C2 notes

Algebraic and Functions

Factor Theorem

- Show that $x - 1$ is a factor of $2x^3 - 3x^2 - x + 2$
  - Rearrange $x - 1$ to find $x, x = 1$
  - Sub $x$ into the equation $[2(1)^3 - 3(1)^2 - 1 + 2]$
  - If equation equals to 0 then $x - 1$ is a factor
  - $[2(1)^3 - 3(1)^2 - 1 + 2] = 0 \therefore x - 1$ is a factor

Long Division

- Find the solutions of $2x^3 - 3x^2 - x + 2$
  - $x - 1$ is a factor $\therefore$ divide equation by $x - 1$

\[
\begin{align*}
2x^2 - x - 2 \\
x - 1 & \overline{2x^3 - 3x^2 - x + 2} \\
- 2x^3 + 2x^2 & \\
\underline{x^2 - x} & \\
- x^2 + x & \\
- 2x + 2 & \\
- 2x + 2 & \\
0 & \\
\end{align*}
\]

- Divide the highest term (the highest power of $x$) by the highest term in the factor $(x)$
- Multiply the answer $(2x^2)$ by the factor
- Subtract the answer from the 2 highest term in the equation $([2x^3 - 3x^2] - [2x^2 - x])$
- The highest terms should cancel out
- Bring down the next term
- Repeat steps from the start until you are left with a remainder or 0

- $2x^3 - 3x^2 - x + 2$ divided by $x - 1$ has 0 has a remainder so $x - 1$ is one of the solutions of the equation. The other 2 can be found by factorising the quadratic equation got from long division $(2x^2 - x - 2)$

- Solutions are $1 + \sqrt{17}/4$, $1 - \sqrt{17}/4$ and $x - 1$ - using the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - If the equation has missing terms for example $3x^4 - 2x^2 + 4$, simply rewrite and put 0 for the co-efficient of the missing $x$ terms and use long division to find solutions; so $3x^4 - 2x^2 + 4 = 3x^4 - 0x^3 + 2x^2 + 0x + 4$

Exponentials

- Exponential functions: $y = a^x$
\[ \text{Area} = \frac{142}{360} \times \pi \times 6^2 = 44.6 \text{cm}^2 \]

\[ \text{Perimeter} = \left( \frac{142}{360} \times 2\pi \times 6 \right) + (6 + 6) = 14.9 + 12 = 26.9 \text{cm} \]

1. Find the area of the minor segment (shaded below), giving your answer to 1 dp.

\[ \text{Area of sector} - \text{Area of triangle} = \text{Area of segment} \]

\[ \text{Area of sector} = \frac{70}{360} \times \pi \times 5.3^2 = 17.2 \text{cm}^2 \]

\[ \text{Area of triangle} = \frac{1}{2} (5.3 \times 5.3) \sin 1.2 \quad = 13.2 \text{cm}^2 \]

\[ \text{Area of segment} = 17.2 - 13.2 = 4.0 \text{cm}^2 \]

2. Find the area of the major segment (shown below), giving your answer to 1 dp.

\[ \text{Area of major segment} = \text{Area of circle} - \text{Area of minor segment} \]

\[ \text{Area of circle} = \pi \times 3.8^2 = 45.36 \text{cm}^2 \]

\[ \text{Area of minor segment} = \text{Area of sector} - \text{Area of triangle} \]

\[ \text{Area of sector} = \frac{12}{2\pi} \times \pi \times 3.8^2 = 3.3 \text{cm}^2 \]

\[ \text{Area of triangle} = \frac{1}{2} (3.8 \times 3.8) \sin 1.2 \quad = 6.72 \text{cm}^2 \]

\[ \text{Area of major segment} = 45.36 \text{cm}^2 - 6.72 \text{cm}^2 = 1.94 \text{cm}^2 \]

Note: The area of the major segment is 1.94 cm^2.