Q.P. Code: 4827

(3 Hours) [Total Marks: 80]

N.B.: (1) Question No.1 is compulsory.
(2) Attempt any three from the remaining six questions.
(3) Figures to the right indicate full marks.

Q1a Find Laplace Transform of \( \frac{\sin t}{t} \)

b Prove that \( f(x) = \sinh x \) is analytic and find its derivative.

c Find Fourier Series for \( f(x) = 9 - x^2 \) over \((-3, 3)\)

d Find \( Z[f(k) + g(k)] \) if \( f(k) = \frac{1}{3^k}, g(k) = \frac{1}{5^k} \)

Q2a Prove that \( \overrightarrow{F} = ye^{xy} \cos z \mathbf{i} + xe^{xy} \sin z \mathbf{j} - e^{xy} \cos z \mathbf{k} \) is rotational. Find Scalar Potential for \( \overrightarrow{F} \)

Hence evaluate \( \int_{C} \overrightarrow{F} \cdot d\overrightarrow{r} \) along the curve \( C \) joining the points \((0, 0, 0)\) and \((-1, 2, \pi)\)

b Find the Fourier series for \( f(x) = \frac{\pi - x}{2}; 0 \leq x \leq 2\pi \).

c Find Inverse Laplace Transform of \( i) \frac{s + 29}{(s^2)(s^2 + 9)} \) \( ii) \frac{e^{-2s}}{s^2 + 8s + 25} \)

Q3a Find the Analytic function \( f(z) = u + iv \) if \( u + v = \frac{x}{x^2 + y^2} \)

b Find Inverse Z transform of \( \frac{1}{(z - 1/2)(z - 1/3)} \) for \( 1/3 < |z| < 1/2 \)

c Solve the Differential Equation \( \frac{d^2y}{dt^2} + y = t \), \( y(0) = 1 \), \( y'(0) = 0 \), using Laplace Transform.

Q4a Find the Orthogonal Trajectory of \( 3x^2y - y^3 = k \)

b Using Green's theorem evaluate \( \int_{C} (xy + y^2)dx + x^2dy \), \( C \) is closed path formed by \( y = x, y = x^2 \)

c Find Fourier Integral of \( f(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & x > \pi \end{cases} \). Hence show that \( \int_{0}^{\pi} \frac{\cos(\lambda \pi/2)}{1 - \lambda^2} d\lambda = \frac{\pi}{2} \)