

where we assume that the outcomes of the experiment are *equally likely*.

We will briefly refer to theoretical probability as probability.

This definition of probability was given by Pierre Simon Laplace in 1795.

Probability theory had its origin in the 16th century when an Italian physician and mathematician J. Cardan wrote the first book on the subject, *The Book on Games of Chance*. Since its inception, the study of probability has attracted the attention of great mathematicians. James Bernoulli (1654 – 1705), A. de Moivre (1667 – 1754), and Pierre Simon Laplace are among those who made significant contributions to this field. Laplace's *Theorie Analytique des Probabilités*, 1812, is considered to be the greatest contribution by a single person to the theory of probability. In recent years, probability has been used extensively in many areas such as biology, economics, genetics, physics, sociology etc.



Pierre Simon Laplace
(1749–1827)

Let us find the probability for some of the events associated with experiments where the equally likely assumption holds.

Example 1 : Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Solution : In the experiment of tossing a coin once, the number of possible outcomes is two — Head (H) and Tail (T). Let E be the event ‘getting a head’. The number of outcomes favourable to E, (i.e., of getting a head) is 1. Therefore,

$$P(E) = P(\text{head}) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}} = \frac{1}{2}$$

Similarly, if F is the event ‘getting a tail’, then

$$P(F) = P(\text{tail}) = \frac{1}{2} \quad (\text{Why ?})$$

Example 2 : A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the

- (i) yellow ball? (ii) red ball? (iii) blue ball?

Solution : Kritika takes out a ball from the bag without looking into it. So, it is equally likely that she takes out any one of them.

Let Y be the event ‘the ball taken out is yellow’, B be the event ‘the ball taken out is blue’, and R be the event ‘the ball taken out is red’.

Now, the number of possible outcomes = 3.

(i) The number of outcomes favourable to the event Y = 1.

So,
$$P(Y) = \frac{1}{3}$$

Similarly, (ii) $P(R) = \frac{1}{3}$ and (iii) $P(B) = \frac{1}{3}$.

Remarks :

1. An event having only one outcome of the experiment is called an *elementary event*. In Example 1, both the events E and F are elementary events. Similarly, in Example 2, all the three events, Y, B and R are elementary events.

2. In Example 1, we note that : $P(E) + P(F) = 1$.

In Example 2, we note that : $P(Y) + P(R) + P(B) = 1$.

Observe that the sum of the probabilities of all the elementary events of an experiment is 1. This is true in general also.

Example 3 : Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?

Solution : (i) Here, let E be the event ‘getting a number greater than 4’. The number of possible outcomes is six : 1, 2, 3, 4, 5 and 6, and the outcomes favourable to E are 5 and 6. Therefore, the number of outcomes favourable to E is 2. So,

$$P(E) = P(\text{number greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

(ii) Let F be the event ‘getting a number less than or equal to 4’.

Number of possible outcomes = 6

Outcomes favourable to the event F are 1, 2, 3, 4.

So, the number of outcomes favourable to F is 4.

Therefore,
$$P(F) = \frac{4}{6} = \frac{2}{3}$$

The number of possible outcomes = 52

Therefore,
$$P(F) = \frac{48}{52} = \frac{12}{13}$$

Remark : Note that F is nothing but $\bar{E}$. Therefore, we can also calculate P(F) as

follows:
$$P(F) = P(\bar{E}) = 1 - P(E) = 1 - \frac{1}{13} = \frac{12}{13}.$$

Example 5 : Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Solution : Let S and R denote the events that Sangeeta wins the match and Reshma wins the match, respectively.

The probability of Sangeeta's winning = $P(S) = 0.62$ (given)

The probability of Reshma's winning = $P(R) = 1 - P(S)$

[As the events R and S are complementary]

$$= 1 - 0.62 = 0.38$$

Example 6 : Savita and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year).

Solution : Out of the two friends, one girl, say, Savita's birthday can be any day of the year. Now, Hamida's birthday can also be any day of 365 days in the year.

We assume that these 365 outcomes are equally likely.

- (i) If Hamida's birthday is different from Savita's, the number of favourable outcomes for her birthday is $365 - 1 = 364$

So,
$$P(\text{Hamida's birthday is different from Savita's birthday}) = \frac{364}{365}$$

- (ii) $P(\text{Savita and Hamida have the same birthday})$

$$= 1 - P(\text{both have different birthdays})$$

$$= 1 - \frac{364}{365} \quad [\text{Using } P(\bar{E}) = 1 - P(E)]$$

$$= \frac{1}{365}$$