Example: Find the range \( \mathcal{R}(T) \) in the above example.

\[
\begin{align*}
T(1,0,0) &= (1,0,-1) \\
T(0,1,0) &= (-1,0,1) \\
T(0,0,1) &= (0,1,0)
\end{align*}
\]

\[
\mathcal{R}(T) = \left[ \begin{array}{ccc}
1 & 0 & -1 \\
-1 & 0 & 1 \\
0 & 1 & 0
\end{array} \right]
\]

\[
= \left[ \begin{array}{ccc}
1 & 0 & -1 \\
0 & 1 & 0
\end{array} \right]
\]

\[
\] 

Let \( A \) be the standard matrix of \( T \).

Then \( T(\alpha_1, \alpha_2, \alpha_3) = 0 \iff A \left[ \begin{array}{l}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{array} \right] = 0_{3 \times 3} \).

\[
\mathcal{N}(T) \text{ is the same as the nullspace of } A.
\]

\[
\dim(\ker(T)) = \text{nullity } A.
\]

\[
A = \left[ \begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{array} \right] = \left[ \begin{array}{ccc}
T(e_1) & T(e_2) & T(e_3)
\end{array} \right]
\]

We know that \( \mathcal{R}(T) \) is the span of \( T(e_1), T(e_2), \) and \( T(e_3) \).

\[
\mathcal{R}(T) \text{ is reaching but the column space of } A.
\]

\[
\dim \mathcal{R}(T) = \text{column space of } A = 2 \text{ for } A.
\]

We call this number the rank \( \text{ of } T \), abbreviated by \( \text{rk}(T) \). Thus we have

\[
\text{rk}(T) + \text{null}(T) = \text{rk}(A) + \text{null}(A) = \text{Number of columns of } A.
\]

**Rank-Nullity Theorem for Linear Transformations:**

Let \( T: \mathbb{R}^m \to \mathbb{R}^n \) be a linear transformation. Then

\[
\text{rk}(T) + \text{null}(T) = n = \text{dimension of the domain space}
\]