Chapter 5

Factorial Designs

5.1 Introduction

An experiment where the effects of two or more factors are studied simultaneously is called a factorial design.

Advantages of Factorial Designs:

1) Economy: Generally to study two or more factors simultaneously will require fewer experimental units than a ‘one-at-a-time’ approach.
2) The interactions between the factors may be explored. Analysing the interactions may yield a valuable insight into the particular conditions under which the experimental variables are most effective.

Terminology

Completely Crossed Design: Each level of each Factor appears with each level of all the other factors in the experiment.

Balanced Design: Each of the treatment combinations is observed the same number of times.

If two Factors are used in a completely crossed design experiment, factor A has 2 levels and Factor B has 4 levels then the design is 2 x 4. Similarly if three Factors are included, A with 3 levels, B with 4 levels and C with 2 levels, the design is 3 x 4 x 2.

Assumptions

i) Normality
ii) Homogeneity of Variance
iii) Independence of the observations

These can be tested as for the one-way ANOVA via options within the ANOVA

5.2 Two Way Completely Crossed Balanced Design

General Notation: Factor A with \(a\) levels
Factor B with \(b\) levels
ANOVA using Type II SS

Tests of Between-Subjects Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Type II Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
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<td>5.24</td>
<td>8.96</td>
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<tr>
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<td>Total</td>
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<tr>
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</table>

a. R Squared = .428 (Adjusted R Squared = .380)

ANOVA using Type I SS

Tests of Between-Subjects Effects

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</table>

a. R Squared = .428 (Adjusted R Squared = .380)

Notice that the three methods produce the same values for SS(Error) and SS(Daycare*Age) but different values for the main effects of Daycare and Age. The different SS used to analyse the data resulted in different conclusions for the hypothesis test of main effect of Daycare, where the p-value based on Type I SS was 0.142 compared to 0.026 for Type II and 0.041 for Type III SS.
### 4. drug

**Dependent Variable:** blood pressure

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<tr>
<th>Drug</th>
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<td>188.2</td>
<td>2.55</td>
<td>183.1</td>
<td>193.3</td>
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### 5. diet * biofeed

**Dependent Variable:** blood pressure

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### 6. diet * drug

**Dependent Variable:** blood pressure

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<th>Drug</th>
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### 7. biofeed * drug

**Dependent Variable:** blood pressure

<table>
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<tr>
<th>Biofeed</th>
<th>Drug</th>
<th>Mean</th>
<th>Std. Error</th>
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</table>
5.6.3 Analysis

Table of treatment means and standard deviations leaves no cause for concern regarding homogeneity of variance.

ANOVA Table

The main effects were all significant:

- Diet: F=33.438, p=0.000
- Biofeed: F=12.943, p=0.001
- Drug: F=11.863, p=0.000

The first-order interactions were not significant:

- Biofeed x Drug: F=0.819, p=0.446
- Biofeed x Diet: F=0.222, p=0.639
- Drug x Diet: F=2.820, p=0.068

The second-order interaction is significant:

- Biofeed x Diet x Drug: F=3.475, p=0.037
Oxygen uptake Experiment
Nine men took part in an experiment. three were non smokers (1), three were moderate smokers (2) and three were heavy smokers (3). They each performed three types of stress test - bicycle (1), Treadmill (2) and Step (3). The time until maximum oxygen uptake was recorded in minutes.

Time to Maximum oxygen uptake in minutes

<table>
<thead>
<tr>
<th>Smoking</th>
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<th>Exercise 2</th>
<th>Exercise 3</th>
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Input the data and analyse the data.

Questions for practical 5.1

General questions

i)  What do we mean by factorial designs? What are the advantages of the factorial designs compared to the simple one way analysis?

ii) How many factors and how many levels in each factor there are in a 4x3x2 factorial design?

iii) What are the three main questions which can be answered in a two way analysis of variance table?

iv) Define what is meant by the term interaction between two factors.

From practical 5: Memory experiment

i)  Comment on the initial Box plot of the number of words recalled. Can you say anything about whether different tasks or age are important or whether there is interaction between task and age?

ii) From the ANOVA table (Tests of Between –Subjects Effects):

   a) Is the variation between the two ages statistical significant?

   b) Is the variation between different tasks statistically significant?