Probability Distributions

Topics:

* Some Special/Standard Probability Distributions
  - Discrete: Binomial, Poisson and Geometric
  - Continuous: Exponential and Normal
  - Properties and applications of these distributions to industrial problems.
* Functions of Random Variables
1.2.2 Mean and Variance of Poisson Distribution

\[ M_X(t) = e^{\lambda(e^t - 1)} \]

First Moment = Mean \[ = E(X) = M'_X(0) \]
\[ M'_X(t) = \lambda e^t e^{\lambda(e^t - 1)} \]
\[ M'_X(0) = \lambda \]

Second Moment \[ = E(X^2) = M''_X(0) \]
\[ M''_X(t) = (\lambda e^t)^2 e^{\lambda(e^t - 1)} + \lambda e^t e^{\lambda(e^t - 1)} \]
\[ M''_X(0) = \lambda^2 + \lambda \]

\[ \text{Var}(X) = E(X^2) - [E(X)]^2 \]
\[ = \lambda^2 + \lambda - \lambda^2 = \lambda \]

Note that for Poisson Distribution, Mean=Variance=\( \lambda \).

1.3 Geometric Distribution

A random variable \( X \) is said to follow Geometric distribution, if it assumes only non-negative values and its probability mass function is given by

\[ P(X = x) = pq^{x-1}p, \quad x = 0, 1, 2, \ldots, \quad 0 < p < 1 \]

where \( q = 1 - p \).

We can also write the pmf of the Geometric distribution as

\[ P(X = x) = q^{x-1}p, \quad x = 1, 2, 3, \ldots, \quad 0 < p < 1 \]

where \( q = 1 - p \).
\[ p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5} \text{ and} \]

\[ (1) \Rightarrow n \cdot \frac{1}{5} = 20 \Rightarrow n = 100 \]

\[ \therefore \text{the parameters are} \ (100, \frac{1}{5}). \]

**Example: 5.** Comment on the following "The mean of a binomial distribution is 3 and variance is 4".

**Hints/Solution:** Let \((n, p)\) be the parameters of the binomial distribution, then mean = \(np\) and variance = \(npq\).

Given \(np = 3\) \(\quad \rightarrow (1)\)
and \(npq = 4\) \(\quad \rightarrow (2)\)

Using \((1)\) in \((2)\), we get
\[ (2) \Rightarrow 3q = 4 \Rightarrow q = \frac{4}{3} > 1, \text{ which is not true, since the probability value can not be greater than 1.} \]

So, there is no binomial distribution with this data, the statement is false.

**Example: 6.** Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show a five or a six?

**Hints/Solution:** Success is getting 5 or 6 in a die. Let \(X\) denote the number of success when 6 dice are thrown.

\[ X \text{ is a binomial random variable with parameter} \ (n, p). \]

\[ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, 2, \cdots, n \]

Given \(n = 6\) and \(p = \text{probability of getting 5 or 6} = \frac{2}{6} = \frac{1}{3}\)

\[ q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3} \]

\[ P(X = x) = 6 \binom{6}{x} \left( \frac{1}{3} \right)^x \left( \frac{2}{3} \right)^{6-x}, \ x = 0, 1, 2, \cdots, 6 \]