

The forms highlighted in red *do not seem to be* equivalent of "if p then q ". What follows is a brief justification of their equivalences.

q is necessary for p (a necessary condition for p is q)

This means that q must be true whenever p is true. However, it does not say anything about truth value of q when p is false. Hence, the statement is *false only* when p is true and q is false, but it is true otherwise. Thus, it is logically equivalent to $p \rightarrow q$.

p only if q

Note that " p only if q " says that p cannot be true when q is not true. That is, the statement is *false only* in the case when q is false and p is true. When p is false, q may be either true or false, because the statement says nothing about the truth value of q in that case. Hence, this is logically equivalent to "if p then q ".

q unless $\neg p$

This means that q is true as long as $\neg p$ is false. Hence, the statement " q unless $\neg p$ " is false only when p is true and q is false, but it is true otherwise. Consequently, " q unless $\neg p$ " and $p \rightarrow q$ always have the same truth value.

Biconditionals

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition " p if and only if q " (abbreviated p iff q). The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

There are some other common ways to express $p \leftrightarrow q$: " p is necessary and sufficient for q " and "if p then q , and conversely"

Example

Let p : "You can take the flight" and q : "You buy a ticket." Then $p \leftrightarrow q$ is the statement "You can take the flight if and only if you buy a ticket."

This statement is true if p and q are either both true or both false, that is, if you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight. It is false when p and q have opposite truth values, that is, when you do not buy a ticket, but you can take the flight (such as when you get a free trip) and when you buy a ticket and cannot take the flight (such as when the airline bumps you).

You should also note that

Propositions p and q are logically equivalent (i.e. $p \equiv q$) iff $p \leftrightarrow q$ is a tautology.

یہ سنگ و خشت نہیں جو تری نگاہ میں ہے
(اقبال)

وہی جہاں ہے ترا جس کو تو کرے پیدا