13. Inverse operations

Suppose we pick a base, 2 say.
Suppose we pick a power, 8 say.
We will now raise the base 2 to the power 8, to give $2^8$.
Suppose we now take logarithms to base 2 of $2^8$.
We then have

$$\log_2 2^8$$

Using the laws of logarithms we can write this as

$$8 \log_2 2$$

Recall that $\log_a a = 1$, so $\log_2 2 = 1$, and so we have simply 8 again, the number we started with.
So, raising the base 2 to a power, and then finding the logarithm to base 2 of the result are inverse operations.
Let’s look at this another way.
Suppose we pick a number, 8 say.
Suppose we find its logarithm to base 2, to evaluate $\log_2 8$.
Suppose we now raise the base 2 to this power: $2^{\log_2 8}$.
Because $8 = 2^3$ we can write this as $2^{3 \log_2 2}$. Using the laws of logarithms this equals $2^{3 \log_2 2}$ which equals $2^3$ or 8, since $\log_2 2 = 1$. We see that raising the base 2 to the logarithm of a number to base 2 results in the original number.
So raising a base to a power, and finding the logarithm to that base are inverse operations. Doing one operation, and then doing the other we end up where we started.

**Example**
Suppose we are working in base $e$. We can pick a number $x$ and evaluate $e^x$. If we follow this by taking logarithms to base $e$ we obtain

$$\ln e^x$$

Using the laws of logarithms this equals

$$x \ln e$$

but $\ln e = 1$ and so we are left with simply $x$ again. So, raising the base $e$ to a power, and then finding logarithms to base $e$ are inverse operations.

**Example**
Suppose we are working in base 10. We can pick a number $x$ and evaluate $10^x$. If we follow this by taking logarithms to base 10 we obtain

$$\log 10^x$$

Using the laws of logarithms this equals

$$x \log 10$$

but $\log 10 = 1$ and so we are left with simply $x$ again. So, raising the base 10 to a power, and then finding logarithms to base 10 are inverse operations.