\[(D - m_1)(D - m_2) \cdots (D - m_n)y = 0\] 

Auxiliary Eqn \(A.E\)

Before it see diff operator

Finding P.I by method of variation of parameters.

So, let,

\[y'' + py' + qy = x^3\]  \[p, q, x \text{ are } f(x)\]

let \(y_1, y_2\) be two of it \& \(C_1, C_2\) are C.F

\[C_1y_1 + C_2y_2 \rightarrow \text{C.F of 0} \]

let \(y_1, y_2\) get by

... AF

Replace \(d^2y/dx^2\) \(d^2y/\partial x^2\)

\(\text{If } m = \pm 1\)

\[C_1e^t + C_2e^{-t}\]

\[u' = \frac{y_2x}{y_1y_2 - y_1'y_2'} - \frac{y_1x}{y_1y_2 - y_1'y_2'}\]

\[v = \frac{y_2y_1}{y_1y_2 - y_1'y_2'}\]
Homogeneous Linear Diff Eq \( -n \) (reduced to linear Diff Eq \( n \))

with constant coefficients

\[
x^n \frac{d^ny}{dx^n} + a_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_n y = x(x).
\]

Suppose \( x = e^t \)

\[
\frac{dy}{dt} = \frac{1}{x} \frac{dy}{dt}.
\]

\[
\frac{dy}{dt} = x \frac{dy}{dt} = D_y
\]

\[\text{llly, } x^2 \frac{d^2y}{dx^2} = \boxed{D(D-1)y}\]

\[x^3 \frac{d^3y}{dx^3} = \boxed{D(D-1)(D-2)y}\]

\[\text{and so on}\]

Diffeq solving using Differential Operator

We know that,

\[(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = x \]

\[D_y y = x(0)\]

Differential Operator

So, the general form is \( y = f(D) y = 0\)

\[D_y^m y = a_m + a_{m-1} D + \cdots + a_1 D^{m-1} + a_0 D^m y = 0\]

So, auxiliary Eq

\[f(m) = 0 \Rightarrow m^n + a_m + a_{m-1} m^{n-1} + \cdots + a_0 = 0\]
4. Higher Ratio Test (Raabe's Test):
\[ \lim_{n \to \infty} n \left( \frac{u_n}{u_{n+1}} \right) = K, \quad K > 1, \text{ cgt.} \]
\[ K < 1, \text{ dgt.} \]

5. Logarithmic Test:
\[ \lim_{n \to \infty} n \left( \log \left( \frac{u_n}{u_{n+1}} \right) \right) = K, \quad K > 1, \text{ cgt.} \]
\[ K < 1, \text{ dgt.} \]

6. Cauchy Integral Test:
For \( x \geq 1 \), \( f(x) \) is non-negative, monotonically increasing function of \( x \):
\[ f(n) = u_n. \]
So, for all values of \( n \), \( \sum_{n=1}^{\infty} f(x) \Delta x \)

\[ \int_{a}^{b} f(x) \, dx = \pm \infty, \quad (\text{dgt. or cgt.}) \]

→ Alternating Series:
\[ \sum_{n=1}^{\infty} (-1)^{n+1} u_n \]

→ Leibniz Test:
- Alternating series is cgt. if:
  - \( u_n > u_{n+1} \) or \( u_n < u_{n+1} \) or \( \lim_{n \to \infty} |u_n| = 0. \]
- If not cgt., then osc. (no case of dgt.)
Absolute C.G.T.
\[ 3! |u| = 1u_1 + 1u_2 + 1u_3 + \ldots \text{ is C.G.T.} \] (Leibnitz also
- Conditionally C.G.T.
  - Leibnitz
  \[ \sum 3! |u| = 1u_1 + 1u_2 \ldots \times \text{C.G.T.} \]

Successive Differentiation

\[ y = f(x), \quad y' = f'(x), \quad \ldots \quad y^{(n)} = f^{(n)}(x). \]

\[ e^x \rightarrow y = e^x, \quad y' = a^n e^x \]

\[ (x+b)^m \rightarrow y = (x+b)^m, \quad y' = (x+b)^{m-1} \cdot m! \cdot x^n \]

\[ \text{If } m = m, \quad y = m! a^m \]

\[ \text{If } m < m, \quad y = m! a^m \]

\[ \text{If } m > m, \quad y = m! a^m \]

\[ y = \log (x+b) \rightarrow y_1 = \frac{a}{x+b} \]

\[ y^{(n)} = \frac{a^n \cdot (-1)^{n-1} \cdot x \cdot (n-1)!}{(x+b)^{n+1}} \]
Reduction Formula

- Connects an integral with other of same type integral, but of lower order.

\( I_n = \int \sin^n x \, dx = \int \sin^{(n-1)} x \sin x \, dx \)

\[ = \frac{\sin^{(n-1)} x \cos x + (n-1) I_{n-2} - (n-1) I_n}{n} \]

\( I_n = \frac{1}{n} \left[ \frac{\sin^{(n-1)} x \cos x + (n-1) I_{n-2}}{n} \right] \]

\[ \int_0^{\pi/2} \sin^n x \, dx = \frac{\pi}{2^n} [I_n] = \frac{(n-1)}{n} [I_{n-2}] \]

\[ \int_0^{\pi/2} \sin^n x \, dx = \frac{(n-1)(n-3) \ldots 3.1}{n(n-2)(n-4) \ldots 2} \times \frac{\pi}{2} \]

\[ \int_0^{\pi/2} \cos^n x \, dx = \frac{(n-1)(n-3) \ldots 4.2 \ldots}{n(n-2)(n-4) \ldots 2} \times 1 \]

\[ \text{if } n \text{ is even} \]

\[ \text{if } n \text{ is odd} \]