Properties of GP:-

(i) $a, b, c$ are in GP $\Rightarrow b^2 = ac$

If $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ are in GP

a) $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ are in GP
b) $\lambda a_{1}, \lambda a_{2}, \lambda a_{3}, \ldots, \lambda a_{n}$ are in GP

(\(\lambda \in R \setminus \{0\}\))

c) $a'_{1}, a'_{2}, a'_{3}, \ldots, a'_{n}$ are in GP for $a'_{k} = \frac{1}{a_{k}}$

d) $\frac{1}{a_{1}}, \frac{1}{a_{2}}, \frac{1}{a_{3}}, \ldots, \frac{1}{a_{n}}$ are in GP

The product of the terms of GP in a given order is always the same and is equal to the product of the first and last term.

(iii) $a_{1}a_{2}a_{3}\cdots a_{n} = a_{n}a_{n-1}\cdots a_{1}$

(iv) If $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ is a GP of non-zero, non-negative terms then

$\log a_{1}, \log a_{2}, \log a_{3}, \ldots, \log a_{n}$ are in AP and vice versa.

Geometric mean (GM):-

(i) The geometric mean $G_{n}$ of any two numbers $a$ and $b$ is given by $\sqrt{ab}$ where $a, G_{n}, b$ are in GP.

(ii) If $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ be $n$ numbers then geometric mean of these numbers is

$G_{n} = \left(\frac{a_{1}a_{2}a_{3}\cdots a_{n}}{n}\right)^{\frac{1}{n}}$

(iii) The $n$ numbers $G_{1}, G_{2}, G_{3}, \ldots, G_{n}$ are said to be geometric means between $a$ and $b$ if $a, G_{1}, G_{2}, G_{3}, \ldots, G_{n}, b$ are in GP.

Here $a =$ First term ; $b = (n + 2)th$ term.

then $r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$ ; $G_{1} = a \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$

$G_{2} = a \left(\frac{b}{a}\right)^{\frac{2}{n-1}}$ ; $G_{n} = a \left(\frac{b}{a}\right)^{\frac{n-1}{n-1}}$

$G_{1}, G_{2}, G_{3}, \ldots, G_{n} = \left(\sqrt[n]{ab}\right)^{n} = \left(GM \ of \ a, b\right)^{n}$

Some facts about GP: -

If $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ and $b_{1}, b_{2}, b_{3}, \ldots, b_{n}$ are two GP's then

(i) $a_{1} \pm b_{1}, a_{2} \pm b_{2}, a_{3} \pm b_{3}, \ldots, a_{n} \pm b_{n}$ are not in GP

(ii) $a_{1}b_{1}, a_{2}b_{2}, a_{3}b_{3}, \ldots, a_{n}b_{n}$ are in GP

Increasing and decreasing GP: -

Let $a, ar, ar^2, \ldots, be$ a GP

a) If $a > 0$, $r > 1$ then it is increasing GP

b) If $a > 0$, $0 < r < 1$ then it is decreasing GP

c) If $a < 0$, $r > 1$ then it is decreasing GP

d) If $a < 0$, $0 < r < 1$ then it is increasing GP

Arithmetic - Geometric progression (APGP): If $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ be in AP and $h_{1}, h_{2}, h_{3}, \ldots, h_{n}$ be in GP then

$a_{1}h_{1}, a_{2}h_{2}, a_{3}h_{3}, \ldots, a_{n}h_{n}$ is said to be APGP.

An APGP is of the form

$ah_{1}(a + d), bh_{2}(a + 2d), cr_{1}(a + 4d), \ldots, dh_{n}(a + nd)$

(i) $n^{th}$ term of an APGP

$T_{n} = [a + (n - 1)d]h_{r}^{n-1}$

\[ S_{n} = \frac{ah_{1}}{1 - r} \left[ \frac{1 - r^{n}}{1 - r} \right] \]

(ii) If $-1 < r < 1$, then sum to infinite terms

$s_{n} = \frac{ah_{1}}{1 - r} + \frac{ah_{n}}{r}$

Harmonic Progression (HP):-

A sequence is an H.P if the reciprocals of its terms form an AP.

H.P is of the form

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots, \frac{1}{a+(n-1)d}$