(iii) Additive inverse of $(2,1)(-4,6)$

Let $z_1 = 2+i \quad z_2 = -4+6i$

$$z_1 z_2 = (2+l)(-4+6i) = -8 + 8i - 6i^2$$

$$= -2+8i$$

Additive inverse is (-2, -8)

4. **If** $z_1 = (6,-3)$ *and* $z_2 = (2,-1)$ **then find** z_1/z_2

Solution: -

Given $z_1 = 6+3i \quad z_2 = 2-i$

$$\frac{z_1}{z_2} = \frac{6+3i}{z-i} = \frac{(6+3i)(2+i)}{4-i^2} = \frac{8+12i+3i^2}{5}$$

$$1+\frac{12}{5}i = \left(1, \frac{12}{5}\right)$$

5. **If** $z = \cos\theta + i\sin\theta$ **then find** $z - \frac{1}{z}$

Solution: -

$$z = \cos\theta + i\sin\theta$$

$$\Rightarrow \frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$$

$$= \cos\theta - i\sin\theta$$

$$z - \frac{1}{z} = 2i\sin\theta$$

iv)z = $\frac{4+3i}{(2+3i)(4-3i)}$

$$=\frac{4+3i}{8-6i+12i+9}$$

$$=\frac{4+3i}{17+6i}$$

$$=\frac{4+3i}{17+6i}\times\frac{17-6i}{17-6i}$$

$$=\frac{(68+18)+i(-24+51)}{(17)^2-(6i)^2}$$

$$=\frac{86+27i}{289+36}$$

$$=\frac{86+27i}{325}$$

$$=\frac{86}{325}+i\frac{27}{325}$$

v) z = $(-5i)\left(\frac{i}{8}\right)$

$$z=\frac{-5}{8}i^2$$

$$z=\frac{5}{8}+0i$$

vi)z = $\frac{2+5i}{3-2i}+\frac{2-5i}{3+2i}$

$$z=\frac{(2+5i)(3+2i)}{(3-2i)(3+2i)}+\frac{(2-5i)(3-2i)}{(3-2i)(3-2i)}$$

$$z=\frac{6+4i+15i+10i^2}{9+4}+\frac{6-4i-15i+10i^2}{9+4}$$

$$z=\frac{-4+19i-4-19i}{13}$$

$$z=\frac{-8}{13}+0i$$

10. Find a square root for the following complex numbers.

i) 7 + 24i

ii) $-47+i8\sqrt{3}$

Sol: i) z = 7 + 24i

Let square root of z be a + ib

$$a+ib=\sqrt{7+24i}$$

$$(a+ib)^2=7+24i$$

$$a^2-b^2+2abi=7+24i$$

$$a^2-b^2=8, 2ab=24 \quad ...(1)$$

$$|a+ib|=|\sqrt{7+24i}|$$

Squaring on both sides,

$$|a+ib|^2=|7+24i|$$

$$a^2+b^2=\sqrt{49+576}$$

$$a^2+b^2=\sqrt{625}=25 \quad ...(2)$$

Adding

$$\begin{array}{l} a^2-b^2=7 \\ \underline{a^2+b^2=25} \\ 2a^2=7+25 \\ a^2=16 \\ a=\pm 4 \end{array}$$

$2b^2 = 25 - 7$

$b^2 = 9$

$b = \pm 3$

$a + ib = \pm(4 + 3i)$.

ii) $-47+i8\sqrt{3}$

Let the square root of z be a + ib,

$$(a+ib)^2=-47+i8\sqrt{3}$$

$$a^2-b^2=-47, 2ab=8\sqrt{3}$$

$$|a+ib|=|\sqrt{-47+i8\sqrt{3}}|$$

19. Show that the points in the Argand diagram represented by the complex numbers 2 + 2i, –2 – 2i, $-2\sqrt{3}+2\sqrt{3}\,i$ are the vertices of an equilateral triangle.

Sol: **Let** A(2, 2), B(–2, –2), C($-2\sqrt{3}, 2\sqrt{3}$) be points represents given complex numbers in the argand plane .

$$AB=\sqrt{(2+2)^2+(2+2)^2}=4\sqrt{2}$$

$$BC=\sqrt{(-2+2\sqrt{3})^2+(-2-2\sqrt{3})^2}$$

$$BC=\sqrt{4+12-8\sqrt{3}+4+12+8\sqrt{3}}=4\sqrt{2}$$

$$AC=\sqrt{(2+2\sqrt{3})^2+(2-2\sqrt{3})^2}=4\sqrt{2}$$

AB = AC = BC

ΔABC is equilateral.

20. Find the eccentricity of the ellipse whose equation is $|z-4|+\left|z-\frac{12}{5}\right|=10$

Sol. Given equation is of the form $SP+S^1P=2a$

Where $S(4,0)$ $S'\left(\frac{12}{5},0\right)$ and $2a=10 \Rightarrow a=5$

SS’= 2ae

$$\Rightarrow 4-\frac{12}{5}=2X\,5e \Rightarrow \frac{8}{5}=10e \Rightarrow e=\frac{4}{5}$$

21. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.

Sol: $\frac{a+ib}{a-ib}=\frac{(a+ib)\,(a+ib)}{(a-ib)\,(a+ib)}$

$$=\frac{(a)^2+(ib)^2+2a(ib)}{(a)^2-(ib)^2}$$

$$=\frac{a^2-b^2+2iab}{a^2+b^2}$$

Short Answer Questions

1.

i) If $(a+ib)^2 = x+iy$**, find $x^2 + y^2$.**

Sol: i) $(a+ib)^2 = x+iy$

$$a^2 - b^2 + 2abi = x+iy$$

$$a^2 - b^2 = x$$

$$2ab = y$$

Now $x^2 = (a^2 - b^2)^2$

$$y^2 = 4a^2 b^2$$

$$x^2 + y^2 = (a^2 - b^2)^2 + 4a^2b^2 = (a^2 + b^2)^2$$

ii) If $x + iy = \dfrac{3}{2+\cos\theta + i\sin\theta}$ **then show that $x^2 + y^2 = 4x - 3$.**

$x + iy = \dfrac{3}{2+\cos\theta + i\sin\theta}$ **rationalizing the dr.**

$$= \frac{3(2+\cos\theta - i\sin\theta)}{(2+\cos\theta)^2 - i^2\sin^2\theta}$$

$$= \frac{3(2+\cos\theta - i\sin\theta)}{4+\cos^2\theta + 4\cos\theta + \sin^2\theta}$$

$$= \frac{6+3\cos\theta - 3i\sin\theta}{5+4\cos\theta}$$

$$= \frac{6+3\cos\theta}{5+4\cos\theta} + \frac{-3i\sin\theta}{5+4\cos\theta}$$

$$x = \frac{6+3\cos\theta}{5+4\cos\theta},\ y = \frac{-3\sin\theta}{5+4\cos\theta}$$

L.H.S. =

$$x^2 + y^2 = \left(\frac{6+3\cos\theta}{5+4\cos\theta}\right)^2 + \left(\frac{-3\sin\theta}{5+4\cos\theta}\right)^2$$

$$= \frac{36+9\cos^2\theta + 36\cos\theta + 9\sin^2\theta}{(5+4\cos\theta)^2}$$

$$\sqrt{x^2+(y+4)^2}+\sqrt{x^2+(y-4)^2}=10$$

$$x^2+(y+4)^2=\left(10-\sqrt{x^2(y-4)^2}\right)^2$$

$$x^2+(y+4)^2=$$

$$100+x^2+(y-4)^2-20\sqrt{x^2+(y-4)^2}$$

Solving we get

$25x^2 + 9y^2 = 225$ is ellipse.

Centre (0, 0)

Eccentricity = e = $\sqrt{\dfrac{a^2-b^2}{a^2}}$

$$=\sqrt{\frac{25-9}{25}}$$

$$e=\frac{4}{5}.$$

8. If z_1, z_2 are two non-zero complex numbers satisfying

i) $|z_1 + z_2| = |z_1| + |z_2|$ then show that Arg z_1 – Arg z_2 = 0.

Sol: i)$|z_1 + z_2| = |z_1| + |z_2|$

Squaring both sides

$$|z_1+z_2|^2=(|z_1|+|z_2|)^2$$

$$(z_1+z_2)(\overline{z}_1+\overline{z}_2)$$

$$=|z_1|^2+|z_2|^2+2|z_1||z_2|$$

$$z_1\overline{z}_1+z_2\overline{z}_2+z_1\overline{z}_2+z_2\overline{z}_1$$

$$=|z_1|^2+|z_2|^2+2|z_1||z_2|$$

$$z_1\overline{z}_2+z_2\overline{z}_1=2|z_1||z_2|$$

$$(x_1+iy_1)(x_2-iy_2)+(x_2+iy_2)(x_1-iy_1)$$

$$=2\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$$

Squaring on both sides we get