Q. 1. (i) The PDE \( \frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2} \) is known as:

(i) wave equation   (ii) heat equation
(iii) Laplace equation (iv) none of these

Ans. (i) Wave equation

Q. 1. (j) The PDE \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \), is:

(i) parabolic   (ii) elliptic
(iii) hyperbolic (iv) circular

Ans. (ii) Elliptic

Section B

Note: Attempt any three parts from this section. Each part carries equal marks: (3 \times 10 = 30)

Q. 2. (a) A particle of mass \( mn \) moves in a straight line under the action of force \( mn^2x \) which is always directed towards a fixed point \( x_0 \) on the line. Determine the displacement \( x(t) \) in terms of distance to the motion \( x \). Given that initially \( x = 0 \), \( \frac{dx}{dt} = x_0 (0 < \lambda < 1) \).

Ans. The differential equation describing this simple harmonic motion is

\[
m \frac{d^2x}{dt^2} = -2\lambda \ mnv - mn^2x, \quad \Rightarrow \quad m \frac{d^2x}{dt^2} = -2\lambda \ mn \frac{dx}{dt} - mn^2x
\]

\[
\frac{d^2x}{dt^2} + 2\lambda n \frac{dx}{dt} + n^2 x = 0
\]

...(1)

The auxiliary equation is \( M^2 + 2\lambda nM + n^2 = 0 \)

\[
M = \frac{-2\lambda n \pm \sqrt{4\lambda^2 n^2 - 4n^2}}{2} = -\lambda n \pm in\sqrt{1-\lambda^2}
\]

The general solution is \( x = e^{-\lambda nt}[c_1 \cos n\omega t + c_2 \sin n\omega t] \) ...

where \( \omega = \sqrt{1-\lambda^2} \),

using I.C. \( x = 0, t = 0 \) \( \Rightarrow c_1 = 0 \)

Differentiating (2), we get

\[
\frac{dx}{dt} = e^{-\lambda nt}[-c_1 n \omega \sin n\omega t + c_2 n \omega \cos n\omega t] - \lambda ne^{-\lambda nt}[c_1 \cos n\omega t + c_2 \sin n\omega t]
\]

Again using I.C. \( \frac{dx}{dt} = x_0, t = 0 \)

\[
x_0 = -c_1 \cdot 0 + c_2 n \omega - \lambda n \{c_1 + c_2 \cdot 0\} \quad \Rightarrow \quad c_2 = \frac{x_0}{n\omega}
\]

From (2), its solution is \( x = e^{-\lambda nt} \left( \frac{x_0}{n\omega} \right) \sin n\omega t \)

Q. 2. (b) Using Frobenius method, obtain a series solution in powers of \( x \) for differential equation \( 2x(1-x) \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} + 3y = 0 \)
Ans. \[ 2x (1 - x) \frac{d^2y}{dx^2} + (1 - x) \frac{dy}{dx} + 3y = 0 \] ...(1)

\[ \therefore x = 0 \text{ is a regular singular point} \]

Let

\[ y = x^m \sum_{n=0}^{\infty} a_n x^n = \sum_{m=0}^{\infty} a_n x^{m+n} \] ...(2)

\[ \frac{dy}{dx} = \sum_{n=0}^{\infty} a_n (m+n) x^{m+n-1} \]

\[ \frac{d^2y}{dx^2} = \sum_{n=0}^{\infty} a_n (m+n)(m+n-1) x^{m+n-2} \]

Substituting these values in equation (1), we get

\[ 2x (1 - x) \sum_{n=0}^{\infty} a_n (m+n)(m+n-1) x^{m+n-2} + (1 - x) \sum_{n=0}^{\infty} a_n (m+n) x^{m+n-1} \]

\[ + 3 \sum_{n=0}^{\infty} a_n x^{m+n} = 0 \]

\[ \sum_{n=0}^{\infty} a_n [2(m+n)x^{m+n-1}] + (m+n) x^{m+n-1} \]

\[ + \sum_{n=0}^{\infty} a_n [-2(m+n)(m+n-1) - (m+n) + 3] x^{m+n} = 0 \]

\[ \sum_{m=0}^{\infty} a_n (m+n)(2m+2n-1) x^{m+n-1} - \sum_{n=0}^{\infty} a_n (m+n+1)(2m+2n-3) x^{m+n} = 0 \] ...(3)

Coefficient of \( x^{m-1} = 0 \)

\[ a_0(m)(2m-1) = 0 \Rightarrow a_0 \neq 0 \]

\[ m(2m-1) = 0 \quad \Rightarrow \quad m = 0, \frac{1}{2} \]

It is the root of indicial equation.

Coefficient of \( x^m = 0 \)

\[ a_1(m+1)(2m+1) - a_0(m+1)(2m-3) = 0 \quad \Rightarrow \quad a_1 = \left( \frac{2m-3}{2m+1} \right) a_0 \]

Coefficient of \( x^{m+n} = 0 \)

\[ a_{n+1} (m+n+1)(2m+2n+1) - a_n (m+n+1)(2m+2n-3) = 0 \]

\[ a_{n+1} = \left( \frac{2m+2n-3}{2m+2n+1} \right) a_n \] ...(4)

\[ n = 1, \quad a_2 = \left( \frac{2m-1}{2m+3} \right) a_1 = \left( \frac{2m-1}{2m+1} \right) \left( \frac{2m-3}{2m+3} \right) a_0 \]

\[ n = 2, \quad a_3 = \left( \frac{2m+1}{2m+5} \right) a_2 = \left( \frac{2m-1}{2m+3} \right) \left( \frac{2m-3}{2m+5} \right) a_0 \]

Substituting these value of \( a_1, a_2, a_3 \ldots \) in equation (2), we get

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]