Basic Concepts Behind the Binary System

To understand binary numbers, begin by recalling elementary school math. When we first learned about numbers, we were taught that, in the decimal system, things are organized into columns:

```
H | T | O
1 | 9 | 3
```

such that "H" is the hundreds column, "T" is the tens column, and "O" is the ones column. So the number "193" is 1-hundreds plus 9-tens plus 3-ones.

Years later, we learned that the ones column meant $10^0$, the tens column meant $10^1$, the hundreds column $10^2$ and so on, such that

```
10^2 | 10^1 | 10^0
1 | 9 | 3
```

the number 193 is really $\{(1*10^2)+(9*10^1)+(3*10^0)\}$.

As you know, the decimal system uses the digits 0-9 to represent numbers. If we wanted to put a larger number in column $10^n$ (e.g., 10), we would have to multiply $10*10^n$, which would give $10^{n+1}$, and be forced to add a column to the left. For example, putting ten in the $10^0$ column is impossible, so we put a 1 in the $10^1$ column, and a 0 in the $10^0$ column, thus using two columns. Twelve would be $12*10^0$, or $10^0(10+2)$, or $10^1+2*10^0$, which also uses an additional column to the left.

The binary system works under the exact same principles as the decimal system, only it operates in base 2 rather than base 10. In other words, instead of columns being

```
10^2 | 10^1 | 10^0
```

they are

```
2^2 | 2^1 | 2^0
```

Instead of using the digits 0-9, we only use 0-1 (again, if we used anything larger it would be like multiplying $2*2^n$ and getting $2^n+1$, which would not fit in the $2^n$ column. Therefore, it would shift you one column to the left. For example, "3" in binary cannot be put into one column. The first column we fill is the right-most column, which is $2^0$, or 1. Since $3>1$, we need to use an extra column to the left, and indicate it as "11" in binary $(1*2^1) + (1*2^0)$.

Examples: What would the binary number 1011 be in decimal notation?
Try a few examples of binary addition:

\[
\begin{array}{ccc}
111 & +110 & 111 \\
111 & +110 & 111 \\
______ & 01 & 1101 \\
\end{array}
\]

\[
\begin{array}{ccc}
101 & +111 & 101 \\
101 & +111 & 101 \\
______ & 00 & 1100 \\
\end{array}
\]

\[
\begin{array}{ccc}
111 & +111 & 111 \\
111 & +111 & 111 \\
______ & 10 & 1110 \\
\end{array}
\]

Click here to return to the question

Using the regular algorithm for binary addition, add \((5+12), (-5+12), (-12+5),\) and \((12+12)\) in each system. Then convert back to decimal numbers.

Signed Magnitude:

\[
\begin{array}{cccc}
00000101 & 00001100 & 00001100 & 00001100 \\
00001100 & 00001100 & 00001100 & 00001100 \\
17 & 16 & 17 & -17 \\
\end{array}
\]

One's Complement:

\[
\begin{array}{cccc}
00000101 & 11111100 & 11110011 & 00001100 \\
00001100 & 00001100 & 11111010 & 11110011 \\
00010001 & 00000110 & 11101101 & 11111111 \\
17 & 6 & -18 & 0 \\
\end{array}
\]

Two's Complement:

\[
\begin{array}{cccc}
00000101 & 11111101 & 11110100 & 00001100 \\
00001100 & 00001100 & 11111011 & 11110100 \\
00010001 & 00000111 & 11101111 & 00000000 \\
17 & 7 & -17 & 0 \\
\end{array}
\]

Signed Magnitude: