Q4. (12 points) Given that \( y_1(x) = x + 1 \) is a solution of the differential equation

\[
(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0,
\]

find a second solution \( y_2(x) \) of the equation.

**Solution:**

The standard form of the equation is

\[
y'' + \frac{2(1+x)}{1-2x-x^2} y' - \frac{2}{1-2x-x^2} y = 0,
\]

1. \( p(x) = \frac{2(1+x)}{1-2x-x^2} \).

The second solution is

2. \( y_2(x) = y_1(x) \int \frac{e^{-\int p(x) dx}}{y_1(x)} dx \)

= \( (x+1) \int \frac{e^{-\int \frac{2(1+x)}{1-2x-x^2} dx}}{y_1(x)} dx \)

= \( (x+1) \int \frac{\ln(x^2+2x-1)}{(x+1)^2} dx \)

= \( (x+1) \int \frac{x^2+2x-1}{(x+1)^2} dx \)

= \( (x+1) \int \left[ 1 - \frac{2}{(x+1)^2} \right] dx \)

= \( (x+1) \left[ x + \frac{2}{x+1} \right] \)

= \( x^2 + x + 2 \)