The minor \( (M_{ij}) \) of \( a_{ij} \) is the determinant of the matrix \((n-1) \times (n-1)\) and is found by canceling row \(i\) and column \(j\) from \(a_{ij}\).

Cofactor \( A_{ij} \) of \( a_{ij} \) is \( A_{ij} = (-1)^{i+j} M_{ij} \)

**Examples of Minors and Cofactors**

1. Find \( M_{12} \) and \( A_{12} \) for the Matrix \[
\begin{bmatrix}
15 & 20 \\
7 & -5
\end{bmatrix}
\]

To find \( M_{12} \) and \( A_{12} \) delete row 1 and column 2

\[
\begin{bmatrix}
15 & 20 \\
7 & -5
\end{bmatrix}
\]

\( M_{12} = 7, \ A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (7) = -7 \)

\( A_{12} = -7 \)

2. Find \( M_{23} \) and \( A_{23} \) for the Matrix \[
\begin{bmatrix}
8 & 2 & 3 \\
16 & 4 & 6 \\
36 & 8 & 12
\end{bmatrix}
\]

To find \( M_{23} \) delete row 2 and column 3

\[
\begin{bmatrix}
8 & 2 & 3 \\
16 & 4 & 6 \\
36 & 8 & 12
\end{bmatrix}
\]

\( M_{23} = |8 \ 2| = 8(8) - 36(2) = -8 \)

\( |36 \ 2| \)

\( A_{23} = (-1)^{2+3} M_{23} = (-1)^5 (8(8) - 36(2)) = 72 - 64 = 8 \)

\( A_{23} = 8 \)

**Determinant of a 3 x 3 Matrix**

The determinant of a 3 x 3 matrix is found by multiplying each value in row 1 by its cofactor then adding the sums. This is referred to as “expanding by the first row”.