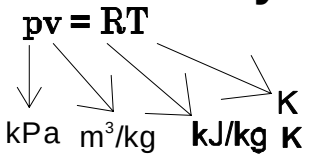
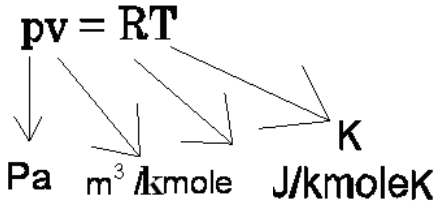
Some special units for Thermodynamics



Note: Physicists use below units



Universal gas constant, R_u= 8.314 kJ/kmole – K

Characteristic gas constant, $R_c = \frac{R_u}{M}$

For Air $R = \frac{8.314}{29} = \frac{\text{kJ/kmole-K}}{\text{kg/kmole}}$

= 0.287 kJ/kg- K

For water $R = \frac{8.314}{18}\ \frac{\text{kJ/kmole-K}}{\text{kg/kmole}}$

= 0.461 kJ/kg -K

Units of heat and work is kJ

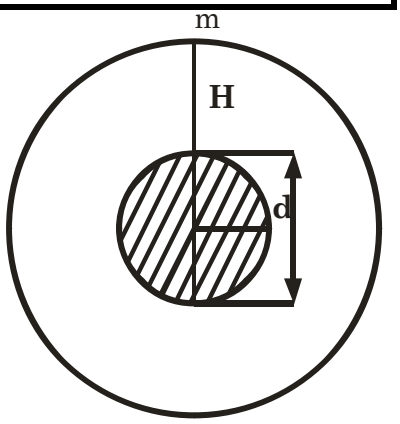
Units of pressure is kPa

1 atm = 101.325 kPa

1 bar = 100 kPa

1 MPa =1000 kPa.

Force of attraction $= \dfrac{GMm}{\left(d/2 + H\right)^2}$ ...



1.3

(i)

If we place it in a surface of earth then

Force of attraction $= \dfrac{GMm}{\left(d/2\right)^2} = mg_o$

or $g_o = \dfrac{GM}{\left(d/2\right)^2}$

$\therefore$ Weight $(W) = \dfrac{GMm}{\left(d/2 + H\right)^2}$

$= \dfrac{mg_o\left(d/2\right)^2}{\left(d/2 + H\right)^2}$ from equation...(i)

$= mg_o\left(\dfrac{d}{d+2H}\right)^2$ Proved

Q1.4 **The first artificial earth satellite is reported to have encircled the earth at a speed of 28,840 km/h and its maximum height above the earth's surface was stated to be 916 km. Taking the mean diameter of the earth to be 12,680 km, and assuming the orbit to be circular, evaluate the value of the gravitational acceleration at this height.**

The mass of the satellite is reported to have been 86 kg at sea-level. Estimate the gravitational force acting on the satellite at the operational altitude.

(**Ans.** 8.9 m/s^2; 765 N)

Solution: Their force of attraction = centrifugal force

Centirfugal force $= \dfrac{mv^2}{r}$

$= \dfrac{86 \times \left(\dfrac{28840 \times 1000}{60 \times 60}\right)^2}{\left(\dfrac{12680 \times 10^3}{2} + 916 \times 10^3\right)}$ N

$= 760.65$ N (Weight)

Q1.5 **Convert the following readings of pressure to kPa, assuming that the barometer reads 760 mmHg:**

(a) 90 cmHg gauge (b) 40 cmHg vacuum

(c) 1.2 m H_2O gauge (d) 3.1 bar

Solution: 760 mm Hg = $0.760 \times 13600 \times 9.81$ Pa

$= 10139.16$ Pa

$\simeq 101.4$ kPa

Solution: $dp = dh\,\rho g$

or $dp = dh \times \frac{1}{v} \times g$

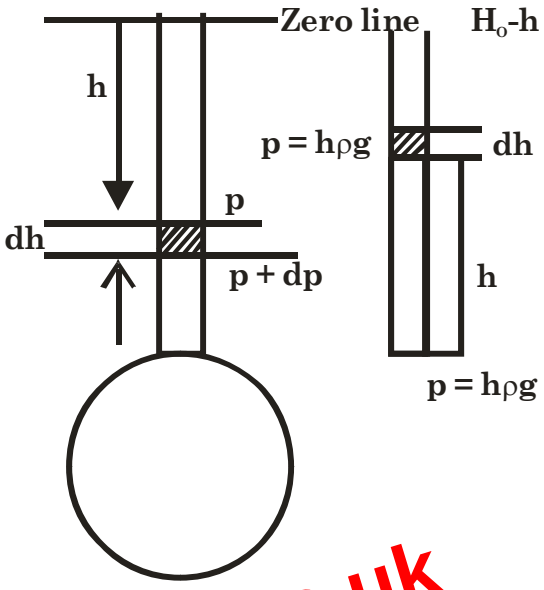
or $v = \frac{g\,dh}{dp}$

$pv^{1.4} = 2.3\times10^3 = 2300$

or $v = \left(\frac{2300}{p}\right)^{\frac{1}{1.4}} = \left(\frac{2300}{p}\right)^{n}$ where $n = \frac{1}{1.4}$

or $\frac{g\,dh}{dp} = \left(\frac{2300}{p}\right)^{n}$

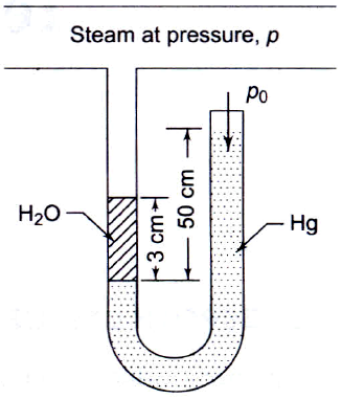
or $g\,dh = \left(\frac{2300}{p}\right)^{n} dp$



or $\int_0^H dh = \frac{2300^n}{g} \int_0^{101320} \frac{dp}{p^n}$

or $h = \frac{2300^n}{g(1-n)}\left[(101320)^{(1-n)} - 0\right] = 2420\,\text{m} = 2.42\ \text{km}$

Q1.8 **The pressure of steam flowing in a pipeline is measured with a mercury manometer, shown in Figure. Some steam condenses into water. Estimate the steam pressure in kPa. Take the density of mercury as 13.6×10^3 kg/m^3, density of water as 10^3 kg/m^3, the barometer reading as 76.1 cmHg, and g as 9.806 m/s^2.**



Solution: $p_o + 0.50\times\rho_{Hg}\times g = 0.03\times\rho_{H_2O}\times g + p$

or $p = 0.761\times13.6\times10^3\times9.806 + 0.5\times13.6\times10^3\times9.806 - 0.03\times1000\times9.806$ Pa.

$= 167.875$ kPa

Q1.9 **A vacuum gauge mounted on a condenser reads 0.66 mHg. What is the absolute pressure in the condenser in kPa when the atmospheric pressure is 101.3 kPa?**

(Ans. 13.3 kPa)

Solution: Absolute = atm. – vacuum

$= 101.3 - 0.66\times13.6\times10^3\times9.81\times10^{-3}$ kPa

$= 13.24$ kPa

Q2.4 **when the reference junction of a thermocouple is kept at the ice point and the test junction is at the Celsius temperature *t,* and e.m.f. *e* of the thermocouple is given by the equation**

$$\varepsilon = at + bt^2$$

Where a = 0.20 mV/deg, and $b = -5.0 \times 10^{-4}$ mV/deg^2

(a) Compute the e.m.f. when t = - 100°C, 200°C, 400°C, and 500°C, and draw graph of ε against t in this range.

(b) Suppose the e.m.f. ε is taken as a thermometric property and that a temperature scale t^* is defined by the linear equation.

$$t^* = a'\varepsilon + b'$$

And that $t^* = 0$ at the ice point and $t^* = 100$ at the steam point. Find the numerical values of a' and b' and draw a graph of ε against t^*.

(c) Find the values of t^* when t = -100°C, 200°C, 400°C, and 500°C, and draw a graph of t^* against t.

(d) Compare the Celsius scale with the t^* scale.

Solution: Try please

Q2.5 **The temperature t on a thermometric scale is defined in terms of a property K by the relation**

$$t = a \ln K + b$$

Where a and b are constants. The values of K are found to be 1.83 and 6.78 at the ice point and the steam point, the temperatures of which are assigned the numbers 0 and 100 respectively. Determine the temperature corresponding to a reading of K equal to 2.42 on the thermometer.

(Ans. 21.346°C)

Solution:

$$t = a \ln x + b$$

$$0 = a \times \ln 1.83 + b \quad \ldots (i)$$

$$100 = a \times \ln 6.78 + b \quad \ldots (ii)$$

Equation {(ii) – (i)} gives

$$a \cdot \ln\left(\frac{6.78}{1.83}\right) = 100$$

or $a = 76.35$

$\therefore$ $b = -a \times \ln 1.83$

$= -46.143$

$\therefore$ $t = 76.35 \ln k - 46.143$

$\therefore$ $t^* = 76.35 \times \ln 2.42 - 46.143$

$= 21.33°C$

Q2.6 **The resistance of the windings in a certain motor is found to be 80 ohms at room temperature (25°C). When operating at full load under steady state conditions, the motor is switched off and the resistance of the windings, immediately measured again, is found to be 93 ohms. The windings are made of copper whose resistance at temperature t°C is given by**

$\therefore$ Power $= \Delta p\dot{v}$

$$= (0.9 - 0.101325) \times 10^3 \text{kPa} \times \frac{1}{60} \text{ m}^3/\text{s}$$

$$= 13.31 \text{ kJ}/\text{s}$$

(b) So that pressure will be 0.9 MPa

$\therefore$ $h\rho g = 0.9$ MPa

or $h = \dfrac{0.9 \times 10^6}{1000 \times 9.81}$ m $= 91.743$ m

(c) $\dfrac{1}{2}\dot{m}\left(V_2^2 - V_1^2\right) = \Delta p\dot{v}$ where $\dot{m} = \dot{v}\rho$

or $\dfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \Delta p$

or $V_2^2 - V_1^2 = 2\dfrac{\Delta p}{\rho}$

or $V_2^2 = V_1^2 + 2\dfrac{\Delta p}{\rho}$

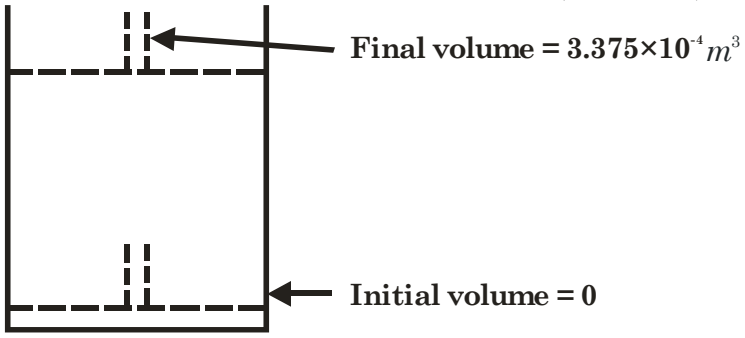
$$= 10^2 + \frac{2 \times (0.9 - 0.101325) \times 10^6}{1000}$$

$V_2 = 41.2$ m/s

Q3.2 The piston of an oil engine, of area 0.0045 m², moves downwards 75 mm, drawing in 0.00028 m³ of fresh air from the atmosphere. The pressure in the cylinder is uniform during the process at 80 kPa, while the atmospheric pressure is 101.325 kPa, the difference being due to the flow resistance in the induction pipe and the inlet valve. Estimate the displacement work done by the air finally in the cylinder.

(Ans. 27 J)

Solution : Volume of piston stroke
$= 0.0045 \times 0.075 \text{m}^3$
$= 0.0003375 \text{m}^3$
$\therefore$ $\Delta V = 0.0003375$ m³
as pressure is constant
$= 80$ kPa
So work done $= p\Delta V$
$= 80 \times 0.0003375$ kJ
$= 0.027$ kJ $= 27$ J



Q3.3 An engine cylinder has a piston of area 0.12 m³ and contains gas at a pressure of 1.5 MPa. The gas expands according to a process which is represented by a straight line on a pressure-volume diagram. The final pressure is 0.15 MPa. Calculate the work done by the gas on the piston if the stroke is 0.30 m.

(Ans. 29.7 kJ)

Solution: Initial pressure (p_1) = 1.5 MPa
Final volume (V_1) = $0.12\text{m}^2 \times 0.3\text{m}$

Solution:

Initial volume $(V_1) = \frac{\pi d^2}{4} \times L$

$= \frac{\pi \times (0.15)^2}{4} \times 0.25 \text{ m}^3$

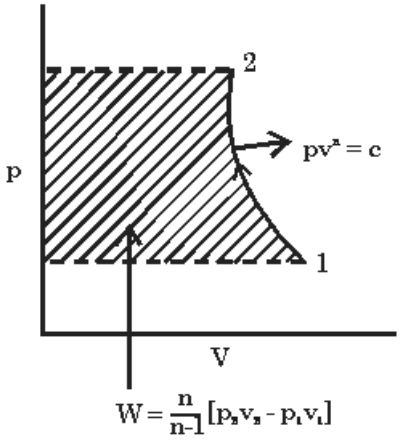
$= 0.00442 \text{ m}^3$

Initial pressure $(p_1) = 101.325$ kPa.

Final volume $(V_2) = \frac{V_1}{5} = 0.000884 \text{ m}^3$

$$p_1 V_1^{1.2} = p_2 V_2^{1.2}$$

Or $\quad p_2 = \frac{p_1 V_1^{1.2}}{V_2^{1.2}} = 699.41 \approx 700 \text{ kPa}$



Work done / unit stroke – unit cylinder (W)

$$= \left(\frac{1.2}{1.2-1}\right) \times [p_1 V_1 - p_2 V_2]$$

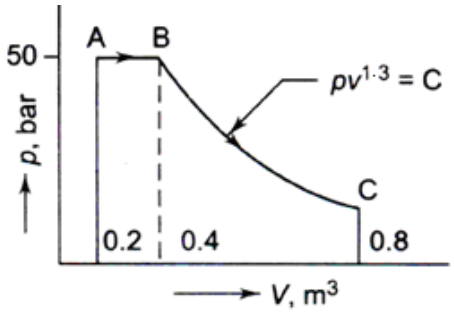
$$= \left(\frac{101.325 \times 0.00442 - 700 \times 0.000884}{1.2-1}\right) \times 1.2$$

(-ive work, as work done on the system)

Power $= \frac{W \times 500 \times 2 \times 1.2}{60}$ kW

$= 17.95$ kW

Q3.13 **Determine the total work done by a gas system following an expansion process as shown in Figure.**



(Ans. 2.253 MJ)

Solution: Area under AB

$= (0.4 - 0.2) \times 50 \times 10^5$ J

$= 10^6 \text{W} = 1 \text{ MJ}$

$$W_{23} = \int_2^3 pdV$$

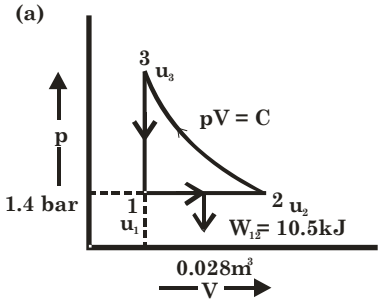
$$= p_2 V_2 \int_2^3 \frac{dV}{V}$$

$$= p_2 V_2 \ln\left(\frac{V_3}{V_2}\right)$$

$$= p_2 V_2 \ln\left(\frac{V_1}{V_2}\right)$$

$$= 1.4 \times 100 \times 0.103 \times \ln\left(\frac{0.028}{0.103}\right)$$

$$= -18.783 \text{kJ} \qquad \left[\begin{array}{ll} \text{as} & W_{12} = p(V_2 - V_1) \\ & 10.5 = 1.4 \times 100 (V_2 - 0.028) \\ \therefore & V_2 = 0.103 \text{ m}^3 \end{array}\right]$$



$u_2 = u_3$
$u_1 - u_3 = -26.4\text{kJ}$
$\therefore\ u_1 = u_3 - 26.4\text{kJ}$
$= u_2 - 26.4\text{kJ}$
$u_2 = u_1 + 26.4\text{kJ}$

$W_{31} = 0$ as constant volume

$\therefore$ Net work output $= -8.283$ kJ ans.(b)

(c) $Q_{12} = U_2 - U_1 + W_{12}$

$= 26.4 + 10.5\text{kJ} = 36.9\text{kJ}$

(d) $Q_{23} = U_3 - U_2 + W_{23}$

$= 0 - 18.783\text{kJ} = -18.783$ kJ

$Q_{31} = U_2 - U_3 + 0 = 26.4\text{kJ}$

$\therefore \sum Q = Q_{12} + Q_{23} + Q_{31} = 36.9\text{kJ} - 18.783 - 26.4$

$= -8.283\text{kJ}$

$\therefore \sum W = \sum Q$ Proved.

If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

(**Ans.** 1.8 MJ)

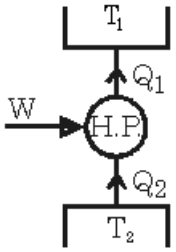
Solution: COP of the Ref. is 5

So for each MJ removed from the cold body we need work

$$= \frac{1\,MJ}{5} = 200\text{ kJ}$$

For 200 kJ work output of heat engine hair $\eta = 30\%$

We have to supply heat = $\frac{200\,kJ}{0.3} = 666.67$ kJ



Now

COP of H.P. = COP of Ref. + 1

= 5 + 1 = 6

Heat input to the H.E. = 1 MJ

$\therefore$ Work output (W) = 1 × 0.3 MJ = 300 kJ

That will be the input to H.P.

$\therefore \left(COP\right)_{H.P} = \frac{Q_1}{W}$

$\therefore$ $Q_1 = (COP)_{H.P.} \times W = 6 \times 300$ kJ = 1.8 MJ

Q6.4 **An electric storage battery which can exchange heat only with a constant temperature atmosphere goes through a complete cycle of two processes. In process 1–2, 2.8 kWh of electrical work flow into the battery while 732 kJ of heat flow out to the atmosphere. During process 2–1, 2.4 kWh of work flow out of the battery.**

(a) Find the heat transfer in process 2–1.

(b) If the process 1–2 has occurred as above, does the first law or the second law limit the maximum possible work of process 2–1? What is the maximum possible work?

(c) If the maximum possible work were obtained in process 2–1, what will be the heat transfer in the process?

(**Ans.** (a) – 708 kJ (b) Second law, $W_{2-1} = 9348$ kJ (c) $Q_{2-1} = 0$)

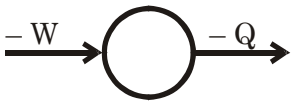
Solution: **From the first Law of thermodynamics**

(a) For process 1–2

$Q_{1-2} = E_2 - E_1 + W_{1-2}$

$-732 = (E_2 - E_1) - 10080$

$\left[2.8\text{ kWh} = 2.8 \times 3600\text{ kJ}\right]$



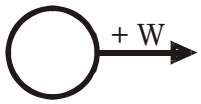
$\therefore$ $E_2 - E_1 = 9348$ kJ

For process 2–1

$Q_{21} = E_1 - E_2 + W_{21}$

$= -9348 + 8640$

$= -708$ kJ i.e. Heat flow out to the atmosphere.



(b) Yes Second Law limits the maximum possible work. As Electric energy stored in a battery is High grade energy so it can be completely converted to the work. Then,

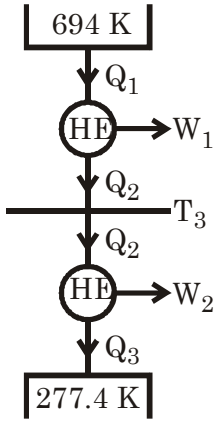
$W = 9348$ kJ

Solution: $\frac{Q_1}{694} = \frac{Q_2}{T} = \frac{Q_1 - Q_2}{694 - T} = \frac{Q_3}{277.4} = \frac{Q_2 - Q_3}{T - 277.4}$

Hence $Q_1 - Q_2 = 2\,W_2$

$Q_2 - Q_3 = W_2$

$\therefore \quad \frac{2}{694 - T} = \frac{1}{T - 277.4}$



or $2T - 277.4 \times 2 = 694 - T$

or $T = 416.27\ K = 143.27^{o}\ C$

(b) $\eta_1 = 40\%$

$\eta_2 = 1 - \frac{277.4}{416.27} = 33.36\%$

(c) $Q_2 = \frac{416.27}{694} \times 200\ kJ = 119.96\ kJ$;

$Q_1 = \frac{277.4}{416.27} \times 119.96 = 79.94\ kJ$

Q6.11 **A heat engine operates between the maximum and minimum temperatures of 671°C and 60°C respectively, with an efficiency of 50% of the appropriate Carnot efficiency. It drives a heat pump which uses river water at 4.4°C to heat a block of flats in which the temperature is to be maintained at 21.1°C. Assuming that a temperature difference of 11.1°C exists between the working fluid and the river water, on the one hand, and the required room temperature on the other, and assuming the heat pump to operate on the reversed Carnot cycle, but with a COP of 50% of the ideal COP, find the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more wasteful?**

(**Ans.** 0.79 kJ/kJ heat input)

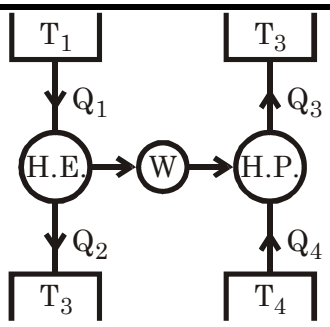
Solution: Carnot efficiency $(\eta) = 1 - \frac{273 + 60}{273 + 671} = 1 - \frac{333}{944} = 0.64725$

Actual $(\eta) = 0.323623 = 1 - \frac{Q_1'}{Q_1}$

Work output (W) = $\frac{Q_1}{T_1}(T_1 - T_2)$

For H.P.

Work input (W) = $\frac{Q_4}{T_4}(T_3 - T_4)$



$\therefore \quad \frac{Q_1}{T_1}(T_1 - T_2) = \frac{Q_4}{T_4}(T_3 - T_4)$

or $$\boxed{\frac{Q_4}{Q_1} = \frac{T_4}{T_1}\left\{\frac{T_1 - T_2}{T_3 - T_4}\right\}}$$

This is the expression.

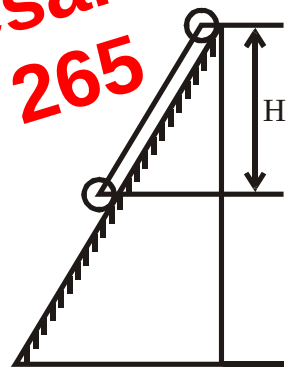
Q6.24 **Prove that the following propositions are logically equivalent:**
(a) A PMM2 is Impossible
(b) A weight sliding at constant velocity down a frictional inclined plane executes an irreversible process.

Solution: Applying First Law of Thermodynamics

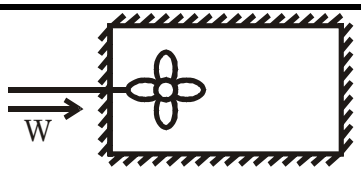
$Q_{12} = E_2 - E_1 + W_{1.2}$

or $0 = E_2 - E_1 - mgh$

or $E_1 - E_2 = mgh$



$$= \int_{273}^{273} m\, c_P \frac{dT}{T} = 0$$

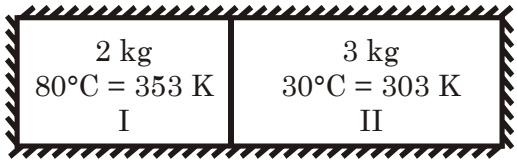


Q7.3 **Two kg of water at 80°C are mixed adiabatically with 3 kg of water at 30°C in a constant pressure process of 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process (c_p of water = 4.187 kJ/kg K).**

(Ans. 0.0576 kJ/K)

Solution: If final temperature of mixing is T_f then

$2 \times c_P (353 - T_f) = 3 \times c_P (T_f - 303)$



or $T_f = 323$ K

$(\Delta S)_{system} = (\Delta S)_I + (\Delta S)_{II}$

$$= \int_{353}^{323} m_1 c_P \frac{dT}{T} + \int_{303}^{323} m_1 c_P \frac{dT}{T}$$

$$= 2 \times 4.187 \ln\left(\frac{323}{353}\right) + 3 \times 4.187 \times \ln\frac{323}{303}$$

$= 0.05915$ kJ/K

Q7.4 **In a Carnot cycle, heat is supplied at 350°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K.**

(a) If the cycle operates on a stationary mass of 1 kg of water, how much is the work done per cycle, and how much is the heat supplied?

(b) If the cycle operates in steady flow with a power output of 20 kW, what is the steam flow rate?

(Ans. (a) 465.12, 897.12 kJ/kg, (b) 0.043 kg/s)

Solution: If heat required for evaporation is Q kJ/kg then

(a) $\frac{Q}{(350 + 273)} = 1.44$

or $Q = 897.12$ kJ/kg

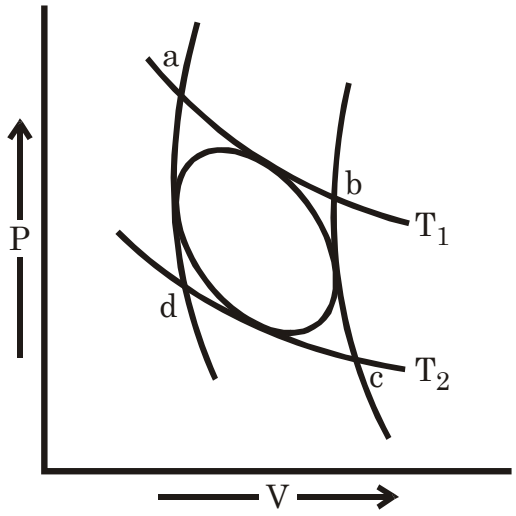
It is a Carnot cycle so $\eta = 1 - \frac{(273 + 27)}{(350 + 273)}$

$\therefore$ $W = \eta . Q = 465.12$ kJ

(b) $P = \dot{m} W$ or $\dot{m} = \frac{P}{W} = \frac{20}{465.12}$ kg/s $= 0.043$ kg/s

Q7.5 **A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $\oint dQ / T$.**

Solution:



Q7.7 **Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam**

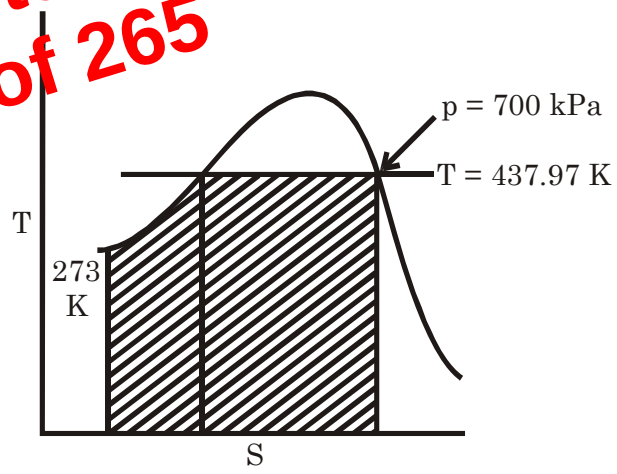
(Ans. 6.6967 kJ/kg K)

Solution: $(\Delta S)_{Water}$

$$= \int_{273}^{437.97} 1 \times 4.187 \times \frac{dT}{T}$$

$$= 4.187 \ln\left(\frac{437.97}{273}\right) kJ/K$$

$$= 1.979 \text{ kJ/K}$$



$(\Delta S)_{Eva\ pour}$

$$= \frac{1 \times 2066.3}{437.97} kJ/K$$

$$= 4.7179 \text{ kJ/K}$$

$(\Delta s)_{system}$

$$= 6.697 \text{ kJ/kg} - \text{K}$$

Q7.8 **One kg of air initially at 0.7 MPa, 20°C changes to 0.35 MPa, 60°C by the three reversible non-flow processes, as shown in Figure. Process 1: *a*-2 consists of a constant pressure expansion followed by a constant volume cooling, process 1: *b*-2 an isothermal expansion followed by a constant pressure expansion, and process 1: *c*-2 an adiabatic**

$$S_4 - S_3 = \int_{273}^{268} \frac{m c_p \, dT}{T} = 0.01 \times \left(\frac{4.2}{2}\right) \times \ln \frac{268}{273} \text{ kJ/ } K$$

$= -0.3882$ J/K

$\therefore$ $S_4 - S_1 = -15.63$ J/K

$\therefore$ Net Entropy change = 15.63 J/K

Q7.10 **Calculate the entropy change of the universe as a result of the following processes:**

(a) A copper block of 600 g mass and with C_p of 150 J/K at 100°C is placed in a lake at 8°C.

(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(c) Two such blocks, at 100 and 0°C, are joined together.

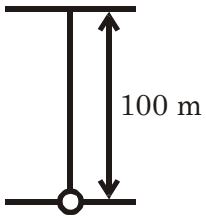
(Ans. (a) 6.63 J/K, (b) 2.095 J/K, (c) 3.64 J/K)

Solution:

(a) $(\Delta S)_{copper} = \int_{373}^{281} m c_P \frac{dT}{T}$

$= 150 \ln \frac{281}{373} J/K$

$= -42.48$ J/K



As unit of C_P is J/K there for

$\therefore$ It is heat capacity

i.e. $C_p = m c_p$

$$(\Delta S)_{lake} = \frac{C_p(100 - 8)}{281} \text{ J/ } K$$

$$= \frac{150(100 - 8)}{281} \text{ J/ } K = 49.11 \text{ J/K}$$

$(\Delta S)_{univ} = (\Delta S)_{COP} + (\Delta S)_{lake} = 6.63$ J/K

(b) Work when it touch water = $0.600 \times 9.81 \times 100$ J = 588.6 J

As work dissipated from the copper

$(\Delta S)_{copper} = 0$

As the work is converted to heat and absorbed by water then

$$(\Delta S)_{lake} = \frac{W = Q}{281} = \frac{588.6}{281} \text{ J/ } K = 2.09466 \text{ J/K}$$

$\therefore$ $(\Delta S)_{univ} = 0 + 2.09466$ J/k = 2.09466 J/K

(c) Final temperature $(T_f) = \frac{100 + 0}{2} = 50^o$ C = 323 K

(ii) the change in entropy for the room air, (iii) the work done by the engine.

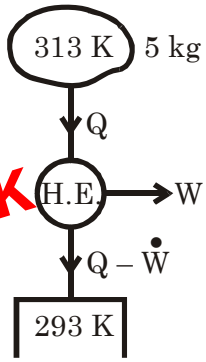
(b) If the aluminium block is allowed to cool by natural convection to room air, compute (i) the change in entropy for the block, (ii) the change in entropy for the room air (iii) the net the change in entropy for the universe.

(Ans. (a) – 134 J/K, + 134 J/K, 740 J;
(b) – 134 J/K, + 136.5 J/K, 2.5 J/K)

Solution:

(a) $$(\Delta S)_{A1} = \int_{313}^{293} \frac{m c_P \, dT}{T}$$

$$5 \times 400 \times \ln \frac{293}{313} \text{ J/ } K = -132.06 \text{ J/K}$$



$$(\Delta S)_{air} = \frac{Q - W}{293}$$

And $Q = m \; c_P \; (313 - 293) = 40000$ J

As heat is reversibly flow then

$(\Delta S)_{Al} + (\Delta S)_{air} = 0$

or $-132.06 + 136.52 - \frac{W}{293} = 0$

or $W = 1.306$ kJ

(b) $(\Delta S)_{Al}$ = Same for reversible or Irreversible = –132.06 J/K

$$(\Delta S)_{air} = \frac{4000}{293} = 136.52 \text{ J/K}$$

$(\Delta S)_{air} = +4.4587$ J/K

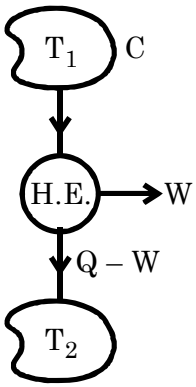
Q7.22 **Two bodies of equal heat capacities C and temperatures T_1 and T_2 form an adiabatically closed system. What will the final temperature be if one lets this system come to equilibrium (a) freely? (b) Reversibly? (c) What is the maximum work which can be obtained from this system?**

Solution:

(a) Freely $T_f = \frac{T_1 + T_2}{2}$

(b) Reversible

Let find temperature be T_f



$$\text{the } (\Delta S)_{hot} = \int_{T_1}^{T_f} C \frac{dT}{T}$$

$$= C \ln \frac{T_f}{T_1}$$

$$(\Delta S)_{cold} = \int_{T_2}^{T_f} C \frac{dT}{T} = C \ln \left(\frac{T_f}{T_2} \right)$$

$$\therefore \; (\Delta S)_{univ.} = (\Delta S)_{hot} = (\Delta S)_{cold} = (\Delta S)_{surroundings}$$

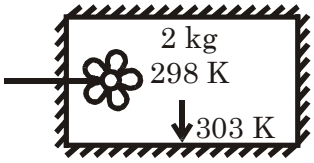
$$= C \ln \frac{T_f}{T_1} + C \ln \frac{T_f}{T_2} = 0$$

Solution:

$(\Delta S)_{surr.} = 0$

$$(\Delta S)_{sys} = \int_{298}^{303} m\,c\,\frac{dT}{T}$$

$$= 2 \times 4.187 \times \ln\frac{303}{298} = 0.13934 \text{ kJ/K}$$



$\therefore \quad (\Delta S)_{univ} = (\Delta S)_{sys} + (\Delta S)_{surr} = 0.13934 + 0 = 0.13934$ kJ/K

Q7.25 **A copper rod is of length 1 m and diameter 0.01 m. One end of the rod is at 100°C, and the other at 0°C. The rod is perfectly insulated along its length and the thermal conductivity of copper is 380 W/mK. Calculate the rate of heat transfer along the rod and the rate of entropy production due to irreversibility of this heat transfer.**

Ans. 2.985 W, 0.00293 W/K)

Solution:

$$\dot{Q} = kA\frac{\Delta T}{\Delta x}$$

$$= 380 \times 7.854 \times 10^{-5} \times \frac{100}{1} \text{ W} = 2.9845 \text{ W}$$

At the 373 K end from surrounding $\dot{Q}$ amount heat is go to the system. So at this end

$$(\Delta \dot{S})_{charge} = -\frac{\dot{Q}}{373}$$

And at the 273 K and from system $\dot{Q}$ amount of heat is rejected to the surroundings.

$$\therefore \quad (\Delta \dot{S})_{charge} = \frac{\dot{Q}}{273}$$

$$\therefore \quad (\Delta \dot{S})_{univ.} = \frac{\dot{Q}}{273} - \frac{\dot{Q}}{373} = 0.00293 \text{ W/K}$$

Q7.26 **A body of constant heat capacity C_p and at a temperature T_i is put in contact with a reservoir at a higher temperature T_f. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to**

Solution: Try please.

Q7.31 **A block of iron weighing 100 kg and having a temperature of 100°C is immersed in 50 kg of water at a temperature of 20°C. What will be the change of entropy of the combined system of iron and water? Specific heats of iron and water are 0.45 and 4.18 kJ/kg K respectively.**

(**Ans.** 1.1328 kJ/K)

Solution: Let final temperature is t_f °C

$\therefore \quad 100 \times 0.45 \times (100 - t_f) = 50 \times 4.18 \times (t_f - 20)$

$100 - t_f = 4.644\ t_f - 20 \times 4.699$

or $\quad 5.644\ t_f = 192.88$

or $\quad t_f = 34.1732^\circ$ C

$\therefore \quad t_f = 307.1732$ K

ENTROPY = $(\Delta S)_{\text{iron}} + (\Delta S)_{\text{water}}$

$$= 100 \times 0.45 \ln\left(\frac{307.1732}{373}\right) + 50 \times 4.180 \times ln\left(\frac{307.1732}{293}\right)$$

$$= 1.1355 \text{ kJ/K}$$

Q7.32 **36 g of water at 30°C are converted into steam at 250°C at constant atmospheric pressure. The specific heat of water is assumed constant at 4.2 J/g K and the latent heat of vaporization at 100°C is 2260 J/g. For water vapour, assume $pV = mRT$ where $R = 0.4619$ kJ/kg K, and**

$\frac{C_p}{R} = a + bT + cT^{-2}$ where $a = 3.634$, $b = 1.195 \times 10^{-3}$ K^{-1} and $c = 0.189 \times 10^{-6}$ K^{-2}

Calculate the entropy change of the system.

(**Ans.** 277.8 J/K)

Solution:

m = 36 g = 0.036 kg

$T_1 = 30^\circ C = 303$ K

$T_2 = 373$ K

$T_3 = 523$ K

$(\Delta S)_{\text{Water}}$

$$= m c_p \ln\left(\frac{373}{303}\right) \text{kJ/ } K$$

$$= 0.03143 \text{ kJ/K}$$

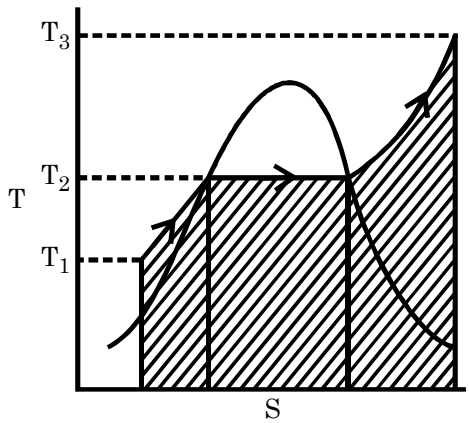
$$(\Delta S)_{\text{Vaporization}} = \frac{mL}{T_2}$$

$$= \frac{0.036 \times 2260}{373}$$

$$= 0.21812 \text{ kJ/K}$$

$$(\Delta S)_{\text{Vapor}} = \int_{373}^{523} m c_p \frac{dT}{T}$$

$$= mR \int_{373}^{523} \left(\frac{a}{T} + b + CT\right) dT$$



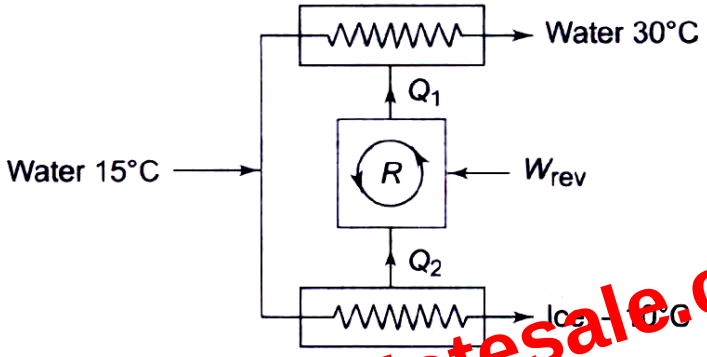
$\therefore$ Required mass of air = $\frac{5200}{363.36}$ kg = 14.311 kg

Specific volume of air at 7 MPa, 25°C then

$$v = \frac{RT}{p} = \frac{0.287 \times 298}{7000} \text{ m}^3/\text{kg} = 0.012218 \text{ m}^3/\text{kg}$$

$\therefore$ Required storage volume (V) = 0.17485 m^3

Q8.8 **Ice is to be made from water supplied at 15°C by the process shown in Figure. The final temperature of the ice is – 10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C. Determine the minimum work required to produce 1000 kg of ice.**



Take c_p for water = 4.187 kJ/kg K, c_p for ice = 2.093 kJ/kg K, and latent heat of fusion of ice = 334 kJ/kg.

(Ans. 33.37 MJ)

Solution: Let us assume that heat reject or temperature is (T_0)

(i) Then for 15°C water to 0° C water if we need W_R work minimum.

Then (COP) = $\frac{Q_2}{W_R} = \frac{T_2}{T_0 - T_2}$

or $W_R = Q_2 \frac{(T_0 - T_2)}{T_2}$

$= Q_2 \left(\frac{T_0}{T_2} - 1 \right)$

When temperature of water is
T if change is dT

Then $dQ_2 = -mc_P\, dT$

(heat rejection so –ve)

$\therefore \quad dW_R = -m c_P\, dT \left(\frac{T_0}{T} - 1 \right)$

$\therefore \quad W_{R_1} = -m c_P \int_{288}^{273} \left(\frac{T_0}{T} - 1 \right) dT$

$= m c_P \left[T_0 \ln \frac{288}{273} - (288 - 273) \right]$

$= 4187 \left[T_0 \ln \frac{288}{273} - 15 \right] \text{kJ}$

(ii) W_R required for 0° C water to 0 ° C ice

$$W_{act} = \frac{181.55}{0.75} = 242 \text{ kJ/kg}$$

(a) $\therefore$ Power required driving the compressor

$= \dot{m} W_{act} = 2.42$ kW

Extra work addede in 2′ to 2 is (242 – 181.55) = 60.85 kJ/kg

$\therefore$ If $C_p(T_2 - T_2') = 60.85$

or $$T_2 = T_2' + \frac{60.85}{1.005} = 529.25 \text{ K}$$

$\therefore$ Availability loss due to cooling

$$= \int_{313}^{529.25} 1 \times 1.005 \left(1 - \frac{288}{T}\right) dT$$

$$= 1.005 \left\{(529.21 - 313) - 288 \ln\left(\frac{529.25}{313}\right)\right\} \text{ kJ/kg}$$

= 65.302 kJ/kg

$\therefore$ Total available energy loss

= (60.85 + 65.302) kJ/kg = 126.15 kJ/kg

$\therefore$ Power loss due to irreversibility = 126.15 kW

Q8.13 **In a rotary compressor, air enters at 1.1 bar, 21 ° C where it is compressed adiabatically to 6.6 bar, 250° C. Calculate the irreversibility and the entropy production for unit mass flow rate. The atmosphere is at 1.03 bar, 20° C. Neglect the K.E. changes.**

(**Ans.** 19 kJ/kg, 0.064 kJ/kg K)

Solution:

$p_1 = 1.1$ bar = 110 kPa

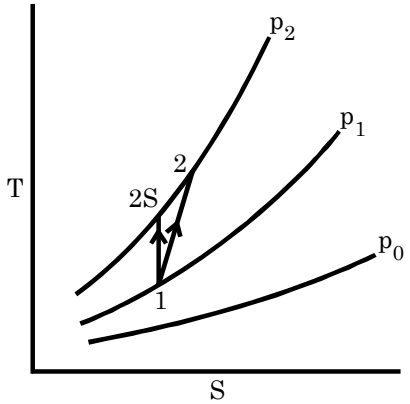
$T_1 = 294$ K

$p_2 = 6.6$ bar = 660 kPa

$T_2 = 523$ K

$p_0 = 103$ kPa

$T_0 = 293$ K



$$\Delta s = s_2 - s_1 = \int_1^2 \left(\frac{dh}{T} - v\frac{dp}{T}\right)$$

$$= \left[C_P \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}\right]$$

$$= \left[1.005 \ln \frac{523}{294} - 0.287 \ln\left(\frac{660}{110}\right)\right]$$

= 0.064647 kJ/kg – K = 64.647 J/kg – K

Minimum work required

W_{min} = Availability increase

$= h_2 - h_1 - T_0 (s_2 - s_1)$

$= mc_P (T_2 - T_1) - T_0 \Delta s$

= 1 × 1.005 (523 – 294) – 293 × 0.064647= 211.2 kJ/kg

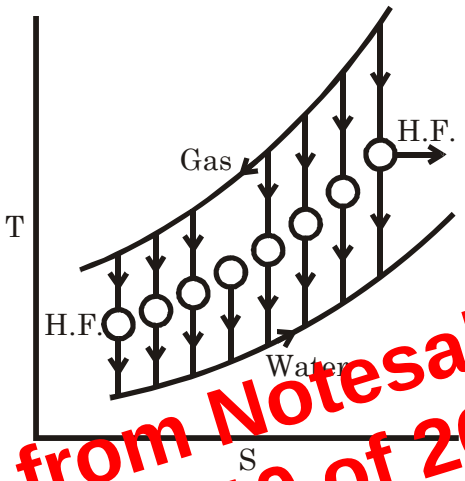
the plant per kg of turbine gas? Take c_p for exhaust gas as 1.08 and for air as 1.05 kJ/kg K. Neglect heat transfer to the surroundings and the changes in kinetic and potential energy.

(Ans. (a) 66 kJ/kg, (b) 0.0731 kJ/kg K, (c) 38.7 kJ/kg)

Solution: (a) Availability decrease of extra gases = $Q\left(1-\frac{T_0}{T}\right)$

$$= \int_{700}^{800} m c_p \left(1-\frac{293}{T}\right) dT = 1\times 1.08\left[(800-700)-293\ln\left(\frac{800}{700}\right)\right]$$

$$= 65.745 \text{ kJ/kg}$$



(b) Exit air temperature T_{exit}

$$2\,m\,c_{pa}\left(T_e - 470\right) = m \times c_{pg}\left(800 - 700\right)$$

or $T_e = 521.5$ K

$\therefore$ Availability increases

$$= 2\times 1.05\times\left[(521.5-470)-293\ln\frac{521.5}{470}\right] = 44.257 \text{ kJ/kg}$$

$\therefore \dot{S}_{gas} = 73.336$ J/K of per kg gas flow

For reversible heat transfer

$(\Delta S)_{univ} = 0$

$(\Delta S)_{Gas} = -(\Delta S)_{water}$

$$m\times 1.08\ln\frac{800}{700}$$

$$= -2m\times 1.05\times\ln\left(\frac{470}{T_o}\right)$$

or $\ln\frac{T_o}{470} = 0.068673$

$\therefore$ $T_o = 503.4$ K

$\therefore$ $Q_1 = m \times 1.08(800 - 700) = 108$ kJ/kg

$Q_2 = 2m \times 1.05\ (503.4 - 470) = 70.162$ kJ/kg of gas

$W = Q_1 - Q_2 = 37.84$ kJ/kg of gas flow [i.e. extra output]

Q8.18 An air preheater is used to heat up the air used for combustion by cooling the outgoing products of combustion from a furnace. The rate of flow of the products is 10 kg/s, and the products are cooled from 300°C to

$$= \frac{x_2 \times 4.2}{4.2 + 0.55} = \frac{0.96216 \times 4.2}{4.2 + 0.55} = 0.85$$

$h_1 = h_{f1} + x\, h_{fg1} = 844.7 + 0.85 \times 1945.2 = 2499.6$ kJ/kg

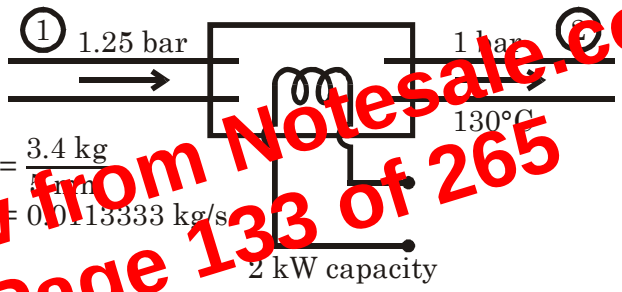
But at 1 bar minimum 5° super heat i.e. 105°C enthalpy is 2686 kJ/kg
So it is not possible to calculate only by throttling calorimeter.

Q9.11 **Steam from an engine exhaust at 1.25 bar flows steadily through an electric calorimeter and comes out at 1 bar, 130°C. The calorimeter has two 1 kW heaters and the flow is measured to be 3.4 kg in 5 min. Find the quality in the engine exhaust. For the same mass flow and pressures, what is the maximum moisture that can be determined if the outlet temperature is at least 105°C?**

(**Ans.** 0.944, 0.921)

Solution: $h_2 = 2676.2 + \frac{30}{50}(2776.4 - 2676.2) = 2736.3$ kJ/kg

$$\dot{m} h_1 = \dot{m} h_2 - \dot{Q}$$



$\dot{m} = \frac{3.4 \text{ kg}}{5 \text{ min}} = 0.0113333$ kg/s

$$h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2560 \text{ kJ/kg}$$

At 1.25 bar: from Steam Table

At 1.2 bar, $h_f = 439.4$ kJ/kg	$h_{fg} = 2244.1$ kJ/kg
At 1.3 bar, $h_f = 449.2$ kJ/kg	$h_{fg} = 2237.8$ kJ/kg
At 1.25 bar $h_f = 444.3$ kJ/kg;	$h_{fg} = 2241$ kJ/kg

If dryness fraction is x
Then $2560 = 444.3 + x \times 2241$
or $x = 0.9441$
If outlet temperature is 105° C then
$h_2 = 2686$ kJ/kg (then from problem 9.9)

$$\therefore \quad h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2509.53 \text{ kJ/kg}$$

Then if dryness fraction is x_2 then
$2509 = 444.3 + x_2 \times 2241$ $\therefore\ x_2 = 0.922$ (min)

Q9.12 **Steam expands isentropically in a nozzle from 1 MPa, 250°C to 10 kPa. The steam flow rate is 1 kg/s. Find the velocity of steam at the exit from the nozzle, and the exit area of the nozzle. Neglect the velocity of steam at the inlet to the nozzle.**

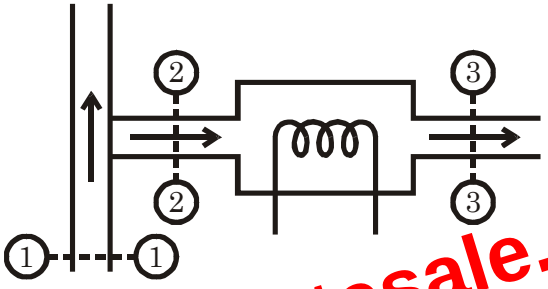
Q9.18 **A sample of wet steam from a steam main flows steadily through a partially open valve into a pipeline in which is fitted an electric coil. The valve and the pipeline are well insulated. The steam mass flow rate is 0.008 kg/s while the coil takes 3.91 amperes at 230 volts. The main pressure is 4 bar, and the pressure and temperature of the steam downstream of the coil are 2 bar and 160°C respectively. Steam velocities may be assumed to be negligible.**

(a) Evaluate the quality of steam in the main.

(b) State, with reasons, whether an insulted throttling calorimeter could be used for this test.

(**Ans.** (a) 0.97, (b) Not suitable)

Solution: (a)



$\dot{m}_2 = 0.008$ kg/s $\quad \dot{Q} = I^2R = \frac{3.91 \times 230}{1000}$ kW $= 6.893$ kW

$p_2 = 4$ bar $\quad p_3 = 2$ bar $\quad t_3 = 160°C$

$t_2 = 143.6°$ C $\quad h_3 = 2768.8 + \frac{10}{50}(2870.5 - 2768.8)$ kJ/kg

$= 2789.14$ kJ/kg

From steady flow energy equation

$$\dot{m}\, h_2 + \dot{Q} = \dot{m}\, h_3 + 0 \;:\; h_2 = h_3 - \frac{\dot{Q}}{\dot{m}} = 2676.73 \text{ kJ/kg}$$

If dryness fraction of steam x then

$h_2 = h_{f2} + x\, h_{fg2}$

or $\quad 2676.73 = 604.7 + x \times 2133 \qquad \therefore x = 0.9714$

(b) For throttling minimum enthalpy required 2686 kJ/kg if after throttling 5°C super heat and atm. Pressure is maintained as here enthalpy is less so it is not possible in **throttling calorimeter.**

Q9.19 **Two insulated tanks, *A* and *B*, are connected by a valve. Tank *A* has a volume of 0.70 m³ and contains steam at 1.5 bar, 200°C. Tank *B* has a volume of 0.35 m³ and contains steam at 6 bar with a quality of 90%. The valve is then opened, and the two tanks come to a uniform state. If there is no heat transfer during the process, what is the final pressure? Compute the entropy change of the universe.**

(**Ans.** 322.6 KPa, 0.1985 kJ/K)

Solution: **From Steam Table** from Steam Table

Sp. Enthalpy (h_A) = 2872.9 kJ/kg $\quad t_B = 158.8°$ C

Sp. Vol (v_A) = 1.193 m³/kg $\quad$ Sp. Enthalpy (h_B)

10. $a = 3\,p_c v_c^2$; $b = \frac{v_c}{3}$; $R = \frac{8}{3}\frac{p_c v_c}{T_c}$; values of Z at critical **point 0.375 for Van der** Waal gas.

11. **Boyle temperature** $(T_B) = \frac{a}{bR}$

12. $\mu = x_1\mu_1 + x_2\mu_2 + \ldots\ldots + x_c\mu_c$

$$R_m = \frac{m_1R_1 + m_2R_2 + \ldots\ldots m_cR_c}{m_1 + m_2 + \ldots\ldots\ldots m_c}$$

$$u_m = \frac{m_1u_1 + m_2u_2 + \ldots\ldots m_cu_c}{m_1 + m_2 + \ldots\ldots\ldots m_c}; \qquad h_m = \frac{m_1h_1 + m_2h_2 + \ldots\ldots m_ch_c}{m_1 + m_2 + \ldots\ldots\ldots m_c}$$

$$c_{pm} = \frac{m_1\,c_{P1} + m_2\,c_{P2} + \ldots\ldots m_c\,c_{Pc}}{m_1 + m_2 + \ldots\ldots\ldots m_c} \qquad c_{vm} = \frac{m_1\,c_{v1} + m_2\,c_{v2} + \ldots\ldots m_c\,c_{vc}}{m_1 + m_2 + \ldots\ldots\ldots m_c}$$

$$s_f - s_i = -\left[m_1R_1\ln\frac{p_1}{p} + m_2R_2\ln\frac{p_2}{p} + \ldots\ldots + m_cR_c\ln\frac{p_c}{p}\right]$$

13. **Gibbs function** $G = \bar{R}T\sum n_x(\phi_k + \ln p + \ln x_k)$

Questions with Solution of R. K. Nag

Q.10·1 **What is the mass of air contained in a room 6 m × 9 m × 4 m if the pressure is 101.325 kPa and the temperature is 25°C?**

(Ans. 256 kg)

Solution: Given pressure (p) = 101.325 kPa
Temperature (T) = 25°C = (25 + 273) K = 298 K
Volume (V) = 6 × 9 × 4 m^3 = 216 m^3
From equation of states
pV = mRT For air R = 0.287 kJ/kg – K, Gas constant mass is m kg

$$\therefore \quad m = \frac{pV}{RT} = \frac{101.325 \times 216}{0.287 \times 298}\ \text{kg} = 255.9\ \text{kg}$$

Q.10.2 **The usual cooking gas (mostly methane) cylinder is about 25 cm in diameter and 80 cm in height. It is changed to 12 MPa at room temperature (27°C).**
(a) Assuming the ideal gas law, find the mass of gas filled in the cylinder.
(b) Explain how the actual cylinder contains nearly 15 kg of gas.
(c) If the cylinder is to be protected against excessive pressure by means of a fusible plug, at what temperature should the plug melt to limit the maximum pressure to 15 MPa?

Solution: Given diameter (D) = 25 cm = 0.25 m
Height (H) = 80 cm = 0.8 m
$\therefore$ Volume of the cylinder $(V) = \frac{\pi D^2}{4} \times H = 0.03927\ m^3$
Gas pressure (p) = 12 MPa = 12000 kPa

or $\frac{T_3}{T_1} = \frac{v_3}{v_1} = \frac{v_3}{v_2} = r$ $\therefore \eta = 1 - \frac{\gamma(\gamma-1)}{r^\gamma - 1} = \frac{r^\gamma - 1 - \gamma(r-1)}{r^\gamma - 1}$ Proved

If $\gamma = \frac{4}{3}$ and r = 8 then $\eta_{cycle} = \frac{r^\gamma - 1 - \gamma(r-1)}{r^\gamma - 1}$

$$\eta = 1 - \frac{\frac{4}{3}(8-1)}{(8^{\frac{4}{3}} - 1)} = 0.37778$$

Q.10.25 **Using the Dietetic equation of state**

$$P = \frac{RT}{v-b}.\exp\left(-\frac{a}{RT_v}\right)$$

(a) Show that

$$p_c = \frac{a}{4e^2b^2}, v_c = 2b, T_c = \frac{a}{4Rb}$$

(b) Expand in the form

$$pv = RT\left(1 + \frac{B}{v} + \frac{C}{v^2} + ...\right)$$

(c) Show that

$$T_B = \frac{a}{bR}$$

Solution : Try please.

Q.10.26 **The number of moles, the pressures, and the temperatures of gases *a, b,* and c are given below**

Gas	m (kg mol)	P (kPa)	t (0C)
N_2	1	350	100
CO	3	420	200
O_2	2	700	300

If the containers are connected, allowing the gases to mix freely, find (a) the pressure and temperature of the resulting mixture at equilibrium, and (b) the change of entropy of each constituent and that of the mixture.

Solution : Try please.

Q.10.27 **Calculate the volume of 2.5 kg moles of steam at 236.4 atm. and 776.76 K with the help of compressibility factor versus reduced pressure graph. At this volume and the given pressure, what would the temperature be in K, if steam behaved like a van der Waals gas?**
The critical pressure, volume, and temperature of steam are 218.2 atm., 57 cm^3/g mole, and 647.3 K respectively.

Solution : Try please.

Q.10.28 **Two vessels, *A* and *B,* each of volume 3 m^3 may be connected together by a tube of negligible volume. Vessel a contains air at 7 bar, 95°C while B**

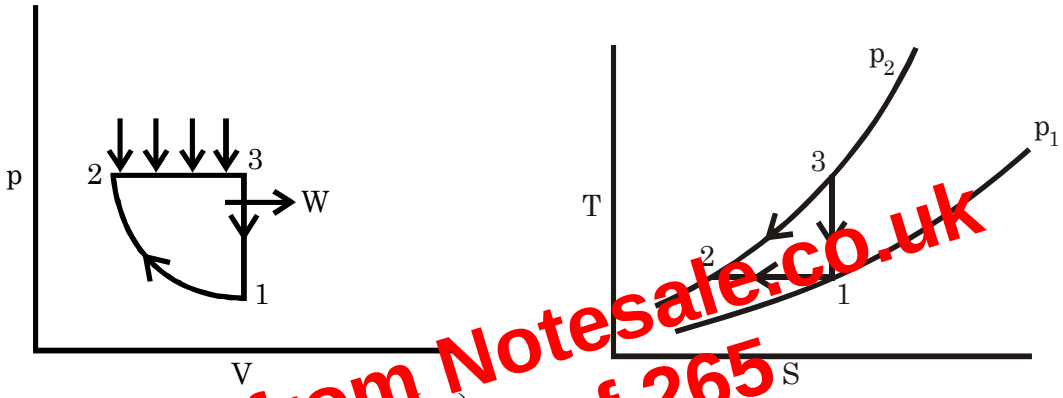
its original state. Draw the cycle on p-v and T'-s coordinates. Show that the cycle efficiency can be expressed in the following form

$$\eta = 1 - \frac{(\gamma - 1)\ln r}{\gamma\left[r^{\gamma - 1/\gamma} - 1\right]}$$

Where r is the pressure ratio, p_2/p_1. Determine the pressure ratio and the cycle efficiency if the initial temperature is 27°C and the maximum temperature is 327°C.

(**Ans.** 13.4, 32.4%)

Solution: Heat addition $(Q_1) = Q_{2-3} = mc_p(T_3 - T_2)$



Heat rejection $(Q_2) = mRT_1 \ln\left(\frac{p_2}{p_1}\right)$

$$\therefore \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{RT_1 \ln\left(\frac{p_2}{p_1}\right)}{C_p(T_3 - T_2)}$$

Here, $\frac{p_2}{p_1} = r$

$$= 1 - \frac{\gamma - 1}{\gamma} \frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\frac{T_3}{T_1} - 1\right)}$$

$$\therefore \quad \frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = r^{\frac{\gamma - 1}{\gamma}}$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$= 1 - \frac{\gamma - 1}{\gamma} \frac{\ln r}{(r^{\frac{\gamma - 1}{\gamma}} - 1)} \quad \text{Proved}$$

$\Rightarrow$ If initial temperature $(T_1) = 27^oC = 300\ K = T_2$

And $T_3 = 327^oC = 600\ K$

$$\therefore \quad r = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{600}{300}\right)^{\frac{1.4}{1.4 - 1}} = 11.314$$

$$\therefore \quad \eta = 1 - \frac{(1.4 - 1)}{(1.4)} \times \frac{\ln(11.314)}{\left[\ (11.314)^{\frac{1.4 - 1}{1.4}} - 1\ \right]} = 0.30686$$

$$\therefore \quad \eta = 1 - \frac{1.4(2.5^{\frac{1}{1.4}} - 1)}{6^{1.4-1}(2.5-1)} = 0.57876$$

$$Q_1 = mc_v(T_3 - T_2)$$
$$= m\,c_v(aT_1r_k^{\gamma-1} - T_1r_k^{\gamma-1})$$
$$= m\,c_v\,T_1r_k^{\gamma-1}(a-1)$$
$$= m \times 0.718 \times 300 \times 6^{0.4} \times (2.5-1)$$
$$= 661.6\ m\ kJ$$
$$\therefore\ W = \eta\, Q_1 = 382.9\ m\ kJ$$

For $V_4 =$; $T_4 = 2.5^{\frac{1}{1.4}} \times 300 = 577.25$ K

$$p_4 = p_1 = 100\ kPa$$

$$V_4 = \frac{mRT_4}{p_4} = \frac{m \times 0.287 \times 577.25}{100}\ m^3$$
$$= 1.6567\ m\ m^3$$

$$V_1 = \frac{m\,R\,T_1}{p_1} = \frac{m \times 0.287 \times 300}{100}$$
$$= 0.861m\ m^3$$

$$\therefore \quad V_4 - V_1 = 0.7957\ m^3$$

Let m.e.p. is p_m then

$$p_m(V_4 - V_1) = W$$
$$p_m = \frac{382.9 \times m}{0.7957\ m}\ kPa$$
$$= 481.21\ kPa = 4.8121\ bar$$

Q10.49 **The relation between u, p and v for many gases is of the form u = a + bpv where *a* and *b* are constants. Show that for a reversible adiabatic process pv^{γ} = constant, where**

$$\gamma = (b + 1)/b.$$

Solution: Try please.

Q10.50 **(a) Show that the slope of a reversible adiabatic process on p-v coordinates is**

$$\frac{dp}{dv} = \frac{1}{kv}\frac{c_p}{c_v} \text{ where } k = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(b) Hence, show that for an ideal *gas,* pv^{γ} = constant, for a reversible adiabatic process.

Solution: Try please.

Q10.51 **A certain gas obeys the Clausius equation of state p (v – b) = RT and has its internal energy given by u = c_VT. Show that the equation for a reversible adiabatic process is $p\,(v-b)^{\gamma}$ = constant, where $\gamma = c_p/c_v$.**

Solution: Try please.

Again

$$dx = \left(\frac{\partial x}{\partial f}\right)_z df + \left(\frac{\partial x}{\partial z}\right)_f dz$$

$$\left(\frac{\partial x}{\partial z}\right)_f = \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1$$

Theorem 3. Among the variables x, y, and z any one variable may be considered as a function of the other two. Thus

x = x(y, z)

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$$

Similarly,

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

$$= \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + \left[\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial x}\right)_y\right] dx$$

$$= \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + dx$$

$$\therefore \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial x}{\partial z}\right)_y = 0$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x = -1$$

Among the thermodynamic variables p, V and T. The following relation holds good

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

Maxwell's Equations

A pure substance existing in a single phase has only two independent variables. Of the eight quantities p, V, T, S, U, H, F (Helmholtz function), and G (Gibbs function) any one may be expressed as a function of any two others.

For a pure substance undergoing an infinitesimal reversible process

(a) dU = TdS - pdV

(b) dH = dU + pdV + VdP = TdS + Vdp

(c) dF = dU - TdS - SdT = - pdT - SdT

(d) dG = dH - TdS - SdT = Vdp - SdT

Since U, H, F and G are thermodynamic properties and exact differentials of the type

dz = M dx + N dy, then

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Applying this to the four equations

Then $S = S(T, p)$

or $dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$

multiplying both side by T

$TdS = T\left(\frac{\partial S}{\partial T}\right)_p dT + T\left(\frac{\partial S}{\partial p}\right)_T dp$

Since $T\left(\frac{\partial S}{\partial T}\right)_p = C_p$, heat capacity at constant pressure

and $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$ by Maxwell's equation

$\therefore$ $TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$ proved.

(iii) Derive:

$$TdS = C_V dT + T\frac{\beta}{k}dV = C_p dT - TV\beta\, dp = \frac{k C_V}{\beta}dp + \frac{C_p}{\beta V}dV$$

[IES-2001]

We know that volume expansivity $(\beta) = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

and isothermal compressibility $(k) = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

$\therefore$ From first TdS equation

$TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

$\frac{\beta}{k} = -\frac{\left(\frac{\partial V}{\partial T}\right)_p}{\left(\frac{\partial V}{\partial p}\right)_T} = -\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T$

As $\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_V \cdot \left(\frac{\partial p}{\partial V}\right)_T = -1$

$\therefore$ $-\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

or $\frac{\beta}{k} = \left(\frac{\partial p}{\partial T}\right)_V$

$\therefore$ $TdS = C_V dT + T \cdot \frac{\beta}{k} \cdot dV$ proved

From second TdS relation

Joule-Kelvin Effect or Joule-Thomson coefficient

The value of the specific heat c_p can be determined from p–v–T data and the Joule–Thomson coefficient. The ***Joule–Thomson coefficient*** μ_J is defined as

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$

Like other partial differential coefficients introduced in this section, the Joule–Thomson coefficient is defined in terms of thermodynamic properties only and thus is itself a property. The units of μ_J are those of temperature divided by pressure.

A relationship between the specific heat c_p and the Joule–Thomson coefficient μ_J can be established to write

$$\left(\frac{\partial T}{\partial p}\right)_h \left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p = -1$$

The first factor in this expression is the Joule–Thomson coefficient and the third is c_p. Thus

$$c_p = \frac{-1}{\mu_J \left(\partial p / \partial h\right)_T}$$

With $\left(\partial h / \partial p\right)_T = 1 / \left(\partial p / \partial h\right)_T$ this can be written as

$$c_p = -\frac{1}{\mu_J}\left(\frac{\partial h}{\partial p}\right)_T$$

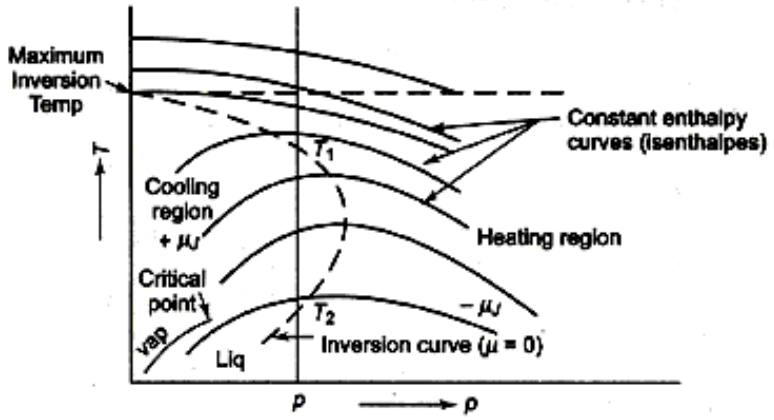
The partial derivative $\left(\partial h / \partial p\right)_T$, called the ***constant-temperature coefficient,*** can be eliminated. The following expression results:

$$c_p = \frac{1}{\mu_J}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]$$

allows the value of c_p at a state to be determined using p–v–T data and the value of the Joule–Thomson coefficient at that state. Let us consider next how the Joule–Thomson coefficient can be found experimentally.

The numerical value of the slope of an isenthalpic on a T-p diagram at any point is called the *Joule-Kelvin coefficient* and is denoted by μ_J. Thus the locus of all points at which μ_J is zero is the inversion curve. The region inside the inversion curve where μ_J is positive is called the *cooling region* and the region outside where μ_J is negative is called the *heating region*. So,

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$



Isenthalpic Curves and the Inversion Curve

Energy Equation

For a system undergoing an infinitesimal reversible process between two equilibrium states, the change of internal energy is

dU = TdS - pdV

Substituting the first TdS equation

$$dU = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_V dV - pdV$$

$$= C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right] dV$$

if $U = f(T,V)$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

This is known as energy equation. Two application of the equation are given below-

(a) For an ideal gas, $p = \frac{n\bar{R}T}{V}$

$$\therefore \left(\frac{\partial p}{\partial T}\right)_V = \frac{n\bar{R}}{V} = \frac{p}{T}$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T.\frac{p}{T} - p = 0$$

U does not change when V changes at T = C.

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T = 0$$

$$\text{since} \quad \left(\frac{\partial p}{\partial V}\right)_T \neq 0, \left(\frac{\partial U}{\partial p}\right)_T = 0$$

U does not change either when p changes at T = C. So the internal energy of an ideal gas is a function of temperature only.

Another important point to note is that for an ideal gas

$$pV = n\bar{R}T \text{ and } T\left(\frac{\partial p}{\partial T}\right) - p = 0$$

Therefore

$$dU = C_v\, dT$$

holds good for an ideal gas in any process (even when the volume changes). But for any other substance

$$dU = C_v\, dT$$

is true only when the volume is constant and dV = 0

Similarly

$$dH = TdS + Vdp$$

and $\quad \mathrm{TdS} = \mathrm{C_p\, dT} - \mathrm{T}\left(\frac{\partial V}{\partial T}\right)_p dp$

$$\therefore dH = C_p dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_p\right] dp$$

$$\therefore \left(\frac{\partial H}{\partial p}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_p$$

As shown for internal energy, it can be similarly proved from Eq. shown in above that the enthalpy of an ideal gas is not a function of either volume or pressure.

$$\left[i.e\left(\frac{\partial H}{\partial p}\right)_T = 0\ and\ \left(\frac{\partial H}{\partial V}\right)_T = 0\right]$$

but a function of temperature alone.

Since for an ideal gas, pV = $n\bar{R}T$

and $\quad V - T\left(\frac{\partial V}{\partial T}\right)_p = 0$

the relation $dH = C_p\, dT$ is true for any process (even when the pressure changes.)

However, for any other substance the relation $dH = C_p\, dT$ holds good only when the pressure remains constant or dp = 0.

(b) Thermal radiation in equilibrium with the enclosing walls processes an energy that depends only on the volume and temperature. The energy density (u), defined as the ratio of energy to volume, is a function of temperature only, or

$$u = \frac{U}{V} = f(T) only.$$

The electromagnetic theory of radiation states that radiation is equivalent to a photon gas and it exerts a pressure, and that the pressure exerted by the black body radiation in an enclosure is given by

$$p = \frac{u}{3}$$

Black body radiation is thus specified by the pressure, volume and temperature of the radiation.

since.

$$U = uV \text{ and } \quad \mathrm{p} = \frac{\mathrm{u}}{3}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = u\ and \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT}$$

By substituting in the energy Eq.

$$u = \frac{T}{3}\frac{du}{dT} - \frac{u}{3}$$

$$\therefore \frac{du}{u} = 4\frac{dT}{T}$$

J. Efficiency of Binary vapour cycle:

$$1 - \eta = (1 - \eta_1)(1 - \eta_2) \ldots\ldots (1 - \eta_n)$$

$\therefore$ For two cycles

$$\boxed{\eta = n_1 + n_2 - n_1 n_2}$$

K. Overall efficiency of a power plant

$$\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{turbine\ (mean)} \times \eta_{generator}$$

Questions with Solution P. K. Nag

Q. 12.1 for the following steam cycles find

(a) W_T in kJ/kg **(b) W_p in kJ/kg,**

(c) Q_1 in kJ/kg, **(d) cycle efficiency,**

(e) steam rate in kg/kW h, and

(f) moisture at the end of the turbine process. Show the results in tabular form with your comments.

Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	1 bar	Ideal Rankine Cycle
-do-	-do-	Neglect W_p
-do-	-do-	Assume 75% pump and Turbine efficiency
-do-	0.1 bar	Ideal Rankine Cycle
10 bar, 300°*c*	-do-	-do-
150 bar, 600°*c*	-do-	-do-
-do-	-do-	Reheat to 600°*C* maximum intermediate pressure to limit end moisture to 15%
-do-	-do-	-do- but with 85% tur- bine efficiencies
10 bar, saturated	0.1 bar	Isentropic pump process ends on satura
Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	0.1 bar	-do- but with 80% machine efficiencies
-do-	-do-	Ideal regenerative cycle
-do-	-do-	Single open heater at 110°*c*
-do-	-do-	Two open heaters at 90°*c* and 135°*c*
-do-	-do-	-do- but the heaters are closed heaters

$$h_f = 191.83 + \left(\frac{15-13.342}{15-10}\right)(225.94 - 191.83) = 203.14 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 + \left(\frac{15-13.342}{15-10}\right)(2373.1 - 2392.8) = 2386.3 \text{ kJ/kg}$$

$$h_{2s} = h_f + x\, h_{fg} = 203.14 + 0.85033 \times 2386.3 = 2232.3 \text{ kJ/kg}$$

$$\eta_{isentropic} = \frac{h_1 - h_{2'}}{h_1 - h_{2s}}$$

$$\therefore \quad h_1 - h_{2'} = \eta_{isentropic} \times (h_1 - h_{2s})$$

$$\therefore \quad h_2' = h_1 - \eta_{isentropic}\,(h_1 - h_{2s})$$

$$= 2803.4 - 0.75\,(2803.4 - 2232.3) = 2375 \text{ kJ/kg.}$$

$$\therefore \quad \text{Turbine work } (W_T) = h_1 - h_{2'} = (2803.4 - 2373)\ \%$$

$$= 428.36 \text{ kJ/kg}$$

$$\therefore \quad \text{Efficiency of the plant} = \frac{W_T}{h_1} = \frac{428.36}{2803.4} \approx 0.1528 = 25.28\%$$

If mass flow rate is $\dot{m}$ kg/s

$$\dot{m}.W_T = 12.5 \times 10^3$$

$$\text{or} \quad \dot{m} = \frac{12.5 \times 10^3}{428.36} = 29.18 \text{ kg/s}$$

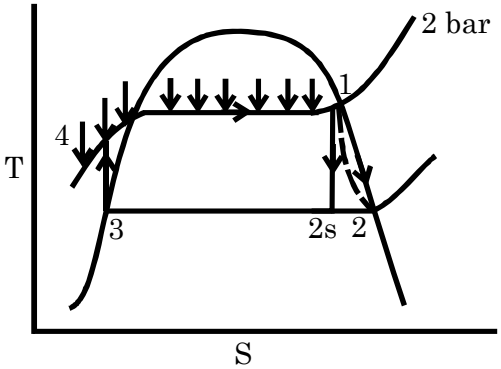
Q.12.3 **A simple steam power cycle uses solar energy for the heat input. Water in the cycle enters the pump as a saturated liquid at 40°C, and is pumped to 2 bar. It then evaporates in the boiler at this pressure, and enters the turbine as saturated vapour. At the turbine exhaust the conditions are 40°C and 10% moisture. The flow rate is 150 kg/h. Determine (a) the turbine isentropic efficiency, (b) the net work output (c) the cycle efficiency, and (d) the area of solar collector needed if the collectors pick up 0.58 kW/m^2.**

(**Ans**. (c) 2.78%, (d) 18.2 m^2)

Solution: From Steam Table $T_1 = 120.23°C = 393.23$ K

$h_1 = 2706.7$ kJ/kg

$s_1 = 7.1271$ kJ/kg – K



At 40°C saturated pressure 7.384 kPa

$h_f = 167.57$ $h_{fg} = 2406.7$

of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute (a) kg of mercury circulated per kg of water, and (b) the efficiency of the combined cycle.

The property values of saturated mercury are given below

p (bar)	T(°C)	h_f (kJ/kg) h_g	s_f (kJ/kg k) s_g	v_f (m^3 /kg) v_g
10	515.5	72.23 363.0	0.1478 0.5167	80.9 x10^{-6} 0.0333
0.2	277.3	38.35 336.55	0.0967 0.6385	77.4 x10^{-6} 1.163

Solution: Try please.

Q.12.10 **In an electric generating station, using a binary vapour cycle with mercury in the upper cycle and steam in the lower, the ratio of mercury flow to steam flow is 10 : 1 on a mass basis. At an evaporation rate of 1,000,000 kg/h for the mercury, its specific enthalpy rises by 356 kJ/kg in passing through the boiler. Superheating the steam in the boiler furnace adds 586 kJ to the steam specific enthalpy. The mercury gives up 251.2 *kJ/kg* during condensation, and the steam gives up 2003 *kJ/kg* in its condenser. The overall boiler efficiency is 85%. The combined turbine metrical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries require 5% of the energy generated by the units. Find the overall efficiency of the plant.**

Solution: Try please

Q.12.11 **A sodium-mercury-steam cycle operates between 1000°C and 40°C. Sodium rejects heat at 670°C to mercury. Mercury boils at 24.6 bar and rejects heat at 0.141 bar. Both the sodium and mercury cycles are saturated. Steam is formed at 30 bar and is superheated in the sodium boiler to 350°C. It rejects heat at 0.0 8 bar. Assume isentropic expansions, no heat losses, and no generation and neglect pumping work. Find (a) the amounts of sodium and mercury used per kg of steam, (b) the heat added and rejected in the composite cycle per kg steam, (c) the total work done per kg steam. (d) the efficiency of the composite cycle, (e) the efficiency of the corresponding Carnot cycle, and (f) the work, heat added, and efficiency of a supercritical pressure steam (single fluid) cycle operating at 250 bar and between the same temperature limits.**

For mercury, at 24.6 bar, h_g = 366.78 kJ/kg

$s_g = 0.48 kJ/kg\,K$ and at 0.141 bar, $s_j = 0.09$

And $s_g = 0.64 kJ/kg\,K$, $h_j = 36.01$ and $h_g = 330.77$ kJ/kg

For sodium, at 1000°C, h_g = 4982.53 kJ/kg

At turbine exhaust = 3914.85 kJ/kg

At 670°C, h_f = 745.29 kJ/kg

For a supercritical steam cycle, the specific enthalpy and entropy at the turbine inlet may be computed by extrapolation from the steam tables.

Solution: Try please.

$s_f = 1.5301,\ s_{fg} = 5.5967$

$s_g = 7.127$ kJ/kg – K so at point (2)

Steam is saturated vapour

So $h_2 = 2706.3$ kJ/kg

At 2 bar saturated temperature is 120.2°C but 65°C liquid

So $h_3 = h_2 - C_P \Delta T$

$= 504.7 - 4.187 \times (120.2 - 65) = 273.6$ kJ/kg

$W_{P_{3-4}} = v_3 (p_1 - p_2) = 0.001 \times (2000 - 200) = 1.8$ kJ/kg

$\therefore\ h_4 = 275.4$ kJ/kg

$\therefore$ Heat input (Q) $= h_1 - h_4 = (3247.6 - 275.4) = 2972.2$ kJ/kg

Turbine work $= (h_1 - h_2)\,\eta = (3247.6 - 2706.3) \times 0.7$ kJ/kg

$= 378.9$ kJ/kg

Heat rejection that utilized $(Q_0) = (h_2 - h_3)\,\eta$

$= (2706.3 - 273.6) \times 0.9 = 2189.4$ kJ/kg

$\therefore$ Net work output $(W_{net}) = W_T - W_P = 378.9 - 1.8$

$= 377.1$ kJ/kg

$\therefore$ Fraction at energy supplied utilized

$$= \frac{W_{net} + Q_0}{Q_1} = \frac{377.1 + 2189.4}{2972.2} \times 100\%$$

$= 86.35\%$

(b) At 0.07 ba

$s_f = 0.559,\ s_{fg} = 7.717$

$\therefore$ Dryness fraction x, $0.559 + x \times 7.717 = 7.127$

$\therefore\ x = 0.85137$

$\therefore\ h_2 = 163.4 + 0.85137 \times 2409.1 = 2214.4$ kJ/kg

$h_3 = 163.4$ kJ/kg

$W_P = 0.001007 \times (2000 - 7) = 2$ kJ/kg

$\therefore\ h_4 = 165.4$ kJ/g.

$\therefore\ W_T = (h_1 - h_2) \times 0.7 = 723.24$ kJ/kg

$W_{net} = W_T - W_P = 721.24$ kJ/kg

Here heat input for power $= (h_1 - h_4) = 3082.2.$ kJ/kg

For same 377.1 kg power we need 0.52285 kg of water

So heat input = 1611.5 kJ for power

Heat input for heating $= \frac{2189.4}{0.9}$ kJ $= 2432.7$ kJ

$\therefore$ Fraction of energy used $= \frac{377.1 + 2189.4}{1611.5 + 2432.7} \times 100\%$

$= 63.46\%$

Q.12.16 In a nuclear power plant saturated steam at 30 bar enters a h.p. turbine and expands isentropically to a pressure at which its quality is 0.841. At this pressure the steam is passed through a moisture separator which removes all the liquid. Saturated vapour leaves the separator and is

$h_f = 0.559, \quad s_{fg} = 7.717$

$\therefore$ If quality is x then

$7.6642 = 0.559 + x \times 7.717 \Rightarrow x = 0.9207$

$\therefore h_4 = 163.4 + 0.9207 \times 2409.1 = 2381.5$ kJ/kg

$h_5 = 163.4$ kJ/kg $\approx h_6, \quad h_7 = 686.8$ kJ/kg $\approx h_8$

$\therefore$ From Heat balance of heater

$m \times h_2 + (1 - m)\, h_6 = h_7 \quad \therefore m = 0.2016$ kg/kg of steam at H.P

$\therefore (1 - m) = 0.7984$

$W_T = h_1 - h_2 + (1 - m)(h_3 - h_4) = 1347.6$kJ/kg

W_P neglected

$Q = (h_1 - h_8) + (1 - m)(h_3 - h_2) = 3118.5$ kJ/kg at H.P

$\therefore$ (a) Reheat pr. 6.6 bar

(b) Steam flow rate at H.P $= \dfrac{80 \times 10^3}{1347.6}$ kg/s $= 59.36$ kg/s

(c) Cycle efficiency $(\eta) = \dfrac{W}{Q} = \dfrac{1347.6}{3118.5} \times 100\% = 43.21\%$

Q.12.18 **Figure shows the arrangement of a steam plant in which steam is also required for an industrial heating process. The steam leaves boiler *B* at 30 bar, 320°C and expands in the H.P. turbine to 2 bar, the efficiency of the H.P. turbine being 75%. At this point one half of the steam passes to the process heater *P* and the remainder enters separator S which removes all the moisture. The dry steam enters the L.P. turbine at 2 bar and expands to the condenser pressure 0.07 bar, the efficiency of the L.P. turbine being 70%. The drainage from the**

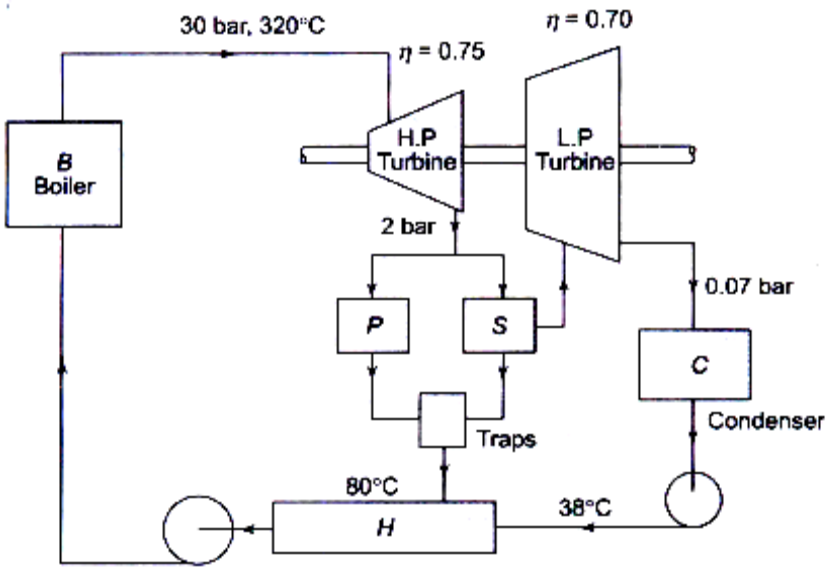


Fig. 12.51

$$\frac{0.025715 \times 0.287\,(813-543)\ln 2}{4.985} \times 100\% = 27.7\%$$

Work done = 1.3812 kJ = 53.71 kJ/kg

For ideal regeneration

$$\eta = 1 - \frac{543}{813} = 33.21\%$$

Q13.2 **An Ericsson cycle operating with an ideal regenerator works between 1100 K and 288 K. The pressure at the beginning of isothermal compression is 1.013 bar. Determine (a) the compressor and turbine work per kg of air, and (b) the cycle efficiency.**

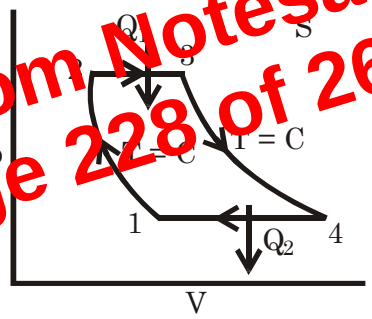
(Ans. (a) w_T = 465 kJ/kg, w_C = 121.8 kJ/kg (b) 0.738)

Solution: Given $T_1 = T_2 = 288$ K

$T_3 = T_4 = 1100$ K

$p_1 = 1.013$ bar = 101.3 kPa

$$\therefore \quad V_1 = \frac{RT_1}{p_1} = 0.81595 \text{ m}^3/\text{kg}$$



$$W_C = RT_1 \ln\left(\frac{V_1}{V_2}\right) \qquad W_T = R\,T_3 \ln\left(\frac{V_4}{V_3}\right)$$

$p_3 = p_2$; $p_1 = p_4$

$$\therefore \eta = 1 - \frac{288}{1100} = 73.82\% \qquad \therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore W = \eta Q_1 = \eta C_P (T_1 - T_2) \qquad \therefore \frac{V_1}{V_2} = \frac{T_1}{T_2} \times \left(\frac{p_2}{p_1}\right) = \left(\frac{p_2}{p_1}\right)$$

$$= 0.7382 \times 1.005\,(1100 - 288) \text{ kJ/kg} = 602.4 \text{ kJ/kg}$$

$$\therefore Q_{2-3} = C_P (T_3 - T_2) = C_v (T_3 - T_2) + p_2 (V_3 - V_2)$$

Q13.5 **An engine equipped with a cylinder having a bore of 15 cm and a stroke of 45 cm operates on an Otto cycle. If the clearance volume is 2000 cm^3, compute the air standard efficiency.**

(Ans.47.4%)

Q13.14 **Obtain an expression for the specific work done by an engine working on the Otto cycle in terms of the maximum and minimum**

Temperatures of the cycle, the compression ratio r_k, and constants of the working fluid (assumed to be an ideal gas).
Hence show that the compression ratio for maximum specific work output is given by

$$r_k = \left(\frac{T_{min}}{T_{max}}\right)^{1/2(1-\gamma)}$$

Solution:

$T_{min} = T_1$
$T_{max} = T_3$

$Q_1 = C_v (T_3 - T_2)$
$Q_2 = C_v (T_4 - T_1)$
$\therefore \quad W = Q_1 - Q_2$
$= C_v [(T_3 - T_2) - (T_4 - T_1)]$

Hence $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$

$\therefore T_2 = T_1 r_c^{\gamma-1}$

And $\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = r_c^{-(\gamma-1)}$ i.e. $r_c^{\gamma-1}$

$= x$

$\therefore T_4 = T_3 . r_c^{-(r-1)} = \frac{T_3}{x}$

Then

$W = C_v \left[T_3 - T_1 x - \frac{T_3}{x} + T_1\right]$

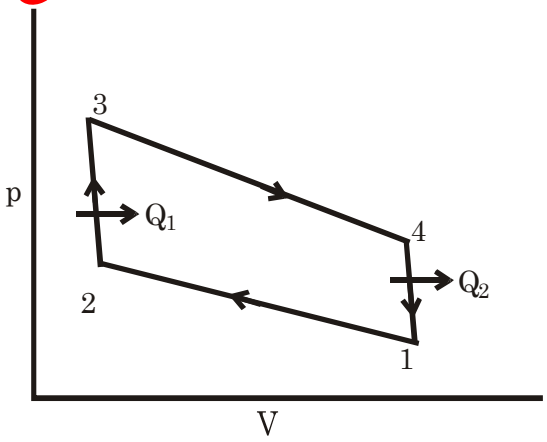
For maximum W, $\frac{dW}{dx} = 0$

$\therefore C_v \left[0 - T_1 + \frac{T_3}{x^2} + 0\right] = 0$

$\therefore x^2 = \frac{T_3}{T_1}$

$\therefore r_c^{\gamma-1} = \sqrt{\frac{T_3}{T_1}} = \sqrt{\frac{T_{max}}{T_{min}}}$

$\therefore \quad r_c = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{1}{2(\gamma-1)}} = \left(\frac{T_{min}}{T_{max}}\right)^{\frac{1}{2(1-\gamma)}}$ Proved.



Q13.15 **A dual combustion cycle operates with a volumetric compression ratio r_k = 12, and with a cut-off ratio 1.615. The maximum pressure is given by p_{max} = 54 p_1' where p_1 is the pressure before compression. Assuming**

indices of compression and expansion of 1.35, show that the m.e.p. of the cycle

$$p_m = 10 p_1$$

Hence evaluate (a) temperatures at cardinal points with T_1 = 335 K, and (b Cycle efficiency.

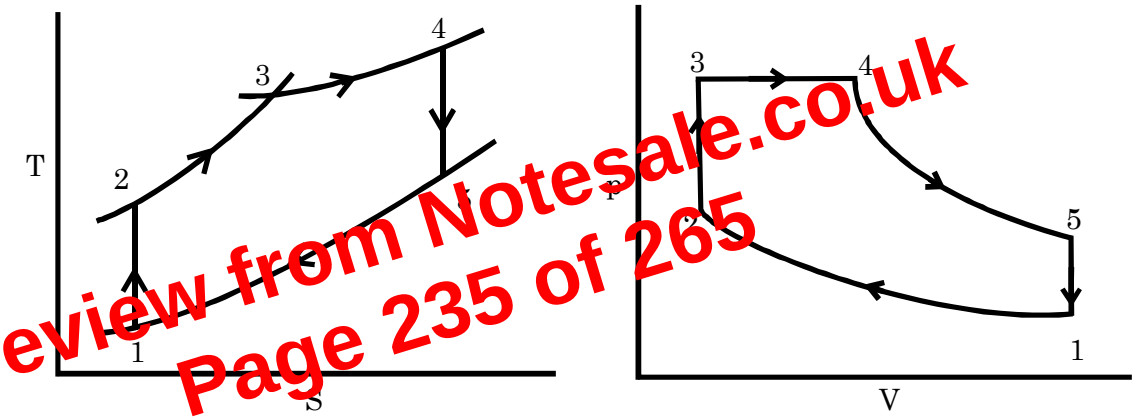
(**Ans.** (a) T_2 = 805 K, p_2 = 29.2 p_1 ' T_3 = 1490 K, T_4 = 2410 K, T_5 = 1200 K, (b) η = 0.67)

Solution:

Here $\frac{v_1}{v_2} = r_c = 12$

$\frac{v_4}{v_3} = \rho = 1.615$ $\qquad pv^{1.35} = C,\ n = 1.35$

$p_{max} = p_3 = p_4 = 54\, p_1$



$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad \therefore T_2 = T_1 \times (12)^{(1.35-1)} = 2.3862\ T_1$$

And $\quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{n} \quad \therefore\ p_2 = p_1 \times (12)^{1.35} = 28.635\ p_1$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \qquad \therefore T_3 = \frac{p_3}{T_2} \times T_2 = \frac{54 p_1}{28.635 p_1} \times 2.3862\ T_1 = 4.5\ T_1$$

$$v_3 = v_2 = \left(\frac{v_1}{12}\right)$$

$$\therefore \quad v_4 = \rho\, v_3 = \frac{1.615}{12} v_1 = 0.13458\ v_1$$

$$\therefore \quad \frac{p_4 v_4}{T_4} = \frac{P_3 v_3}{T_3} \qquad p_3 = p_4$$

$$T_4 = T_3 \times \frac{v_4}{v_3} = 1.615\ T_3 = 1.615 \times 4.5\ T_1 = 7.2675\ T_1$$

$$\therefore \quad \frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{n-1} = \left(\frac{v_4}{v_1}\right)^{n-1}$$

$$\therefore \quad T_5 = 3.6019\ T_1$$

$$\therefore \quad W = [C_v (T_3 - T_2) + C_P (T_4 - T_3) - C_v (T_5 - T_1) = 2.4308\ T_1\ \text{kJ/kg}.$$

(b) Turbine work $\left(W_T\right) = (h_1 - h_2) = C_p(T_1 - T_2) = 351.75$ kJ/kg

(c) Heat supplied $(Q_1) = C_p(T_1 - T_4) = 517.6$ kJ/kg

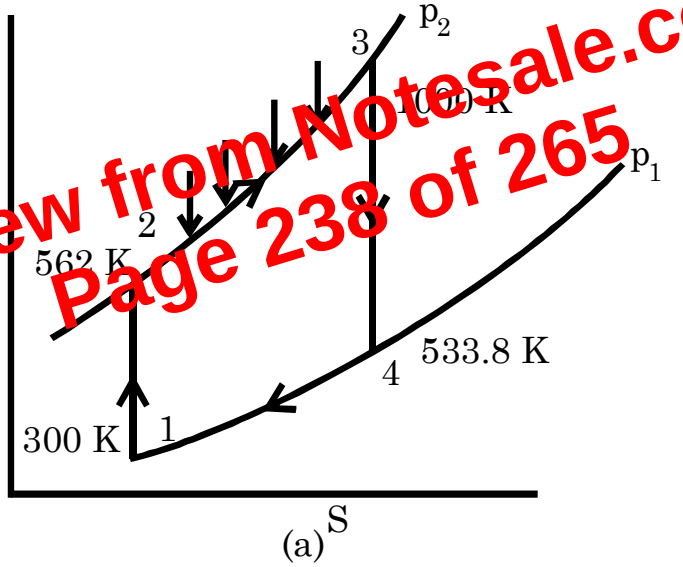
(d) Cycle efficiency $(\eta) = \dfrac{W_T - W_C}{Q_1} = 17.86\%$

(e) Turbine exhaust temperature $(T_2) = 723$ K

Q13.27 **A simple gas turbine plant operating on the Brayton cycle has air inlet temperature 27°C, pressure ratio 9, and maximum cycle temperature 727°C. What will be the improvement in cycle efficiency and output if the turbine process Is divided into two stages each of pressure ratio 3, with intermediate reheating to 727°C?**

(Ans. - 18.3%, 30.6%)

Solution:



(a)

For (a) $T_1 = 300$ K

$\dfrac{p_2}{p_1} = 9$

$T_3 = 1000$ K

$\therefore \quad T_2 = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = 562$ k

$\dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{1}{9}\right)^{\frac{\gamma-1}{\gamma}} \qquad \therefore T_4 = \dfrac{T_3}{9^{\frac{\gamma-1}{\gamma}}} = 533.8$ K

$$\therefore \eta = \frac{99.18}{583.5} \times 100\% = 17\%$$

Q13.29 **A gas turbine plant draws in air at 1.013 bar, 10°C and has a pressure ratio of 5.5. The maximum temperature in the cycle is limited to 750°C. Compression is conducted in an uncooled rotary compressor having an isentropic efficiency of 82%, and expansion takes place in a turbine with an isentropic efficiency of 85%. A heat exchanger with an efficiency of 70% is fitted between the compressor outlet and combustion chamber. For an air flow of 40 kg/s, find (a) the overall cycle efficiency, (b) the turbine output, and (c) the air-fuel ratio if the calorific value of the fuel used is 45.22 MJ/kg.**

(**Ans.** (a) 30.4%, (b) 4272 kW, (c) 115)

Solution: $p_1 = 101.3$ kPa

$T_1 = 283$ K

$\frac{p_2}{p_1} = 5.5$ kPa

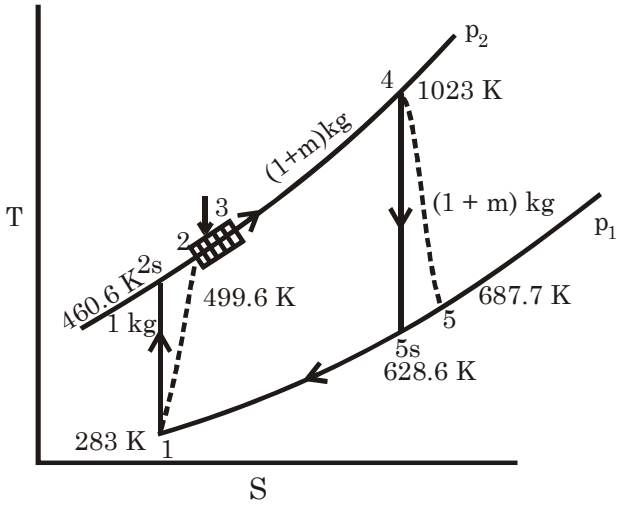
$T_4 = 750°C = 1023$ K

$$\therefore \frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2s} = 460.6 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \therefore T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

$= 499.6$ K

$$\therefore \frac{T_{5s}}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5.5}\right)^{\frac{\gamma-1}{\gamma}}$$



$$\therefore \quad T_{5s} = T_4 \times \left(\frac{1}{5.5}\right)^{\frac{1.4-1}{1.4}} = 628.6\text{K}$$

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5s}} \therefore T_4 - T_5 = \eta_T \ (T_4 - T_{5s}) = 335.3 \text{ K}$$

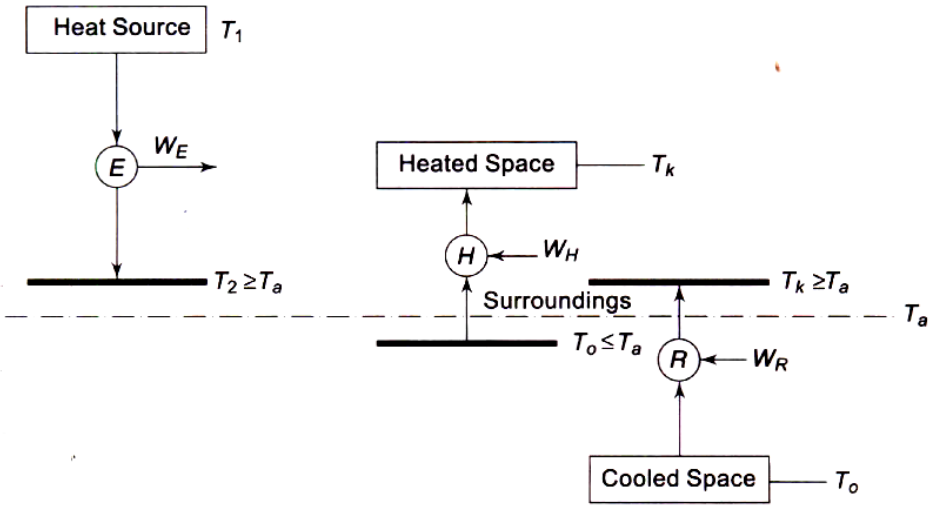


Fig. Comparison of heat engine, heat pump and refrigerating machine

$$COP_{Carnot,HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$COP_{Carnot,R} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

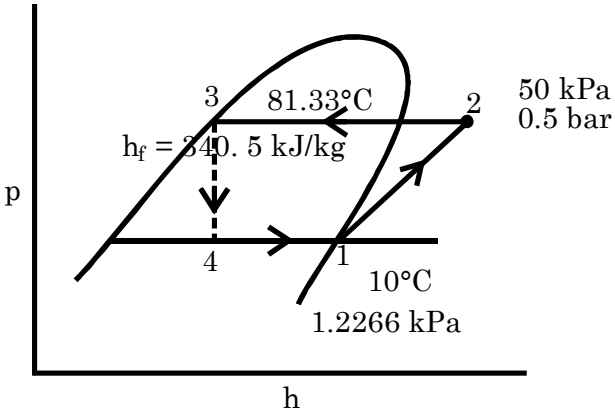
Where

W_{cycle} = work input to the reversible heat pump and refrigerator
Q_H = heat transferred between the system and the hot reservoir
Q_C = heat transferred between the system and cold reservoir
T_H = temperature of the hot reservoir.
T_C = temperature of the cold reservoir.

Solution: $h_1 = 2519.8$ kJ/kg

$s_1 = 8.9008$ kJ/kg-K

$v_1 = 106.38$ m³/kg



400°C 50 kPa s = 8.88642, h = 3278.9

500°C 50 kPa s = 9.1546, h = 3488.7

$$\therefore \quad h_2 = (3488.7 - 3278.9) \times \left(\frac{8.9008 - 8.8642}{9.1546 - 8.8642}\right) + 3278.9 = 3305.3 \text{ kJ/kg}$$

$$\therefore \quad \text{Compressor work (W)} \quad h_2 - h_1 = (3305.3 - 2519.8)$$
$$= 785.5 \text{ kJ/kg}$$

Heating (Q) $= h_2 - h_{f3} = (3305.3 - 340.5)$ kJ/kg $= 2964.8$ kJ/kg

$\dot{m} \times Q = 30$

$$\therefore \quad \dot{m} = \frac{30}{2964.8} = 0.0101187 \text{ kg/s}$$

$$\therefore \quad \text{COP} = \frac{2964.8}{785.5} = 3.77$$

Compressor power = $\dot{m}W$ = 7.95 KW

Q14.26 **A 100 tonne low temperature R-12 system is to operate on a 2-stage vapour compression refrigeration cycle with a flash chamber, with the refrigerant evaporating at – 40°C, an intermediate pressure of 2.1912 bar, and condensation at 30°C. Saturated vapour enters both the compressors and saturated liquid enters each expansion valve. Consider both stages of compression to be isentropic. Determine:**

(a) The flow rate of refrigerant handled by each compressor

(b) The total power required to drive the compressor

(c) The piston displacement of each compressor, if the clearance is 2.5% for each machine

(d) The COP of the system

(e) What would have been the refrigerant flow rate, the total work of compression, the piston displacement in each compressor and the compressor and the COP, if the compression had occurred in a single stage? .

(Ans. (a) 2.464, 3.387 kg/s, (b) 123 kW, (c) 0.6274, 0.314 m³/s, (d) 2.86, (e) 3.349 kg/s, 144.54 kW, 1.0236 m³/s, 2.433)