FP2 Revision Notes

Inequalities
Key Words: sketch, positive
- Use a sketch to best evaluate points of intersection
- Only multiply by POSITIVE values

Series
Key Words: method of differences, partial fractions, sigma notation rules
- When evaluating \( \sum_{r=1}^{\infty} f(r) \) consider \( r=1, r=2, r=3 \ldots \) then sum and terms will cancel!
- If the general term \( u_r = f(r) - f(r+1) \) then \( \sum u_r = \sum f(r) - f(r+1) \)

Further Complex Numbers
Key Words: modulus-argument form, principal argument, complex exponential form, de Moivre's theorem, binominal expansion, locus of points: circle; perpendicular bisector,
- If \( z = x + iy \) then the complex number can be written as \( z = r = r \cos \theta + i \sin \theta \)
- Principal argument: \(-\pi < \theta \leq \pi \)
- \( e^{i\theta} = \cos \theta + i \sin \theta \) (can be proved using Maclaurin and Taylor Series expansion of \( \sin x \) and \( \cos x \))
- Thus a complex number \( z = r \cos \theta + i \sin \theta \)
- \( \cos x \) = real part
- \( \sin x \) = imaginary part
- \( z^x = r^x \cos x + i \sin x \) (can be proved using induction)
- Remember the following identities
  - \( z + \frac{1}{z} = 2 \cos \theta \)
  - \( z^x + \frac{1}{z^x} = 2 \cos x \)
  - \( z - \frac{1}{z} = 2i \sin \theta \)
  - \( z^x - \frac{1}{z^x} = 2i \sin x \)
  - Can be proved using \( z = r \cos \theta + i \sin \theta \)

- For a complex number \( w, w = u + iv \)
- \( z = r \cos \theta + i \sin \theta \)
- To remove a modulus (use Pythagoras' theorem):
  - \( |z| = k \)
  - \( x + iy = k \)
  - \( x^2 + y^2 = k^2 \)
- To remove an argument:
  - \( \arg(z) = \theta \)
  - \( \arg(x + iy) = \theta \)
  - \( z = \tan \theta \) (adjust accordingly depending on quadrant)
- For a transformation \( T \) from the \( z \)-plane to the \( w \)-plane:
  - \( w = z + a + ib \) is a translation \( (a, b) \)
  - \( w = Kz \) is an enlargement scale factor \( k \) centre \((0,0)\)
  - \( w = Kz + a + ib \) is an enlargement scale factor \( k \) centre \((0,0)\) followed by translation \((a, b)\)

First order differential equations
Key Words: family of solution curves, separating the variables, integrating factor, transformations
- If \( \frac{dy}{dx} = f(x)g(y) \), then \( \int \frac{1}{g(y)} dy = \int f(x) dx + c \)
- For a 1st order D.E. in the form \( \frac{dy}{dx} + Py = Q \) where \( P \) and \( Q \) are functions of \( x \), multiply through by the integrating factor to obtain general solution
- When using substitutions get \( y \) and \( \frac{dy}{dx} \) in terms of other variables and it should drop out!

Second order differential equations
Key Words: auxiliary quadratic, general solution, complementary function, particular integral
- For 2nd order D.E. a \( \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0 \) aux equation is \( am^2 + bm + c = 0 \)
- For roots to the aux equation, the general solution to the 2nd order D.E. is...
  - \( y = Ae^{mx} + Be^{nx} \) (distinct roots \( a \) and \( b \))
  - \( y = (A + Bx)e^{mx} \) (repeated root \( a \))
  - \( y = Acoswx + Bsinwx \) (imaginary roots \pm i\omega\)
  - \( y = e^{mx}(Acoswx + Bsinnx) \) (complex roots \( p \pm iq \))
- \( \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x) \)
  - Solve for complementary function \( \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0 \)
  - Then solve for particular integral
  - If \( f(x) \) is in the form... then try...
    - \( k \rightarrow a \)
    - \( kx \rightarrow ax + b \)
    - \( kx^2 \rightarrow ax^2 + bx + c \)
    - \( ke^{mx} \rightarrow Ae^{mx} \)
    - \( mcoswx \rightarrow acoswx + bsinwx \)
    - \( msinwx \rightarrow acoswx + bsinwx \)
    - \( mcoswx + nisinwx \rightarrow acoswx + bsinwx \)
  - General solution is \( y = C.F. + P.I. \)
- When using substitutions get \( y, \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) in terms of other variables and it should drop out!

Maclaurin and Taylor Series
Key Words: look at formula booklet

Polar coordinates
Key Words: polar, Cartesian, converting
- \( r \cos \theta = x \)
- \( r \sin \theta = y \)
- \( x^2 + y^2 = r^2 \)
- \( \theta = \arctan \frac{y}{x} \)
- Area \( = \frac{1}{2} \int_0^{2\pi} r^2 d\theta \)
- For tangents parallel to initial line \( \frac{d}{d\theta} (r \sin \theta) = 0 \)
- For tangents perpendicular to initial line \( \frac{d}{d\theta} (r \cos \theta) = 0 \)
- For \( r = p + q \cos \theta \): conditions for a 'dimple' \( q \leq p < 2q \)