5.1. Mathematical Induction

The Principle of Mathematical Induction

Given a statement $P$ concerning the integer $n$, suppose that
1. $P$ is true for some particular integer $n_0$;
2. If $P$ is true for some particular integer $k \geq n_0$, then it is true for the next integer $k+1$.

Then $P$ is true for all integers $n \geq n_0$.

Problem. Prove that $1+3+5+\ldots+(2n-1)=n^2$.

Solution. Put $n_0=1$. Then

\[ 1+3+5+\ldots+(2n-1)=0=(n_0)^2. \]

Now suppose that $k$ is an integer, $k \geq 1$, and that the statement is true for $n=k$:

\[ 1+3+\ldots+(2k-1)=k^2. \]

We must show that

\[ 1+3+\ldots+(2(k+1)-1)=(k+1)^2. \]

Indeed,

\[ 1+3+\ldots+(2k-1)+(2(k+1)-1) = \\
= k^2 + (2k+1) = (k+1)^2. \]

By the Principle of Math. Induction, $1+3+5+\ldots+(2n-1)=n^2$

for all integers $n \geq 1$. 