Lecture Notes

15MA301-Probability & Statistics

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Unit-2

Probability Distributions

TOPICS:

⋆ Some Special/Standard Probability Distributions

  – Discrete: Binomial, Poisson and Geometric

  – Continuous: Exponential and Normal

  – Properties and applications of these distributions to industrial problems.

⋆ Functions of Random Variables

1.2.2 Mean and Variance of Poisson Distribution

$$\begin{aligned}
M_X(t) &= e^{\lambda(e^t-1)} \\
\\
\text{First Moment = Mean} &= E(X) = M'_X(0) \\
M'_X(t) &= \lambda e^t e^{\lambda(e^t-1)} \\
M'_X(0) &= \lambda
\end{aligned}$$

$$\begin{aligned}
\text{Second Moment} &= E(X^2) = M''_X(0) \\
M''_X(t) &= (\lambda e^t)^2 e^{\lambda(e^t-1)} + \lambda e^t e^{\lambda(e^t-1)} \\
M''_X(0) &= \lambda^2 + \lambda \\
\\
\text{Var}(X) &= E(X^2) - [E(X)]^2 \\
&= \lambda^2 + \lambda - \lambda^2 = \lambda
\end{aligned}$$

Note that for Poisson Distribution, Mean=Variance= λ.

1.3 Geometric Distribution

A random variable X is said to follow Geometric distribution, if it assumes only non-negative values and its probability mass function is given by

$$P(X = x) = q^x p, \quad x = 0, 1, 2, \ldots, \quad 0 < p < 1$$

where $q = 1 - p$.

We can also write the pmf of the Geometric distribution as

$$P(X = x) = q^{x-1} p, \quad x = 1, 2, 3, \ldots, \quad 0 < p < 1$$

where $q = 1 - p$.

$\therefore \ p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$ and

$\therefore \ (1) \Rightarrow n.\frac{1}{5} = 20 \Rightarrow n = 100$

$\therefore$ the parameters are $\left(100, \frac{1}{5}\right)$.

Example: 5. *Comment on the following "The mean of a binomial distribution is 3 and variance is 4".*

Hints/Solution: Let (n,p) be the parameters of the binomial distribution,
then mean =np and variance= npq.

Given $np = 3 - - - - (1)$
and $npq = 4 - - - - (2)$
Using (1) in (2), we get
$(2) \Rightarrow 3q = 4 \Rightarrow q = \frac{4}{3} > 1$, which is not true, since the probability value can not be greater than 1.

So, there is no binomial distribution with this data, the statement is false.

Example: 6. *Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show a five or six.*

Hints/Solution: Success is getting 5 or 6 in a die. Let X denote the number of success when 6 dice are thrown.
$\therefore X$ is a binomial random variable with parameter (n, p).
$\therefore P(X = x) = ^nC_x p^x q^{n-x}, \ x = 0, 1, 2, \cdots, n$

Given $n = 6$ and p =probability of getting 5 or 6 $= \frac{2}{6} = \frac{1}{3}$

$\therefore \ q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore \ P(X = x) = ^6C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}, \ x = 0, 1, 2, \cdots, 6$