engines \( E_1, E_2 \) and \( E_3 \). The engines function independently of each other with respective probabilities \( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \). For the ship to be operational at least two of its engines must function. Let \( X \) denote the event that the ship is operational and \( X_1, X_2 \) and \( X_3 \) denote respectively the events that the engines \( E_1, E_2 \) and \( E_3 \) are functioning then find \( P\left( \frac{X}{X_1} \right) \).

   Ans. \( \frac{7}{16} \)