Starting from symmetry
The starting point is that the result of simple collisions can be predicted without knowing anything much at all. If two objects come together at the same speed in front of you, and stick together, there is no way the combined object can go off to the left or right.

Just moving makes no difference
The second starting point is that observing a collision from a differently moving point of view can’t change the actual collision. But it can change how it looks. An object which was previously seen moving will be seen standing still, if you are travelling along with it. Result: predictions for some simple collisions can be automatically extended to others.
Also, if just moving while observing a collision changes nothing essential, it must be that the rules for how collisions work can depend only on changes of the velocity of objects, not on their velocities. Changes of velocity of an object don’t alter when you add or subtract an observer’s velocity from the object’s velocity before and after the collision.

Changes of velocity
The argument so far is that the outcome of a collision must be described in terms of changes of the velocity of the colliding objects. But this isn’t enough. Big hefty objects don’t change velocity much when they collide with small light ones, which do then change velocity a lot.

Introducing mass
We can define the idea of mass by the ratio of the changes of velocity of two objects in a collision. That is:

\[
\frac{m_1}{m_2} = \frac{\Delta v_2}{\Delta v_1}.
\]

The more massive of the two has the smaller velocity change. Knowing only ratios in this way, a standard unit is needed. It is the standard kilogram, a lump of metal kept in Paris.

Introducing momentum
Symmetry says that two identical objects must have equal and opposite changes of velocity in a collision. When the masses are unequal, symmetry now says that the changes in the quantity \( m \Delta v \) must be equal and opposite. In a collision there is no overall change in the quantity of \( m v \), called momentum. Give momentum the symbol \( p \).

Inside a collision
Inside a collision, the two objects deform one another. Forces act on each. To think about this, split the collision into two parts, one for each object, and part forget about the other object. Just represent the effect of the other object as a force acting on the object you are concerned with.

Force is rate of change of momentum
The effect of the collision is to change the momentum \( p \) of an object, over the time \( dt \) of the collision. Define the force \( F \) which represents the action of the other object as

\[
F = \frac{dp}{dt}.
\]

This can be written

\[
F = \frac{d(mv)}{dt}
\]

which if the mass is constant can be expressed as

\[
F = m \frac{dv}{dt} = ma.
\]

Thus we reach Newton’s second law.

Equal and opposite forces
We now see why ‘action’ and ‘reaction’ are equal and opposite. Two forces, one on each object, changing the momentum of both by equal but opposite amounts \( \Delta p \), represent one single collision. It must last the same time for both objects so the pairs of forces introduced to split the collision must be equal and opposite.

Look for the other object
Newton’s law \( F = ma \) splits one interaction into two pieces. Talk of a force on an object should now make you ask where the other interacting object is. Sometimes it’s obvious – a fist for example. Sometimes it’s obvious when you think about it – the Earth pulling things down for example. And sometimes it isn’t obvious at all. The discovery of outer planets by their effects on inner ones is one example. The inference that there may be a black hole at the centre of our galaxy is another.