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Hence,

$$\text{work done} = (1000\,\text{kg} \times 9.81\,\text{m/s}^2) \times (10\,\text{m})$$
$$= 98\,100\,\text{Nm}$$
$$= \mathbf{98.1\,kNm} \text{ or } \mathbf{98.1\,kJ}$$

(b) $\text{Power} = \dfrac{\text{work done}}{\text{time taken}} = \dfrac{98100\,\text{J}}{20\,\text{s}}$

$$= 4905\,\text{J/s} = \mathbf{4905\,W} \text{ or } \mathbf{4.905\,kW}$$

Now try the following exercise

Exercise 1 Further problems on charge, force, work and power

(Take $g = 9.81\,\text{m/s}^2$ where appropriate)

1. What quantity of electricity is carried by 6.24×10^{21} electrons? [1000 C]
2. In what time would a current of 1 A transfer a charge of 30 C? [30 s]
3. A current of 3 A flows for 5 minutes. What charge is transferred? [900 C]
4. How long must a current of 0.1 A flow so as to transfer a charge of 30 C? [5 minutes]
5. What force is required to give a mass of 20 kg an acceleration of 30 m/s^2? [600 N]
6. Find the accelerating force when a car having a mass of 1.7 Mg increases its speed with a constant acceleration of 3 m/s^2. [5.1 kN]
7. A force of 40 N accelerates a mass at 5 m/s^2. Determine the mass. [8 kg]
8. Determine the force acting downwards on a mass of 1500 g suspended on a string. [14.72 N]
9. A force of 4 N moves an object 200 cm in the direction of the force. What amount of work is done? [8 J]
10. A force of 2.5 kN is required to lift a load. How much work is done if the load is lifted through 500 cm? [12.5 kJ]
11. An electromagnet exerts a force of 12 N and moves a soft iron armature through a distance of 1.5 cm in 40 ms. Find the power consumed. [4.5 W]
12. A mass of 500 kg is raised to a height of 6 m in 30 s. Find (a) the work done and (b) the power developed. [(a) 29.43 kNm (b) 981 W]
13. Rewrite the following as indicated:
    (a) 1000 pF = ... nF
    (b) 0.02 μF = ... pF
    (c) 5000 kHz = ... MHz
    (d) 47 kΩ = ... MΩ
    (e) 0.32 mA = ... μA
    [(a) 1 nF (b) 20000 pF (c) 5 MHz (d) 0.047 MΩ (e) 320 μA]

1.6 Electrical potential and e.m.f.

The **unit of electric potential** is the volt (V), where one volt is one joule per coulomb. One volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$\text{volts} = \frac{\text{watts}}{\text{amperes}} = \frac{\text{joules/second}}{\text{amperes}}$$
$$= \frac{\text{joules}}{\text{ampere seconds}} = \frac{\text{joules}}{\text{coulombs}}$$

A change in electric potential between two points in an electric circuit is called a **potential difference**. The **electromotive force (e.m.f.)** provided by a source of energy such as a battery or a generator is measured in volts.

1.7 Resistance and conductance

The **unit of electric resistance** is the **ohm**(Ω), where one ohm is one volt per ampere. It is defined as the resistance between two points in a conductor when a constant electric potential of one volt applied at the two points produces a current flow of one ampere in the conductor. Thus,

$$\textbf{resistance, in ohms} \quad R = \frac{V}{I}$$

where V is the potential difference across the two points, in volts, and I is the current flowing between the two points, in amperes.

The reciprocal of resistance is called **conductance** and is measured in siemens (S). Thus

$$\textbf{conductance, in siemens } G = \frac{1}{R}$$

where R is the resistance in ohms.

Problem 6. Find the conductance of a conductor of resistance: (a) 10 Ω (b) 5 kΩ (c) 100 mΩ.

(a) Conductance $G = \frac{1}{R} = \frac{1}{10}$ siemen $= \mathbf{0.1\,S}$

(b) $G = \frac{1}{R} = \frac{1}{5 \times 10^3}\,\text{S} = 0.2 \times 10^{-3}\,\text{S} = \mathbf{0.2\,mS}$

(c) $G = \frac{1}{R} = \frac{1}{100 \times 10^{-3}}\,\text{S} = \frac{10^3}{100}\,\text{S} = \mathbf{10\,S}$

1.8 Electrical power and energy

When a direct current of I amperes is flowing in an electric circuit and the voltage across the circuit is V volts, then

$$\textbf{power, in watts } P = VI$$

$$\textbf{Electrical energy} = \textbf{Power} \times \textbf{time}$$
$$= \mathbf{VIt \text{ joules}}$$

Although the unit of energy is the joule, when dealing with large amounts of energy, the unit used is the **kilowatt hour (kWh)** where

$$1\,\text{kWh} = 1000 \text{ watt hour}$$
$$= 1000 \times 3600 \text{ watt seconds or joules}$$
$$= 3\,600\,000\,\text{J}$$

Problem 7. A source e.m.f. of 5 V supplies a current of 3 A for 10 minutes. How much energy is provided in this time?

Energy = power × time, and power = voltage × current. Hence

$$\textbf{Energy} = VIt = 5 \times 3 \times (10 \times 60)$$
$$= 9000\,\text{Ws or J} = \mathbf{9\,kJ}$$

Problem 8. An electric heater consumes 1.8 MJ when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{1.8 \times 10^6\,\text{J}}{30 \times 60\,\text{s}}$$
$$= 1000\,\text{J/s} = 1000\,\text{W}$$

i.e. **power rating of heater = 1 kW**

$$\text{Power } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1000}{250} = 4\,\text{A}$$

Hence the current taken from the supply is 4 A.

Now try the following exercise

Exercise 2 Further problems on e.m.f., resistance, conductance, power and energy

1. Find the conductance of a resistor of resistance (a) 10 Ω (b) 2 kΩ (c) 2 mΩ
[(a) 0.1 S (b) 0.5 mS (c) 500 S]

2. A conductor has a conductance of 50 μS. What is its resistance? [20 kΩ]

3. An e.m.f. of 250 V is connected across a resistance and the current flowing through the resistance is 4 A. What is the power developed? [1 kW]

4. 450 J of energy are converted into heat in 1 minute. What power is dissipated?
[7.5 W]

5. A current of 10 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor? [1 V]

6. A battery of e.m.f. 12 V supplies a current of 5 A for 2 minutes. How much energy is supplied in this time? [7.2 kJ]

7. A d.c. electric motor consumes 36 MJ when connected to a 250 V supply for 1 hour. Find the power rating of the motor and the current taken from the supply. [10 kW, 40 A]

Exercise 10 Multi-choice problems on the introduction to electric circuits
(Answers on page 398)

1. 60 μs is equivalent to:
   (a) 0.06 (b) 0.00006 s
   (c) 1000 minutes (d) 0.6 s

2. The current which flows when 0.1 coulomb is transferred in 10 ms is:
   (a) 1 A (b) 10 A
   (c) 10 mA (d) 100 mA

3. The p.d. applied to a 1 kΩ resistance in order that a current of 100 μA may flow is:
   (a) 1 V (b) 100 V (c) 0.1 V
   (d) 10 V

4. Which of the following formulae for electrical power is incorrect?
   (a) VI (b) $\frac{V}{I}$ (c) I^2R (d) $\frac{V^2}{R}$

5. The power dissipated by a resistor of 4 Ω when a current of 5 A passes through it is:
   (a) 6.25 W (b) 20 W
   (c) 80 W (d) 100 W

6. Which of the following statements is true?
   (a) Electric current is measured in volts
   (b) 200 kΩ resistance is equivalent to 2 MΩ
   (c) An ammeter has a low resistance and must be connected in parallel with a circuit
   (d) An electrical insulator has a high esistance

7. A current of 3 A flows for 50 h through a 6 Ω resistor. The energy consumed by the resistor is:
   (a) 0.9 kWh (b) 2.7 kWh
   (c) 9 kWh (d) 27 kWh

8. What must be known in order to calculate the energy used by an electrical appliance?
   (a) voltage and current
   (b) current and time of operation
   (c) power and time of operation
   (d) current and resistance

9. Voltage drop is the:
   (a) maximum potential
   (b) difference in potential between two points
   (c) voltage produced by a source
   (d) voltage at the end of a circuit

10. A 240 V, 60 W lamp has a working resistance of:
    (a) 1400 ohm (b) 60 ohm
    (c) 960 ohm (d) 325 ohm

11. The largest number of 100 W electric light bulbs which can be operated from a 240 V supply fitted with a 13 A fuse is:
    (a) 2 (b) 7 (c) 31 (d) 18

12. The energy used by a 1.5 kW heater in 5 minutes is:
    (a) 7.5 J (b) 450 J
    (c) 7500 J (d) 450 000 J

13. When an atom loses an electron, the atom:
    (a) becomes positively charged
    (b) disintegrates
    (c) experiences no effect at all
    (d) becomes negatively charged

Chapter 3

Resistance variation

At the end of this chapter you should be able to:

- appreciate that electrical resistance depends on four factors
- appreciate that resistance $R = \rho l/a$, where ρ is the resistivity
- recognize typical values of resistivity and its unit
- perform calculations using $R = \rho l/a$
- define the temperature coefficient of resistance, α
- recognize typical values for α
- perform calculations using $R_\theta = R_0(1 + \alpha\theta)$
- determine the resistance and tolerance of a fixed resistor from its colour code
- determine the resistance and tolerance of a fixed resistor from its letter and digit code

3.1 Resistance and resistivity

The resistance of an electrical conductor depends on four factors, these being: (a) the length of the conductor, (b) the cross-sectional area of the conductor, (c) the type of material and (d) the temperature of the material. Resistance, R, is directly proportional to length, l, of a conductor, i.e. $R \propto l$. Thus, for example, if the length of a piece of wire is doubled, then the resistance is doubled.

Resistance, R, is inversely proportional to cross-sectional area, a, of a conductor, i.e. $R \propto 1/a$. Thus, for example, if the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since $R \propto l$ and $R \propto 1/a$ then $R \propto l/a$. By inserting a constant of proportionality into this relationship the type of material used may be taken into account. The constant of proportionality is known as the **resistivity** of the material and is given the symbol ρ (Greek rho). Thus,

$$\textbf{resistance} \quad R = \frac{\rho l}{a} \textbf{ ohms}$$

ρ is measured in ohm metres (Ω m). The value of the resistivity is that resistance of a unit cube of the material measured between opposite faces of the cube.

Resistivity varies with temperature and some typical values of resistivities measured at about room temperature are given below:

Copper 1.7×10^{-8} Ω m (or 0.017 $\mu\Omega$ m)

Aluminium 2.6×10^{-8} Ω m (or 0.026 $\mu\Omega$ m)

Carbon (graphite) 10×10^{-8} Ω m (0.10 $\mu\Omega$ m)

Glass 1×10^{10} Ω m (or 10^4 $\mu\Omega$ m)

Mica 1×10^{13} Ω m (or 10^7 $\mu\Omega$ m)

Note that good conductors of electricity have a low value of resistivity and good insulators have a high value of resistivity.

Problem 1. The resistance of a 5 m length of wire is 600 Ω. Determine (a) the resistance of an 8 m length of the same wire, and (b) the length of the same wire when the resistance is 420 Ω.

(a) Resistance, R, is directly proportional to length, l, i.e. $R \propto l$. Hence, $600\,\Omega \propto 5$ m or $600 = (k)(5)$, orange-green-red-yellow-brown is a five-band fixed resistor and from Table 3.1, indicates: $352 \times 10^4\ \Omega$ with a tolerance of $\pm 1\%$

$$352 \times 10^4\ \Omega = 3.52 \times 10^6\ \Omega, \text{ i.e. } 3.52\ \text{M}\Omega$$

Hence orange-green-red-yellow-brown indicates **3.52 MΩ ± 1%**

(b) Letter and digit code for resistors

Another way of indicating the value of resistors is the letter and digit code shown in Table 3.2.

Table 3.2

Resistance Value	Marked as:
0.47 Ω	R47
1 Ω	1R0
4.7 Ω	4R7
47 Ω	47R
100 Ω	100R
1 kΩ	1K0
10 kΩ	10K
10 MΩ	10M

Tolerance is indicated as follows: $F = \pm 1\%$, $G = \pm 2\%$, $J = \pm 5\%$, $K = \pm 10\%$ and $M = \pm 20\%$

Thus, for example,

$$\text{R33M} = 0.33\ \Omega \pm 20\%$$
$$\text{4R7K} = 4.7\ \Omega \pm 10\%$$
$$\text{390RJ} = 390\ \Omega \pm 5\%$$

Problem 19. Determine the value of a resistor marked as 6K8F.

From Table 3.2, 6K8F is equivalent to: **6.8 kΩ ± 1%**

Problem 20. Determine the value of a resistor marked as 4M7M.

From Table 3.2, 4M7M is equivalent to: **4.7 MΩ ± 20%**

Problem 21. Determine the letter and digit code for a resistor having a value of 68 kΩ ± 10%.

From Table 3.2, 68 kΩ ± 10% has a letter and digit code of: **68 KK**

Now try the following exercises

Exercise 13 Further problems on resistor colour coding and ohmic values

1. Determine the value and tolerance of a resistor having a colour coding of: blue-grey-orange-red [68 kΩ ± 2%]
2. Determine the value and tolerance of a resistor having a colour coding of: yellow-violet-gold [4.7 Ω ± 20%]
3. Determine the value and tolerance of a resistor having a colour coding of: blue-white-black-black-gold [690 Ω ± 5%]
4. Determine the colour coding for a 51 kΩ four-band resistor having a tolerance of ±2% [green-brown-orange-red]
5. Determine the colour coding for a 1 MΩ four-band resistor having a tolerance of ±10% [brown-black-green-silver]
6. Determine the range of values expected for a resistor with colour coding: red-black-green-silver [1.8 MΩ to 2.2 MΩ]
7. Determine the range of values expected for a resistor with colour coding: yellow-black-orange-brown [39.6 kΩ to 40.4 kΩ]
8. Determine the value of a resistor marked as (a) R22G (b) 4K7F [(a) 0.22 Ω ± 2% (b) 4.7 kΩ ± 1%]
9. Determine the letter and digit code for a resistor having a value of 100 kΩ ± 5% [100 KJ]
10. Determine the letter and digit code for a resistor having a value of 6.8 MΩ ± 20% [6 M8 M]

Problem 2. A cell has an internal resistance of 0.02 Ω and an e.m.f. of 2.0 V. Calculate its terminal p.d. if it delivers (a) 5 A (b) 50 A.

(a) Terminal p.d. $V = E - Ir$ where E = e.m.f. of cell, I = current flowing and r = internal resistance of cell

$E = 2.0\,\text{V}, I = 5\,\text{A}$ and $r = 0.02\,\Omega$

Hence **terminal p.d.**

$$\mathbf{V} = 2.0 - (5)(0.02) = 2.0 - 0.1 = \mathbf{1.9\,V}$$

(b) When the current is 50 A, terminal p.d.,

$$V = E - Ir = 2.0 - 50(0.02)$$

i.e. $$\mathbf{V} = 2.0 - 1.0 = \mathbf{1.0\,V}$$

Thus the terminal p.d. decreases as the current drawn increases.

Problem 3. The p.d. at the terminals of a battery is 25 V when no load is connected and 24 V when a load taking 10 A is connected. Determine the internal resistance of the battery.

When no load is connected the e.m.f. of the battery, E, is equal to the terminal p.d., V, i.e. $E = 25\,\text{V}$

When current $I = 10\,\text{A}$ and terminal p.d.

$$V = 24\,\text{V}, \text{ then } V = E - Ir$$

i.e. $$24 = 25 - (10)r$$

Hence, rearranging, gives

$$10r = 25 - 24 = 1$$

and the internal resistance,

$$\mathbf{r} = \frac{1}{10} = \mathbf{0.1\,\Omega}$$

Problem 4. Ten 1.5 V cells, each having an internal resistance of 0.2 Ω, are connected in series to a load of 58 Ω. Determine (a) the current flowing in the circuit and (b) the p.d. at the battery terminals.

(a) For ten cells, battery e.m.f., $E = 10 \times 1.5 = 15\,\text{V}$, and the total internal resistance, $r = 10 \times 0.2 = 2\,\Omega$. When connected to a 58 Ω load the circuit is as shown in Fig. 4.4

$$\text{Current } I = \frac{\text{e.m.f.}}{\text{total resistance}} = \frac{15}{58 + 2} = \frac{15}{60} = \mathbf{0.25\,A}$$

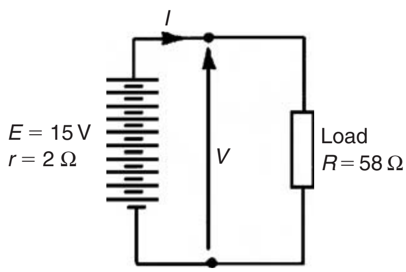


Figure 4.4

(b) P.d. at battery terminals, $V = E - Ir$

i.e. $V = 15 - (0.25)(2) = \mathbf{14.5\,V}$

Now try the following exercise

Exercise 16 Further problems on e.m.f. and internal resistance of a cell

1. Twelve cells, each with an internal resistance of 0.24 Ω and an e.m.f. of 1.5 V are connected (a) in series, (b) in parallel. Determine the e.m.f. and internal resistance of the batteries so formed.

   [(a) 18 V, 2.88 Ω (b) 1.5 V, 0.02 Ω]

2. A cell has an internal resistance of 0.03 Ω and an e.m.f. of 2.2 V. Calculate its terminal p.d. if it delivers

   (a) 1 A (b) 20 A (c) 50 A

   [(a) 2.17 V (b) 1.6 V (c) 0.7 V]

3. The p.d. at the terminals of a battery is 16 V when no load is connected and 14 V when a load taking 8 A is connected. Determine the internal resistance of the battery. [0.25 Ω]

4. A battery of e.m.f. 20 V and internal resistance 0.2 Ω supplies a load taking 10 A. Determine the p.d. at the battery terminals and the resistance of the load. [18 V, 1.8 Ω]

4.7 Secondary cells

Secondary cells can be recharged after use, that is, the conversion of chemical energy to electrical energy is reversible and the cell may be used many times. Examples of secondary cells include the lead–acid cell and the nickel cadmium and nickel-metal cells. Practical applications of such cells include car batteries, telephone circuits and for traction purposes – such as milk delivery vans and fork lift trucks.

Lead–acid cell

A typical lead–acid cell is constructed of:

(i) A container made of glass, ebonite or plastic.

(ii) **Lead plates**
  (a) the negative plate (cathode) consists of spongy lead
  (b) the positive plate (anode) is formed by pressing lead peroxide into the lead grid.

  The plates are interleaved as shown in the plan view of Fig. 4.8 to increase their effective cross-sectional area and to minimize internal resistance.

Figure 4.8

(iii) **Separators** made of glass, celluloid or wood.

(iv) An **electrolyte** which is a mixture of sulphuric acid and distilled water.

The relative density (or specific gravity) of a lead–acid cell, which may be measured using a hydrometer, varies between about 1.26 when the cell is fully charged to about 1.19 when discharged. The terminal p.d. of a lead–acid cell is about 2 V.

When a cell supplies current to a load it is said to be **discharging**. During discharge:

(i) the lead peroxide (positive plate) and the spongy lead (negative plate) are converted into lead sulphate, and

(ii) the oxygen in the lead peroxide combines with hydrogen in the electrolyte to form water. The electrolyte is therefore weakened and the relative density falls.

The terminal p.d. of a lead–acid cell when fully discharged is about 1.8 V. A cell is **charged** by connecting a d.c. supply to its terminals, the positive terminal of the cell being connected to the positive terminal of the supply. The charging current flows in the reverse direction to the discharge current and the chemical action is reversed. During charging:

(i) the lead sulphate on the positive and negative plates is converted back to lead peroxide and lead respectively, and

(ii) the water content of the electrolyte decreases as the oxygen released from the electrolyte combines with the lead of the positive plate. The relative density of the electrolyte thus increases.

The colour of the positive plate when fully charged is dark brown and when discharged is light brown. The colour of the negative plate when fully charged is grey and when discharged is light grey.

Nickel cadmium and nickel-metal cells

In both types the positive plate is made of nickel hydroxide enclosed in finely perforated steel tubes, the resistance being reduced by the addition of pure nickel or graphite. The tubes are assembled into nickel–steel plates.

In the nickel–metal cell, (sometimes called the **Edison cell** or **nife cell**), the negative plate is made of iron oxide, with the resistance being reduced by a little mercuric oxide, the whole being enclosed in perforated steel tubes and assembled in steel plates. In the nickel–cadmium cell the negative plate is made of cadmium. The electrolyte in each type of cell is a solution of potassium hydroxide which does not undergo any chemical change and thus the quantity can be reduced to a minimum. The plates are separated by insulating rods and assembled in steel containers which are then enclosed in a non-metallic crate to insulate the cells from one another. The average discharge p.d. of an alkaline cell is about 1.2 V.

Advantages of a nickel cadmium cell or a nickel–metal cell over a lead–acid cell include:

(i) More robust construction

(ii) Capable of withstanding heavy charging and discharging currents without damage

Revision test 1

This revision test covers the material contained in Chapters 1 to 4. *The marks for each question are shown in brackets at the end of each question.*

1. An electromagnet exerts a force of 15 N and moves a soft iron armature through a distance of 12 mm in 50 ms. Determine the power consumed. (5)

2. A d.c. motor consumes 47.25 MJ when connected to a 250 V supply for 1 hour 45 minutes. Determine the power rating of the motor and the current taken from the supply. (5)

3. A 100 W electric light bulb is connected to a 200 V supply. Calculate (a) the current flowing in the bulb, and (b) the resistance of the bulb. (4)

4. Determine the charge transferred when a current of 5 mA flows for 10 minutes. (2)

5. A current of 12 A flows in the element of an electric fire of resistance 25 Ω. Determine the power dissipated by the element. If the fire is on for 5 hours every day, calculate for a one week period (a) the energy used, and (b) cost of using the fire if electricity cost 13.5p per unit. (6)

6. Calculate the resistance of 1200 m of copper cable of cross-sectional area 15 mm². Take the resistivity of copper as 0.02 μΩm (5)

7. At a temperature of 40°C, an aluminium cable has a resistance of 25 Ω. If the temperature coefficient of resistance at 0°C is 0.0038/°C, calculate its resistance at 0°C (5)

8. (a) Determine the values of the resistors with the following colour coding:
   (i) red-red-orange-silver
   (ii) orange-orange-black-blue-green
   (b) What is the value of a resistor marked as 47 KK? (6)

9. Four cells, each with an internal resistance of 0.40 Ω and an e.m.f. of 2.5 V are connected in series to a load of 38.4 Ω. (a) Determine the current flowing in the circuit and the p.d. at the battery terminals. (b) If the cells are connected in parallel instead of in series, determine the current flowing and the p.d. at the battery terminals. (10)

10. (a) State six typical applications of primary cells
    (b) State six typical applications of secondary cells
    (c) State the advantages of a fuel cell over a conventional battery and state three practical applications. (12)

11. Name five alternative, renewable energy sources, and give a brief description of each. (15)

(b) The current flowing through R_1 and R_4 is 25 A. The current flowing through R_2

$$= \left(\frac{R_3}{R_2 + R_3}\right) I = \left(\frac{2}{6+2}\right) 25$$
$$= \mathbf{6.25\,A}$$

The current flowing through R_3

$$= \left(\frac{R_2}{R_2 + R_3}\right) I$$
$$= \left(\frac{6}{6+2}\right) 25 = \mathbf{18.75\,A}$$

(Note that the currents flowing through R_2 and R_3 must add up to the total current flowing into the parallel arrangement, i.e. 25 A)

(c) The equivalent circuit of Fig. 5.24 is shown in Fig. 5.25

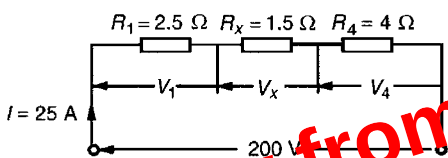


Figure 5.25

p.d. across R_1, i.e.

$$V_1 = IR_1 = (25)(2.5) = \mathbf{62.5\,V}$$

p.d. across R_x, i.e.

$$V_x = IR_x = (25)(1.5) = \mathbf{37.5\,V}$$

p.d. across R_4, i.e.

$$V_4 = IR_4 = (25)(4) = \mathbf{100\,V}$$

Hence the p.d. across R_2

$$= \text{p.d. across } R_3 = \mathbf{37.5\,V}$$

Problem 13. For the circuit shown in Fig. 5.26 calculate (a) the value of resistor R_x such that the total power dissipated in the circuit is 2.5 kW, (b) the current flowing in each of the four resistors.

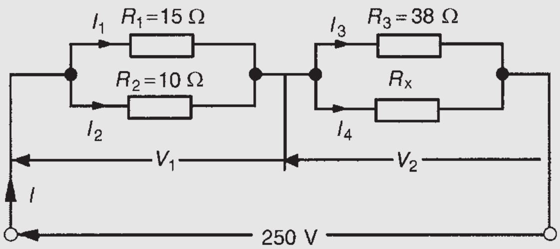


Figure 5.26

(a) Power dissipated $P = VI$ watts, hence

$$2500 = (250)(I)$$
$$\text{i.e. } I = \frac{2500}{250} = 10\,\text{A}$$

From Ohm's law,

$$R_T = \frac{V}{I} = \frac{250}{10} = 25\,\Omega,$$

where R_T is the equivalent circuit resistance. The equivalent resistance of R_1 and R_2 in parallel is

$$\frac{15 \times 10}{15 + 10} = \frac{150}{25} = 6\,\Omega$$

The equivalent resistance of resistors R_3 and R_x in parallel is equal to 25 Ω − 6 Ω, i.e. 19 Ω. There are three methods whereby R_x can be determined.

Method 1

The voltage $V_1 = IR$, where R is 6 Ω, from above, i.e. $V_1 = (10)(6) = 60$ V. Hence

$$V_2 = 250\,\text{V} - 60\,\text{V} = 190\,\text{V}$$
$$= \text{p.d. across } R_3$$
$$= \text{p.d. across } R_x$$
$$I_3 = \frac{V_2}{R_3} = \frac{190}{38} = 5\,\text{A}.$$

Thus $I_4 = 5$ A also, since $I = 10$ A. Thus

$$\mathbf{R_x} = \frac{V_2}{I_4} = \frac{190}{5} = \mathbf{38\,\Omega}$$

Method 2

Since the equivalent resistance of R_3 and R_x in parallel is 19 Ω,

$$19 = \frac{38R_x}{38 + R_x} \quad \left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$$

and $\varepsilon_r = 2.5$. Since

$$C = \frac{\varepsilon_0 \varepsilon_A A}{d} \quad \text{then} \quad d = \frac{\varepsilon_0 \varepsilon_r A}{C}$$

i.e.
$$d = \frac{8.85 \times 10^{-12} \times 2.5 \times 0.08}{4425 \times 10^{-12}} = 0.0004\,\text{m}$$

Hence, the thickness of the paper is 0.4 mm.

Problem 9. A parallel plate capacitor has nineteen interleaved plates each 75 mm by 75 mm separated by mica sheets 0.2 mm thick. Assuming the relative permittivity of the mica is 5, calculate the capacitance of the capacitor.

$n = 19$ thus $n - 1 = 18$, $A = 75 \times 75 = 5625\,\text{mm}^2 = 5625 \times 10^{-6}\,\text{m}^2$, $\varepsilon_r = 5$, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and $d = 0.2\,\text{mm} = 0.2 \times 10^{-3}$ m. Capacitance,

$$C = \frac{\varepsilon_0 \varepsilon_r A(n-1)}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 5 \times 5625 \times 10^{-6} \times 18}{0.2 \times 10^{-3}}\,\text{F}$$

$$= \mathbf{0.0224\,\mu F} \text{ or } \mathbf{22.4\,nF}$$

Now try the following exercise

Exercise 27 Further problems on parallel plate capacitors

(Where appropriate take ε_0 as 8.85×10^{-12} F/m)

1. A capacitor consists of two parallel plates each of area $0.01\,\text{m}^2$, spaced 0.1 mm in air. Calculate the capacitance in picofarads. [885 pF]

2. A waxed paper capacitor has two parallel plates, each of effective area $0.2\,\text{m}^2$. If the capacitance is 4000 pF determine the effective thickness of the paper if its relative permittivity is 2 [0.885 mm]

3. Calculate the capacitance of a parallel plate capacitor having 5 plates, each 30 mm by 20 mm and separated by a dielectric 0.75 mm thick having a relative permittivity of 2.3 [65.14 pF]

4. How many plates has a parallel plate capacitor having a capacitance of 5 nF, if each plate is 40 mm by 40 mm and each dielectric is 0.102 mm thick with a relative permittivity of 6. [7]

5. A parallel plate capacitor is made from 25 plates, each 70 mm by 120 mm interleaved with mica of relative permittivity 5. If the capacitance of the capacitor is 3000 pF determine the thickness of the mica sheet. [2.97 mm]

6. A capacitor is constructed with parallel plates and has a value of 50 pF. What would be the capacitance of the capacitor if the plate area is doubled and the plate spacing is halved? [200 pF]

7. The capacitance of a parallel plate capacitor is 1000 pF. It has 19 plates, each 50 mm by 30 mm separated by a dielectric of thickness 0.40 mm. Determine the relative permittivity of the dielectric. [1.67]

8. The charge on the square plates of a multi-plate capacitor is 80 μC when the potential between them is 5 kV. If the capacitor has twenty-five plates separated by a dielectric of thickness 0.102 mm and relative permittivity 4.8, determine the width of a plate. [40 mm]

9. A capacitor is to be constructed so that its capacitance is 4250 pF and to operate at a p.d. of 100 V across its terminals. The dielectric is to be polythene ($\varepsilon_r = 2.3$) which, after allowing a safety factor, has a dielectric strength of 20 MV/m. Find (a) the thickness of the polythene needed, and (b) the area of a plate. [(a) 0.005 mm (b) $10.44\,\text{cm}^2$]

6.9 Capacitors connected in parallel and series

(a) Capacitors connected in parallel

Figure 6.6 shows three capacitors, C_1, C_2 and C_3, connected in parallel with a supply voltage V applied across the arrangement.

When the charging current I reaches point A it divides, some flowing into C_1, some flowing into C_2 and some into C_3. Hence the total charge $Q_T (= I \times t)$ is divided

The voltage across the 12 µF capacitor,

$$V_3 = \frac{Q}{C_3}$$

$$= \frac{0.6 \times 10^{-3}}{12 \times 10^{-6}} = \mathbf{50\,V}$$

[Check: In a series circuit $V = V_1 + V_2 + V_3$. $V_1 + V_2 + V_3 = 200 + 100 + 50 = 350\,V$ = supply voltage]

In practice, capacitors are rarely connected in series unless they are of the same capacitance. The reason for this can be seen from the above problem where the lowest valued capacitor (i.e. 3 µF) has the highest p.d. across it (i.e. 200 V) which means that if all the capacitors have an identical construction they must all be rated at the highest voltage.

Problem 14. For the arrangement shown in Fig. 6.9 find (a) the equivalent capacitance of the circuit, (b) the voltage across QR, and (c) the charge on each capacitor.

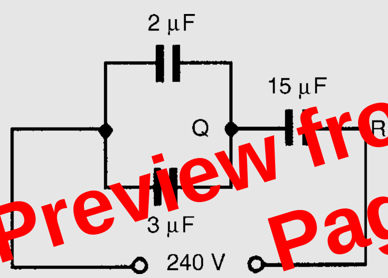


Figure 6.9

(a) 2 µF in parallel with 3 µF gives an equivalent capacitance of 2 µF + 3 µF = 5 µF. The circuit is now as shown in Fig. 6.10.

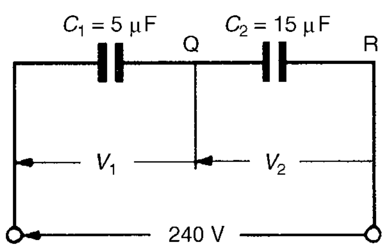


Figure 6.10

The **equivalent capacitance** of 5 µF in series with 15 µF is given by

$$\frac{5 \times 15}{5 + 15}\,\mu F = \frac{75}{20}\,\mu F = \mathbf{3.75\,\mu F}$$

(b) The charge on each of the capacitors shown in Fig. 6.10 will be the same since they are connected in series. Let this charge be Q coulombs.

Then $Q = C_1 V_1 = C_2 V_2$

i.e. $5V_1 = 15V_2$

$$V_1 = 3V_2 \quad (1)$$

Also $V_1 + V_2 = 240\,V$

Hence $3V_2 + V_2 = 240\,V$ from equation (1)

Thus $V_2 = 60\,V$ and $V_1 = 180\,V$

Hence the voltage across QR is 60 V

(c) The charge on the 15 µF capacitor is

$C_2 V_2 = 15 \times 10^{-6} \times 60 = \mathbf{0.9\,mC}$

The charge on the 2 µF capacitor is

$2 \times 10^{-6} \times 180 = \mathbf{0.36\,mC}$

The charge on the 3 µF capacitor is

$3 \times 10^{-6} \times 180 = \mathbf{0.54\,mC}$

Now try the following exercise

Exercise 28 Further problems on capacitors in parallel and series

1. Capacitors of 2 µF and 6 µF are connected (a) in parallel and (b) in series. Determine the equivalent capacitance in each case. [(a) 8 µF (b) 1.5 µF]

2. Find the capacitance to be connected in series with a 10 µF capacitor for the equivalent capacitance to be 6 µF [15 µF]

3. What value of capacitance would be obtained if capacitors of 0.15 µF and 0.10 µF are connected (a) in series and (b) in parallel [(a) 0.06 µF (b) 0.25 µF]

4. Two 6 µF capacitors are connected in series with one having a capacitance of 12 µF. Find the total equivalent circuit capacitance. What

3. **Paper capacitors**. A typical paper capacitor is shown in Fig. 6.16 where the length of the roll corresponds to the capacitance required.

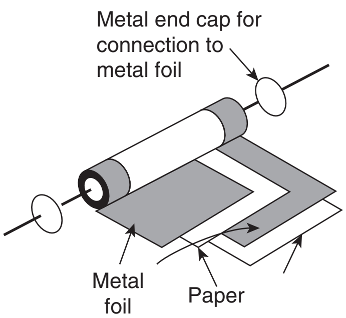


Figure 6.16

The whole is usually impregnated with oil or wax to exclude moisture, and then placed in a plastic or aluminium container for protection. Paper capacitors are made in various working voltages up to about 150 kV and are used where loss is not very important. The maximum value of this type of capacitor is between 500 pF and 10 μF. Disadvantages of paper capacitors include variation in capacitance with temperature change and a shorter service life than most other types of capacitor.

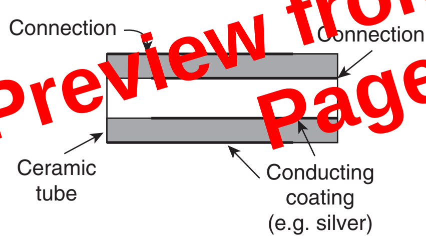


Figure 6.17

4. **Ceramic capacitors**. These are made in various forms, each type of construction depending on the value of capacitance required. For high values, a tube of ceramic material is used as shown in the cross section of Fig. 6.17. For smaller values the cup construction is used as shown in Fig. 6.18, and for still smaller values the disc construction shown in Fig. 6.19 is used. Certain ceramic materials have a very high permittivity and this enables capacitors of high capacitance to be made which are of small physical size with a high working voltage rating. Ceramic capacitors are available in the range 1 pF to 0.1 μF and may be used in high frequency electronic circuits subject to a wide range of temperatures.

5. **Plastic capacitors**. Some plastic materials such as polystyrene and Teflon can be used as dielectrics. Construction is similar to the paper capacitor but using a plastic film instead of paper. Plastic capacitors operate well under conditions of high temperature, provide a precise value of capacitance, a very long service life and high reliability.

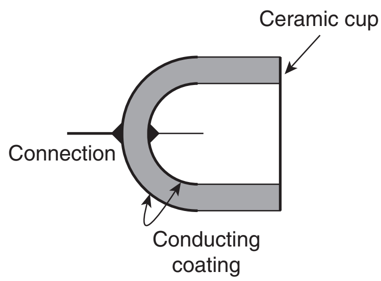


Figure 6.18

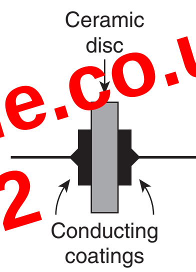


Figure 6.19

6. **Titanium oxide capacitors** have a very high capacitance with a small physical size when used at a low temperature.

7. **Electrolytic capacitors**. Construction is similar to the paper capacitor with aluminium foil used for the plates and with a thick absorbent material, such as paper, impregnated with an electrolyte (ammonium borate), separating the plates. The finished capacitor is usually assembled in an aluminium container and hermetically sealed. Its operation depends on the formation of a thin aluminium oxide layer on the positive plate by electrolytic action when a suitable direct potential is maintained between the plates. This oxide layer is very thin and forms the dielectric. (The absorbent paper between the plates is a conductor and does not act as a dielectric.) Such capacitors **must always be used on d.c.** and must be connected with the correct polarity; if this is not done the capacitor will be destroyed since the oxide layer will be destroyed. Electrolytic capacitors are manufactured with working voltage from

Chapter 7

Magnetic circuits

At the end of this chapter you should be able to:

- appreciate some applications of magnets
- describe the magnetic field around a permanent magnet
- state the laws of magnetic attraction and repulsion for two magnets in close proximity
- define magnetic flux, Φ, and magnetic flux density, B, and state their units
- perform simple calculations involving $B = \Phi/A$
- define magnetomotive force, F_m, and magnetic field strength, H, and state their units
- perform simple calculations involving $F_m = NI$ and $H = NI/l$
- define permeability, distinguishing between μ_0, μ_r and μ
- understand the B–H curves for different magnetic materials
- appreciate typical values of μ_r
- perform calculations involving $B = \mu_0\mu_r H$
- define reluctance, S, and state its units
- perform calculations involving

$$S = \frac{\text{m.m.f.}}{\Phi} = \frac{l}{\mu_0\mu_r A}$$

- perform calculations on composite series magnetic circuits
- compare electrical and magnetic quantities
- appreciate how a hysteresis loop is obtained and that hysteresis loss is proportional to its area

7.1 Introduction to magnetism and magnetic circuits

The study of magnetism began in the thirteenth century with many eminent scientists and physicists such as William Gilbert, Hans Christian Oersted, Michael Faraday, James Maxwell, André Ampère and Wilhelm Weber all having some input on the subject since. The association between electricity and magnetism is a fairly recent finding in comparison with the very first understanding of basic magnetism.

Today, magnets have **many varied practical applications**. For example, they are used in motors and generators, telephones, relays, loudspeakers, computer hard drives and floppy disks, anti-lock brakes, cameras, fishing reels, electronic ignition systems, keyboards, t.v. and radio components and in transmission equipment.

For the 6 cm long path:

$$\text{Reluctance } S_1 = \frac{l_1}{\mu_0 \mu_r A_1}$$

$$= \frac{6 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(1 \times 10^{-4})}$$

$$= 6.366 \times 10^5/\text{H}$$

For the 2 cm long path:

$$\text{Reluctance } S_2 = \frac{l_2}{\mu_0 \mu_r A_2}$$

$$= \frac{2 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(0.5 \times 10^{-4})}$$

$$= 4.244 \times 10^5/\text{H}$$

Total circuit reluctance $S = S_1 + S_2$

$$= (6.366 + 4.244) \times 10^5$$

$$= 10.61 \times 10^5/\text{H}$$

$$S = \frac{\text{m.m.f.}}{\Phi} \text{ i.e. } \Phi = \frac{\text{m.m.f.}}{S} = \frac{NI}{S}$$

$$= \frac{200 \times 0.4}{10.61 \times 10^5} = 7.54 \times 10^{-5}\,\text{Wb}$$

Flux density in the 2 cm path,

$$B = \frac{\Phi}{A} = \frac{7.54 \times 10^{-5}}{0.5 \times 10^{-4}} = \mathbf{1.51\,T}$$

Problem 13. A silicon iron ring of cross-sectional area 5 cm^2 has a radial air gap of 2 mm cut into it. If the mean length of the silicon iron path is 40 cm calculate the magnetomotive force to produce a flux of 0.7 mWb. The magnetisation curve for silicon is shown on page 74.

There are two parts to the circuit — the silicon iron and the air gap. The total m.m.f. will be the sum of the m.m.f.'s of each part.

For the silicon iron:

$$B = \frac{\Phi}{A} = \frac{0.7 \times 10^{-3}}{5 \times 10^{-4}} = 1.4\,\text{T}$$

From the B–H curve for silicon iron on page 74, when $B = 1.4$ T, $H = 1650$ A/m. Hence the m.m.f. for the iron path $= Hl = 1650 \times 0.4 = 660$ A

For the air gap:

The flux density will be the same in the air gap as in the iron, i.e. 1.4 T (This assumes no leakage or fringing occurring). For air,

$$H = \frac{B}{\mu_0} = \frac{1.4}{4\pi \times 10^{-7}} = 1\,114\,000\,\text{A/m}$$

Hence the m.m.f. for the air gap $= Hl = 1\,114\,000 \times 2 \times 10^{-3} = 2228$ A.
Total m.m.f. to produce a flux of 0.6 mWb $= 660 + 2228 =$ **2888 A**.

A tabular method could have been used as shown in Table 7.3 at top of next page.

Problem 14. Figure 7.5 shows a ring formed with two different materials — cast steel and mild steel.

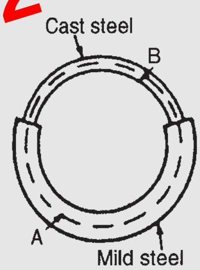


Figure 7.5

The dimensions are:

	mean length	cross-sectional area
Mild steel	400 mm	500 mm^2
Cast steel	300 mm	312.5 mm^2

Find the total m.m.f. required to cause a flux of 500 µWb in the magnetic circuit. Determine also the total circuit reluctance.

Chapter 8

Electromagnetism

At the end of this chapter you should be able to:

- understand that magnetic fields are produced by electric currents
- apply the screw rule to determine direction of magnetic field
- recognize that the magnetic field around a solenoid is similar to a magnet
- apply the screw rule or grip rule to a solenoid to determine magnetic field direction
- recognize and describe practical applications of an electromagnet, i.e. electric bell, relay, lifting magnet, telephone receiver
- appreciate factors upon which the force F on a current-carrying conductor depends
- perform calculations using $F = BIl$ and $F = BIl \sin\theta$
- recognize that a loudspeaker is a practical application of force F
- use Fleming's left-hand rule to pre-determine direction of force in a current carrying conductor
- describe the principle of operation of a simple d.c. motor
- describe the principle of operation and construction of a moving coil instrument
- appreciate that force F on a charge in a magnetic field is given by $F = QvB$
- perform calculations using $F = QvB$

8.1 Magnetic field due to an electric current

Magnetic fields can be set up not only by permanent magnets, as shown in Chapter 7, but also by electric currents.

Let a piece of wire be arranged to pass vertically through a horizontal sheet of cardboard on which is placed some iron filings, as shown in Fig. 8.1(a). If a current is now passed through the wire, then the iron filings will form a definite circular field pattern with the wire at the centre, when the cardboard is gently tapped. By placing a compass in different positions the lines of flux are seen to have a definite direction as shown in Fig. 8.1(b).

If the current direction is reversed, the direction of the lines of flux is also reversed. The effect on both the iron filings and the compass needle disappears when the current is switched off. The magnetic field is thus produced

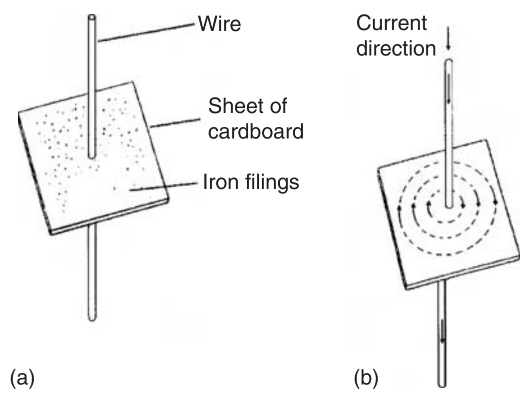


Figure 8.1

by the electric current. The magnetic flux produced has the same properties as the flux produced by a permanent magnet. If the current is increased the strength of the field increases and, as for the permanent magnet,

Chapter 9

Electromagnetic induction

At the end of this chapter you should be able to:

- understand how an e.m.f. may be induced in a conductor
- state Faraday's laws of electromagnetic induction
- state Lenz's law
- use Fleming's right-hand rule for relative directions
- appreciate that the induced e.m.f., $E = Blv$ or $E = Blv \sin\theta$
- calculate induced e.m.f. given B, l, v and θ and determine relative directions
- understands and performs calculations on rotation of a loop in a magnetic field
- define inductance L and state its unit
- define mutual inductance
- appreciate that emf

$$E = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

- calculate induced e.m.f. given N, t, L, change of flux or change of current
- appreciate factors which affect the inductance of an inductor
- draw the circuit diagram symbols for inductors
- calculate the energy stored in an inductor using $W = \frac{1}{2}LI^2$ joules
- calculate inductance L of a coil, given $L = \frac{N\Phi}{I}$ and $L = \frac{N^2}{S}$
- calculate mutual inductance using $E_2 = -M\frac{dI_1}{dt}$ and $M = \frac{N_1N_2}{S}$

9.1 Introduction to electromagnetic induction

When a conductor is moved across a magnetic field so as to cut through the lines of force (or flux), an electromotive force (e.m.f.) is produced in the conductor. If the conductor forms part of a closed circuit then the e.m.f. produced causes an electric current to flow round the circuit. Hence an e.m.f. (and thus current) is 'induced' in the conductor as a result of its movement across the magnetic field. This effect is known as '**electromagnetic induction**'.

Figure 9.1(a) shows a coil of wire connected to a centre-zero galvanometer, which is a sensitive ammeter with the zero-current position in the centre of the scale.

(a) When the magnet is moved at constant speed towards the coil (Fig. 9.1(a)), a deflection is noted

In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an e.m.f. is induced in the conductors and thus a source of e.m.f. is created. A generator converts mechanical energy into electrical energy. (The action of a simple a.c. generator is described in Chapter 14).

The induced e.m.f. E set up between the ends of the conductor shown in Fig. 9.3 is given by:

$$E = Blv \text{ volts}$$

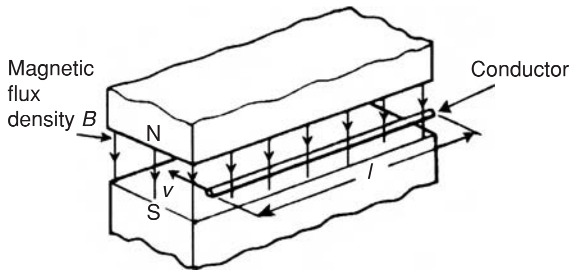


Figure 9.3

where B, the flux density, is measured in teslas, l, the length of conductor in the magnetic field, is measured in metres, and v, the conductor velocity, is measured in metres per second.

If the conductor moves at an angle $\theta°$ to the magnetic field (instead of at 90° as assumed above) then

$$E = Blv \sin\theta \text{ volts}$$

Problem 1. A conductor 300 mm long moves at a uniform speed of 4 m/s at right-angles to a uniform magnetic field of flux density 1.25 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 20 Ω resistance.

When a conductor moves in a magnetic field it will have an e.m.f. induced in it but this e.m.f. can only produce a current if there is a closed circuit. Induced e.m.f.

$$E = Blv = (1.25)\left(\frac{300}{1000}\right)(4) = 1.5 \text{ V}$$

(a) If the ends of the conductor are open circuited **no current will flow** even though 1.5 V has been induced.

(b) From Ohm's law,

$$I = \frac{E}{R} = \frac{1.5}{20} = \mathbf{0.075\,A \text{ or } 75\,mA}$$

Problem 2. At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an e.m.f. of 9 V is to be induced in it? Assume the conductor, the field and the direction of motion are mutually perpendicular.

Induced e.m.f. $E = Blv$, hence velocity $v = E/Bl$

Thus

$$v = \frac{9}{(0.6)(75 \times 10^{-3})} = \frac{9 \times 10^3}{0.6 \times 75} = \mathbf{200\,m/s}$$

Problem 3. A conductor moves with a velocity of 15 m/s at an angle of (a) 90° (b) 60° and (c) 30° to a magnetic field produced between two square-faced poles of side length 2 cm. If the flux leaving a pole face is 5 μWb, find the magnitude of the induced e.m.f. in each case.

$v = 15$ m/s, length of conductor in magnetic field, $l = 2\text{ cm} = 0.02$ m, $A = 2 \times 2\text{ cm}^2 = 4 \times 10^{-4}\text{ m}^2$ and $\Phi = 5 \times 10^{-6}$ Wb

(a) $$E_{90} = Blv \sin 90° = \left(\frac{\Phi}{A}\right) lv \sin 90° = \left(\frac{5 \times 10^{-6}}{4 \times 10^{-4}}\right)(0.02)(15)(1) = \mathbf{3.75\,mV}$$

(b) $$E_{60} = Blv \sin 60° = E_{90} \sin 60° = 3.75 \sin 60° = \mathbf{3.25\,mV}$$

(c) $$E_{30} = Blv \sin 30° = E_{90} \sin 30° = 3.75 \sin 30° = \mathbf{1.875\,mV}$$

Problem 4. The wing span of a metal aeroplane is 36 m. If the aeroplane is flying at 400 km/h, determine the e.m.f. induced between its wing tips. Assume the vertical component of the earth's magnetic field is 40 μT.

11. If a current of I amperes flowing in a coil of N turns produces a flux of Φ webers, the coil inductance L is given by $L = \ldots\ldots$ henrys

12. The energy W stored by an inductor is given by $W = \ldots\ldots$ joules

13. If the number of turns of a coil is N and its reluctance is S, then the inductance, L, is given by: $L = \ldots\ldots\ldots$

14. What is mutual inductance? State its symbol

15. The mutual inductance between two coils is M. The e.m.f. E_2 induced in one coil by the current changing at (dI_1/dt) in the other is given by $E_2 = \ldots\ldots$ volts

16. Two coils wound on an iron ring of reluctance S have N_A and N_B turns respectively. The mutual inductance, M, is given by: $M = \ldots\ldots\ldots$

Exercise 49 Multi-choice questions on electromagnetic induction
(Answers on page 398)

1. A current changing at a rate of 5 A/s in a coil of inductance 5 H induces an e.m.f. of:
   (a) 25 V in the same direction as the applied voltage
   (b) 1 V in the same direction as the applied voltage
   (c) 25 V in the opposite direction to the applied voltage
   (d) 1 V in the opposite direction to the applied voltage

2. A bar magnet is moved at a steady speed of 1.0 m/s towards a coil of wire which is connected to a centre-zero galvanometer. The magnet is now withdrawn along the same path at 0.5 m/s. The deflection of the galvanometer is in the:
   (a) same direction as previously, with the magnitude of the deflection doubled
   (b) opposite direction as previously, with the magnitude of the deflection halved
   (c) same direction as previously, with the magnitude of the deflection halved
   (d) opposite direction as previously, with the magnitude of the deflection doubled

3. When a magnetic flux of 10 Wb links with a circuit of 20 turns in 2 s, the induced e.m.f. is:
   (a) 1 V (b) 4 V
   (c) 100 V (d) 400 V

4. A current of 10 A in a coil of 1000 turns produces a flux of 10 mWb linking with the coil. The coil inductance is:
   (a) 10^6 H (b) 1 H
   (c) 1 μH (d) 1 mH

5. An e.m.f. of 1 V is induced in a conductor moving at 10 cm/s in a magnetic field of 0.5 T. The effective length of the conductor in the magnetic field is:
   (a) 20 cm (b) 5 m
   (c) 20 m (d) 50 m

6. Which of the following is false ?
   (a) Fleming's left-hand rule or Lenz's law may be used to determine the direction of an induced e.m.f.
   (b) An induced e.m.f. is set up whenever the magnetic field linking that circuit changes
   (c) The direction of an induced e.m.f. is always such as to oppose the effect producing it
   (d) The induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit

7. The effect of inductance occurs in an electrical circuit when:
   (a) the resistance is changing
   (b) the flux is changing
   (c) the current is changing

8. Which of the following statements is false? The inductance of an inductor increases:
   (a) with a short, thick coil
   (b) when wound on an iron core
   (c) as the number of turns increases
   (d) as the cross-sectional area of the coil decreases

Chapter 10

Electrical measuring instruments and measurements

At the end of this chapter you should be able to:

- recognize the importance of testing and measurements in electric circuits
- appreciate the essential devices comprising an analogue instrument
- explain the operation of an attraction and a repulsion type of moving-iron instrument
- explain the operation of a moving-coil rectifier instrument
- compare moving-coil, moving-iron and moving-coil rectifier instruments
- calculate values of shunts for ammeters and multipliers for voltmeters
- understand the advantages of electronic instruments
- understand the operation of an ohmmeter/megger
- appreciate the operation of multimeters/Avometers/Flukes
- understand the operation of a wattmeter
- appreciate instrument 'loading' effect
- understand the operation of an oscilloscope for d.c. and a.c. measurements
- calculate periodic time, frequency, peak to peak values from waveforms on an oscilloscope
- appreciate virtual test and measuring instruments
- recognize harmonics present in complex waveforms
- determine ratios of powers, currents and voltages in decibels
- understand null methods of measurement for a Wheatstone bridge and d.c. potentiometer
- understand the operation of a.c. bridges
- understand the operation of a Q-meter
- appreciate the most likely source of errors in measurements
- appreciate calibration accuracy of instruments

Problem 8. For the oscilloscope display of a pulse waveform shown in Fig. 10.18 the 'time/cm' switch is on 50 ms/cm and the 'volts/cm' switch is on 0.2 V/cm. Determine (a) the periodic time, (b) the frequency, (c) the magnitude of the pulse voltage.

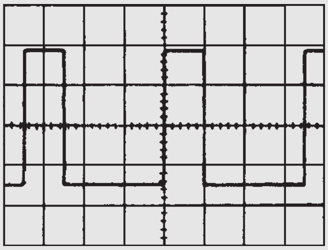


Figure 10.18

(a) The width of one complete cycle is 3.5 cm. Hence the periodic time,

$$T = 3.5\text{ cm} \times 50\text{ ms/cm} = \mathbf{175\text{ ms}}.$$

(b) Frequency, $f = \frac{1}{T} = \frac{1}{0.52 \times 10^{-3}} = \mathbf{5.71\text{ Hz}}.$

(c) The height of a pulse is 3.4 cm hence the magnitude of the pulse voltage

$$= 3.4\text{ cm} \times 0.2\text{ V/cm} = \mathbf{0.68\text{ V}}.$$

Problem 9. A sinusoidal voltage trace displayed by an oscilloscope is shown in Fig. 10.19. If the 'time/cm' switch is on 500 μs/cm and the 'volts/cm' switch is on 5 V/cm, find, for the waveform, (a) the frequency, (b) the peak-to-peak voltage, (c) the amplitude, (d) the r.m.s. value.

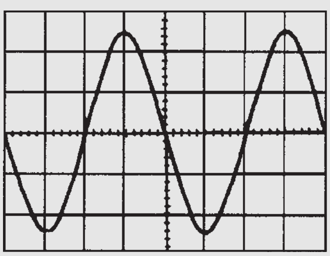


Figure 10.19

(a) The width of one complete cycle is 4 cm. Hence the periodic time, T is 4 cm × 500 μs/cm, i.e. 2 ms.

$$\text{Frequency}, f = \frac{1}{T} = \frac{1}{2 \times 10^{-3}} = \mathbf{500\text{ Hz}}$$

(b) The peak-to-peak height of the waveform is 5 cm. Hence the peak-to-peak voltage

$$= 5\text{ cm} \times 5\text{ V/cm} = \mathbf{25\text{ V}}.$$

(c) Amplitude $= \frac{1}{2} \times 25\text{ V} = \mathbf{12.5\text{ V}}$

(d) The peak value of voltage is the amplitude, i.e. 12.5 V, and r.m.s.

$$\text{voltage} = \frac{\text{peak voltage}}{\sqrt{2}} = \frac{12.5}{\sqrt{2}} = \mathbf{8.84\text{ V}}$$

Problem 10. For the double-beam oscilloscope displays shown in Fig. 10.20 determine (a) their frequency, (b) their r.m.s. values, (c) their phase difference. The 'time/cm' switch is on 100 μs/cm and the 'volts/cm' switch on 2 V/cm.

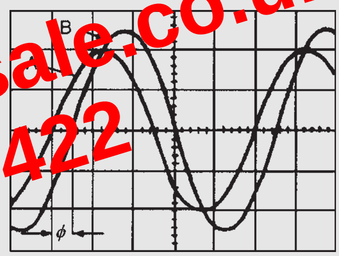


Figure 10.20

(a) The width of each complete cycle is 5 cm for both waveforms. Hence the periodic time, T, of each waveform is 5 cm × 100 μs/cm, i.e. 0.5 ms. Frequency of each waveform,

$$f = \frac{1}{T} = \frac{1}{0.5 \times 10^{-3}} = \mathbf{2\text{ kHz}}$$

(b) The peak value of waveform A is 2 cm × 2 V/cm = **4 V**, hence the r.m.s. value of waveform A

$$= 4/(\sqrt{2}) = \mathbf{2.83\text{ V}}$$

The peak value of waveform B is 2.5 cm × 2 V/cm = 5 V, hence the r.m.s. value of waveform B

$$= 5/(\sqrt{2}) = \mathbf{3.54\text{ V}}$$

- frequency spectrum display and analysis
- data logging (stored waveform data can be exported in formats that are compatible with conventional spreadsheet packages, e.g. as .xls files)
- ability to save/print waveforms and other information in graphical format (e.g. as .jpg or .bmp files).

Virtual instruments can take various forms including:

- internal hardware in the form of a conventional PCI expansion card
- external hardware unit which is connected to the PC by means of either a conventional 25-pin parallel port connector or by means of a serial USB connector

The software (and any necessary drivers) is invariably supplied on CD-ROM or can be downloaded from the manufacturer's web site. Some manufacturers also supply software drivers together with sufficient accompanying documentation in order to allow users to control virtual test instruments from their own software developed using popular programming languages such as VisualBASIC or C++.

10.14 Virtual digital storage oscilloscopes

Several types of virtual DSO are currently available. These can be conveniently arranged into three different categories according to their application:

- Low-cost DSO
- High-speed DSO
- High-resolution DSO

Unfortunately, there is often some confusion between the last two categories. A high-speed DSO is designed for examining waveforms that are rapidly changing. Such an instrument does not necessarily provide high-resolution measurement. Similarly, a high-resolution DSO is useful for displaying waveforms with a high degree of precision but it may not be suitable for examining fast waveforms. The difference between these two types of DSO should become a little clearer later on.

Low-cost DSO are primarily designed for low frequency signals (typically signals up to around 20 kHz) and are usually able to sample their signals at rates of between 10K and 100K samples per second. Resolution is usually limited to either 8-bits or 12-bits (corresponding to 256 and 4096 discrete voltage levels respectively).

High-speed DSOs are rapidly replacing CRT-based oscilloscopes. They are invariably dual-channel instruments and provide all the features associated with a conventional 'scope including trigger selection, time-base and voltage ranges, and an ability to operate in X-Y mode.

Additional features available with a computer-based instrument include the ability to capture transient signals (as with a conventional digital storage 'scope') and save waveforms for future analysis. The ability to analyse a signal in terms of its frequency spectrum is yet another feature that is only possible with a DSO (see later).

Upper frequency limit

The upper signal frequency limit of a DSO is determined primarily by the rate at which it can sample an incoming signal. Typical sampling rates for different types of virtual instrument are:

Type of DSO	Typical sampling rate
Low-cost DSO	20K to 100K per second
High-speed DSO	100M to 1000M per second
High-resolution DSO	20M to 100M per second

In order to display waveforms with reasonable accuracy it is normally suggested that the sampling rate should be *at least* twice and *preferably more* than five times the highest signal frequency. Thus, in order to display a 10 MHz signal with any degree of accuracy a sampling rate of 50M samples per second will be required.

The 'five times rule' merits a little explanation. When sampling signals in a digital to analogue converter we usually apply the Nyquist criterion that the sampling frequency must be at least twice the highest analogue signal frequency. Unfortunately, this no longer applies in the case of a DSO where we need to sample at an even faster rate if we are to accurately display the signal. In practice we would need a minimum of about five points within a single cycle of a sampled waveform in order to reproduce it with approximate fidelity. Hence the sampling rate should be at least five times that of

It is important to remember that, if the input signal is not a pure sine wave it will contain a number of higher frequency harmonics. For example, a square wave will contain odd harmonics that have levels that become progressively reduced as their frequency increases. Thus, to display a 1 MHz square wave accurately you need to take into account the fact that there will be signal components present at 3 MHz, 5 MHz, 7 MHz, 9 MHz, 11 MHz, and so on.

Spectrum analysis

The technique of Fast Fourier Transformation (FFT) calculated using software algorithms using data captured by a virtual DSO has made it possible to produce frequency spectrum displays. Such displays can be to investigate the harmonic content of waveforms as well as the relationship between several signals within a composite waveform.

Figure 10.27 shows the frequency spectrum of the 1 kHz sine wave signal from a low-distortion signal generator. Here the virtual DSO has been set to capture samples at a rate of 4096 per second within a frequency range of d.c. to 12.2 kHz. The display clearly shows the second harmonic (at a level of −50 dB or −70 dB relative to the fundamental), plus further harmonics at 3 kHz, 5 kHz and 7 kHz (all of which are greater than 75 dB down on the fundamental).

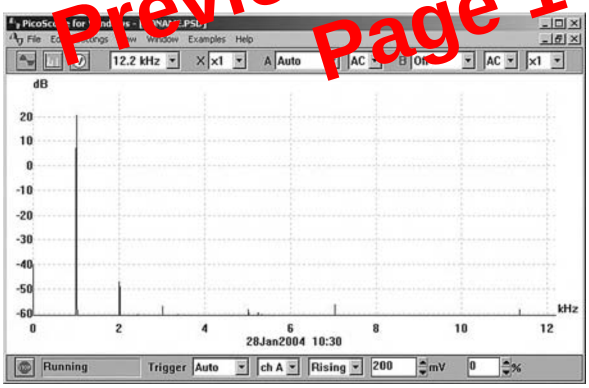


Figure 10.27

Problem 11. Figure 10.28 shows the frequency spectrum of a signal at 1184 kHz displayed by a high-speed virtual DSO. Determine (a) the harmonic relationship between the signals marked 'o' and 'x', (b) the difference in amplitude (expressed in dB) between the signals marked 'o' and 'x', and (c) the amplitude of the second harmonic relative to the fundamental signal 'o'.

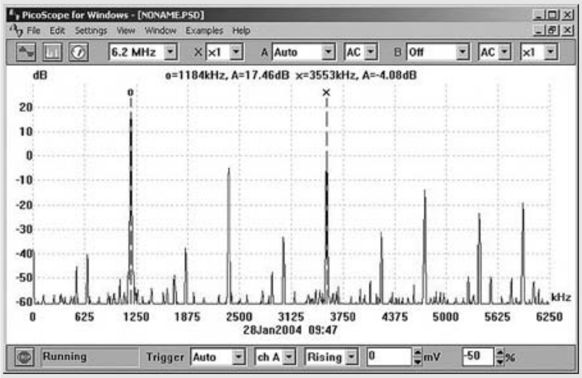


Figure 10.28

(a) The signal x is at a frequency of 3553 kHz. This is three times the frequency of the signal at 'o' which is at 1184 kHz. Thus, ***x* is the third harmonic of the signal 'o'**

(b) The signal at 'o' has an amplitude of +17.46 dB whilst the signal at 'x' has an amplitude of −4.08 dB. Thus, **the difference in level** = (+17.46) − (−4.08) = **21.54 dB**

(c) **The amplitude of the second harmonic** (shown at approximately 2270 kHz) = **−5 dB**

10.15 Waveform harmonics

(i) Let an instantaneous voltage v be represented by $v = V_m \sin 2\pi ft$ volts. This is a waveform which varies sinusoidally with time t, has a frequency f, and a maximum value V_m. Alternating voltages are usually assumed to have wave-shapes which are sinusoidal where only one frequency is present. If the waveform is not sinusoidal it is called a **complex wave**, and, whatever its shape, it may be split up mathematically into components called the **fundamental** and a number of **harmonics**. This process is called harmonic analysis. The fundamental (or first harmonic) is sinusoidal and has the supply frequency, f; the other harmonics are also sine waves having frequencies which are integer multiples of f. Thus, if the supply frequency is 50 Hz, then the third harmonic frequency is 150 Hz, the fifth 250 Hz, and so on.

(ii) A complex waveform comprising the sum of the fundamental and a third harmonic of about half the amplitude of the fundamental is shown in Fig. 10.29(a), both waveforms being initially in phase with each other. If further odd harmonic waveforms of the appropriate amplitudes are added, a good approximation to a square wave results. In Fig. 10.29(b), the third harmonic is shown having an initial phase displacement from the fundamental. The positive and negative half cycles of each of the complex waveforms shown in Figs. 10.29(a) and (b) are identical in shape, and this is a feature of waveforms containing the fundamental and only odd harmonics.

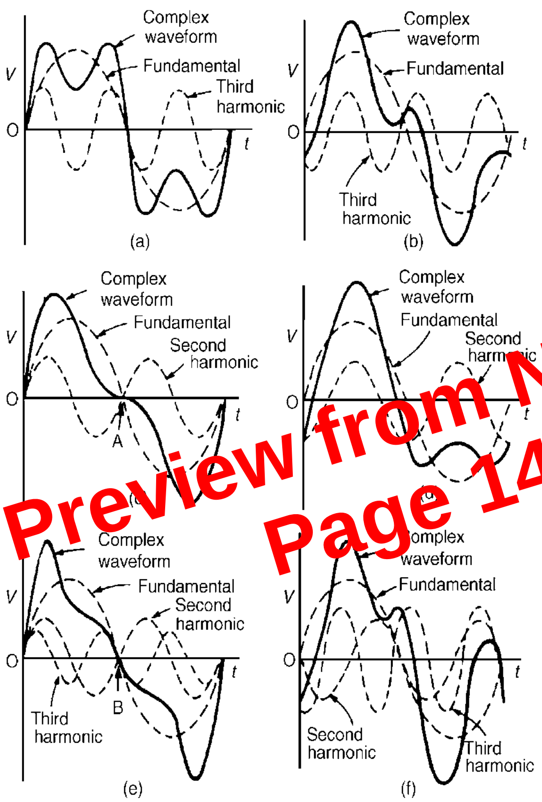


Figure 10.29

(iii) A complex waveform comprising the sum of the fundamental and a second harmonic of about half the amplitude of the fundamental is shown in Fig. 10.29(c), each waveform being initially in phase with each other. If further even harmonics of appropriate amplitudes are added a good approximation to a triangular wave results. In Fig. 10.29(c), the negative cycle, if reversed, appears as a mirror image of the positive cycle about point A. In Fig. 10.29(d) the second harmonic is shown with an initial phase displacement from the fundamental and the positive and negative half cycles are dissimilar.

(iv) A complex waveform comprising the sum of the fundamental, a second harmonic and a third harmonic is shown in Fig. 10.29(e), each waveform being initially 'in-phase'. The negative half cycle, if reversed, appears as a mirror image of the positive cycle about point B. In Fig. 10.29(f), a complex waveform comprising the sum of the fundamental, a second harmonic and a third harmonic are shown with initial phase displacement. The positive and negative half cycles are seen to be dissimilar.

The features mentioned relative to Figs. 10.29(a) to (f) make it possible to recognize the harmonics present in a complex waveform displayed on a CRO.

10.16 Logarithmic ratios

In electronic systems, the ratio of two similar quantities measured at different points in the system, are often expressed in logarithmic units. By definition, if the ratio of two powers P_1 and P_2 is to be expressed in **decibel (dB) units** then the number of decibels, X, is given by:

$$X = 10\lg\left(\frac{P_2}{P_1}\right)\ \text{dB} \qquad (1)$$

Thus, when the power ratio, $P_2/P_1 = 1$ then the decibel power ratio $= 10\lg 1 = 0$, when the power ratio, $P_2/P_1 = 100$ then the decibel power ratio $= 10\lg 100 = +20$ (i.e. a power gain), and when the power ratio, $P_2/P_1 = 1/100$ then the decibel power ratio $= 10\lg 1/100 = -20$ (i.e. a power loss or attenuation).

Logarithmic units may also be used for voltage and current ratios. Power, P, is given by $P = I^2R$ or $P = V^2/R$. Substituting in equation (1) gives:

$$X = 10\lg\left(\frac{I_2^2 R_2}{I_1^2 R_1}\right)\ \text{dB}$$

or $$X = 10\lg\left(\frac{V_2^2/R_2}{V_1^2/R_1}\right)\ \text{dB}$$

If $R_1 = R_2$,

then $$X = 10\lg\left(\frac{I_2^2}{I_1^2}\right)\ \text{dB}$$

or

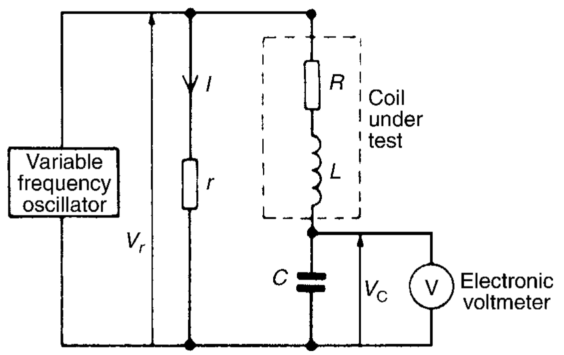


Figure 10.36

Q-meters operate at various frequencies and instruments exist with frequency ranges from 1 kHz to 50 MHz. Errors in measurement can exist with Q-meters since the coil has an effective parallel self capacitance due to capacitance between turns. The accuracy of a Q-meter is approximately ±5%.

Problem 21. When connected to a Q-meter an inductor is made to resonate at 400 kHz. The Q-factor of the circuit is found to be 100 and the capacitance of the Q-meter capacitor is set to 400 pF. Determine (a) the inductance, and (b) the resistance of the inductor.

Resonant frequency, $f_r = 400$ kHz $= 400 \times 10^3$ Hz, Q-factor = 100 and capacitance, $C = 400$ pF $= 400 \times 10^{-12}$ F. The circuit diagram of a Q-meter is shown in Fig. 10.36

(a) At resonance,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

for a series L–C–R circuit.

Hence $$2\pi f_r = \frac{1}{\sqrt{LC}}$$

from which

$$(2\pi f_r)^2 = \frac{1}{LC}$$

and **inductance,**

$$\boldsymbol{L} = \frac{1}{(2\pi f_r)^2 C}$$

$$= \frac{1}{(2\pi \times 400 \times 10^3)^2 (400 \times 10^{-12})} \text{ H}$$

$$= \mathbf{396\ \mu H \text{ or } 0.396\ mH}$$

(b) Q-factor at resonance $= 2\pi f_r L/R$ from which resistance

$$R = \frac{2\pi f_r L}{Q}$$

$$= \frac{2\pi(400 \times 10^3)(0.396 \times 10^{-3})}{100}$$

$$= \mathbf{9.95\ \Omega}$$

Now try the following exercise

Exercise 56 Further problem on the Q-meter

1. A Q-meter measures the Q-factor of a series L-C-R circuit to be 200 at a resonant frequency of 250 kHz. If the capacitance of the Q-meter capacitor is set to 300 pF determine (a) the inductance L, and (b) the resistance R of the inductor. [(a) 1.351 mH (b) 10.61 Ω]

10.22 Measurement errors

Errors are always introduced when using instruments to measure electrical quantities. The errors most likely to occur in measurements are those due to:

(i) the limitations of the instrument;
(ii) the operator;
(iii) the instrument disturbing the circuit.

(i) Errors in the limitations of the instrument

The **calibration accuracy** of an instrument depends on the precision with which it is constructed. Every instrument has a margin of error which is expressed as a percentage of the instruments full scale deflection. For example, industrial grade instruments have an accuracy of ±2% of f.s.d. Thus if a voltmeter has a f.s.d. of 100 V and it indicates 40 V say, then the actual voltage may be anywhere between 40 ± (2% of 100), or 40 ± 2, i.e. between 38 V and 42 V.

When an instrument is calibrated, it is compared against a standard instrument and a graph is drawn of 'error' against 'meter deflection'. A typical graph is shown in Fig. 10.37 where it is seen that the accuracy varies over the scale length. Thus a meter with a ±2% f.s.d. accuracy would tend to have an accuracy which is much better than ±2% f.s.d. over much of the range.

Problem 24. The arms of a Wheatstone bridge ABCD have the following resistances: AB: $R_1 = 1000\,\Omega \pm 1.0\%$; BC: $R_2 = 100\,\Omega \pm 0.5\%$; CD: unknown resistance R_x; DA: $R_3 = 432.5\,\Omega \pm 0.2\%$. Determine the value of the unknown resistance and its accuracy of measurement.

The Wheatstone bridge network is shown in Fig. 10.39 and at balance:

$$R_1 R_x = R_2 R_3,$$

i.e. $$R_x = \frac{R_2 R_3}{R_1} = \frac{(100)(432.5)}{1000} = 43.25\,\Omega$$

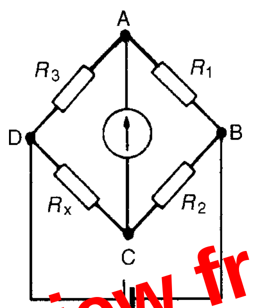


Figure 10.39

The maximum relative error of R_x is given by the sum of the three individual errors,
i.e. $1.0\% + 0.5\% + 0.2\% = 1.7\%$.

Hence $\boldsymbol{R_x = 43.25\,\Omega \pm 1.7\%}$

1.7% of $43.25\,\Omega = 0.74\,\Omega$ (rounding off). Thus R_x may also be expressed as

$$\boldsymbol{R_x = 43.25 \pm 0.74\,\Omega}$$

Now try the following exercises

Exercise 57 Further problems on measurement errors

1. The p.d. across a resistor is measured as 37.5 V with an accuracy of ±0.5%. The value of the resistor is $6\,k\Omega \pm 0.8\%$. Determine the current flowing in the resistor and its accuracy of measurement.
 [6.25 mA ± 1.3% or 6.25 ± 0.08 mA]

2. The voltage across a resistor is measured by a 75 V f.s.d. voltmeter which gives an indication of 52 V. The current flowing in the resistor is measured by a 20 A f.s.d. ammeter which gives an indication of 12.5 A. Determine the resistance of the resistor and its accuracy if both instruments have an accuracy of ±2% of f.s.d.
 [4.16 Ω ± 6.08% or 4.16 ± 0.25 Ω]

3. A Wheatstone bridge PQRS has the following arm resistances: PQ, $1\,k\Omega \pm 2\%$; QR, $100\,\Omega \pm 0.5\%$; RS, unknown resistance; SP, $273.6\,\Omega \pm 0.1\%$. Determine the value of the unknown resistance, and its accuracy of measurement.
 [27.36 Ω ± 2.6% or 27.36 Ω ± 0.71 Ω]

Exercise 58 Short answer questions on electrical measuring instruments and measurements

1. What is the main difference between an analogue and a digital type of measuring instrument?

2. Name the three essential devices for all analogue electrical indicating instruments

3. Complete the following statements:
   (a) An ammeter has a . . . . . . resistance and is connected . . . . . . with the circuit
   (b) A voltmeter has a . . . . . . resistance and is connected . . . . . . with the circuit

4. State two advantages and two disadvantages of a moving coil instrument

5. What effect does the connection of (a) a shunt (b) a multiplier have on a milliammeter?

6. State two advantages and two disadvantages of a moving coil instrument

22. A voltmeter has a f.s.d. of 100 V, a sensitivity of 1 kΩ/V and an accuracy of ±2% of f.s.d. When the voltmeter is connected into a circuit it indicates 50 V. Which of the following statements is false?
    (a) Voltage reading is 50 ± 2 V
    (b) Voltmeter resistance is 100 kΩ
    (c) Voltage reading is 50 V ± 2%
    (d) Voltage reading is 50 V ± 4%

23. A potentiometer is used to:
    (a) compare voltages
    (b) measure power factor
    (c) compare currents
    (d) measure phase sequence

Chapter 11

Semiconductor diodes

At the end of this chapter you should be able to:

- classify materials as conductors, semiconductors or insulators
- appreciate the importance of silicon and germanium
- understand n-type and p-type materials
- understand the p-n junction
- appreciate forward and reverse bias of p-n junctions
- recognise the symbols used to represent diodes in circuit diagrams
- understand the importance of diode characteristics and maximum ratings
- know the characteristics and applications of various types of diode – signal diodes, rectifiers, zener diodes, silicon controlled rectifiers, light emitting diodes, varactor diodes and Schottky diodes.

11.1 Types of material

Materials may be classified as conductors, semiconductors or insulators. The classification depends on the value of resistivity of the material. Good conductors are usually metals and have resistivities in the order of 10^{-7} to 10^{-8} Ωm, semiconductors have resistivities in the order of 10^{-3} to 3×10^{3} Ωm, and the resistivities of insulators are in the order of 10^{4} to 10^{14} Ωm. Some typical approximate values at normal room temperatures are:

Conductors:

Aluminium	2.7×10^{-8} Ωm
Brass (70 Cu/30 Zn)	8×10^{-8} Ωm
Copper (pure annealed)	1.7×10^{-8} Ωm
Steel (mild)	15×10^{-8} Ωm

Semiconductors: (at 27°C)

Silicon	2.3×10^{3} Ωm
Germanium	0.45 Ωm

Insulators:

Glass	$\geq 10^{10}$ Ωm
Mica	$\geq 10^{11}$ Ωm
PVC	$\geq 10^{13}$ Ωm
Rubber (pure)	10^{12} to 10^{14} Ωm

In general, over a limited range of temperatures, the resistance of a conductor increases with temperature increase, the resistance of insulators remains approximately constant with variation of temperature and the resistance of semiconductor materials decreases as the temperature increases. For a specimen of each of these materials, having the same resistance (and thus completely different dimensions), at say, 15°C, the variation for a small increase in temperature to t°C is as shown in Fig. 11.1.

As the temperature of semiconductor materials is raised above room temperature, the resistivity is reduced and ultimately a point is reached where they effectively become conductors. For this reason, silicon should not operate at a working temperature in excess of 150°C to 200°C, depending on its purity, and germanium should not operate at a working temperature in excess of 75°C to 90°C, depending on its purity. As

Chapter 12

Transistors

At the end of this chapter you should be able to:

- understand the structure of bipolar junction transistors (BJT) and junction gate field effect transistors (JFET)
- understand the action of BJT and JFET devices
- appreciate different classes and applications for BJT and JFET devices
- draw the circuit symbols for BJT and JFET devices
- appreciate common base, common emitter and common collector connections
- appreciate common gate, common source and common drain connections
- interpret characteristics for BJT and JFET devices
- appreciate how transistors are used as Class-A amplifiers
- use a load line to determine the performance of a transistor amplifier
- estimate quiescent operating conditions and gain from transistor characteristics and other data

12.1 Transistor classification

Transistors fall into **two main classes** – **bipolar** and **field effect**. They are also classified according to the semiconductor material employed – silicon or germanium, and to their field of application (for example, general purpose, switching, high frequency, and so on). Transistors are also classified according to the application that they are designed for, as shown in Table 12.1. Note that these classifications can be combined so

Table 12.1 Transistor classification

Low-frequency	Transistors designed specifically for audio low-frequency applications (below 100 kHz)
High-frequency	Transistors designed specifically for high radio-frequency applications (100 kHz and above)
Switching	Transistors designed for switching applications
Low-noise	Transistors that have low-noise characteristics and which are intended primarily for the amplification of low-amplitude signals
High-voltage	Transistors designed specifically to handle high voltages
Driver	Transistors that operate at medium power and voltage levels and which are often used to precede a final (power) stage which operates at an appreciable power level
Small-signal	Transistors designed for amplifying small voltages in amplifiers and radio receivers
Power	Transistor designed to handle high currents and voltages

that it is possible, for example, to classify a transistor as a 'low-frequency power transistor' or as a 'low-noise high-frequency transistor'.

12.2 Bipolar junction transistors (BJT)

Bipolar transistors generally comprise n-p-n or p-n-p junctions of either silicon (Si) or germanium (Ge) material. The junctions are, in fact, produced in a single slice of silicon by diffusing impurities through a photographically reduced mask. Silicon transistors are superior when compared with germanium transistors in the vast majority of applications (particularly at high temperatures) and thus germanium devices are very rarely encountered in modern electronic equipment.

The construction of typical n-p-n and p-n-p transistors is shown in Figs. 12.1 and 12.2. In order to conduct the heat away from the junction (important in medium- and high-power applications) the collector is connected to the metal case of the transistor.

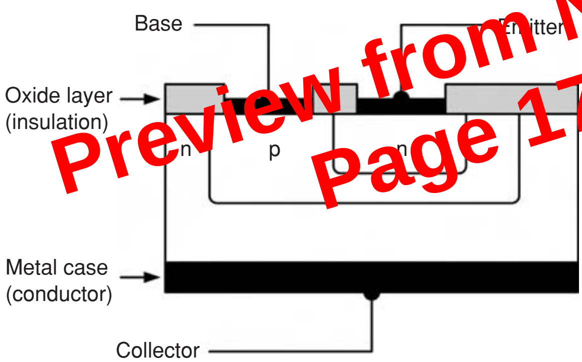


Figure 12.1

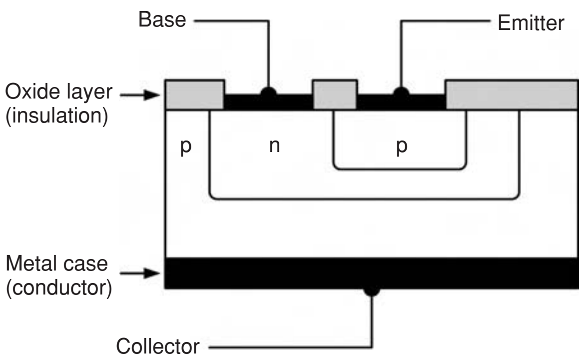
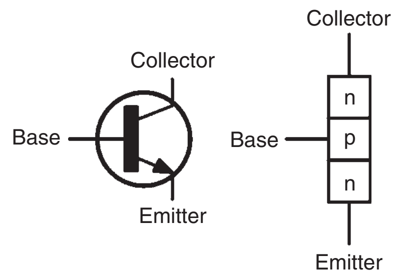
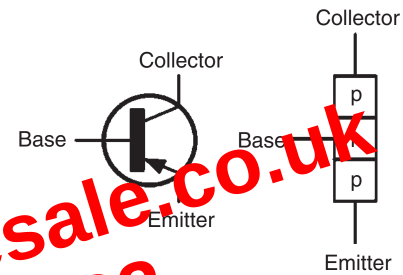


Figure 12.2

The **symbols** and simplified junction models for n-p-n and p-n-p transistors are shown in Fig. 12.3. It is important to note that the base region (p-type material in the case of an n-p-n transistor or n-type material in the case of a p-n-p transistor) is extremely narrow.



(a) n-p-n bipolar junction transistor (BJT)



(b) p-n-p bipolar junction transistor (BJT)

Figure 12.3

12.3 Transistor action

In the **n-p-n transistor**, connected as shown in Fig. 12.4(a), transistor action is accounted for as follows:

(a) the majority carriers in the n-type emitter material are electrons

(b) the base-emitter junction is forward biased to these majority carriers and electrons cross the junction and appear in the base region

(c) the base region is very thin and only lightly doped with holes, so some recombination with holes occurs but many electrons are left in the base region

(d) the base-collector junction is reverse biased to holes in the base region and electrons in the collector region, but is forward biased to electrons in the base region; these electrons are attracted by the positive potential at the collector terminal

18 V supply, determine (a) the quiescent values of collector voltage and current (V_{CQ} and I_{CQ}), and (b) the peak-peak output voltage that would be produced by an input signal of 40 μA peak-peak.

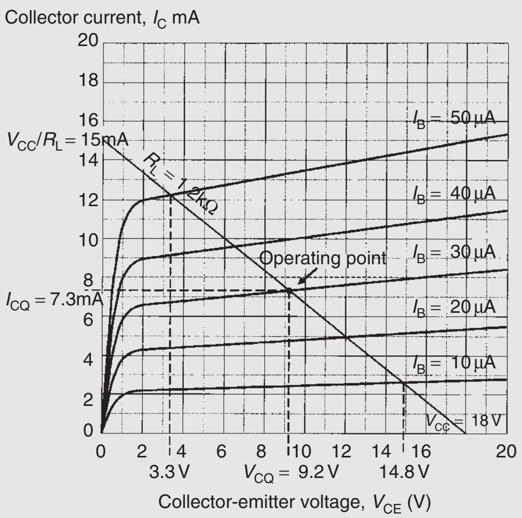


Figure 12.25

(a) First we need to construct the load line on Fig. 12.25. The two ends of the load line will correspond to V_{CC}, the 18 V supply, on the collector-emitter voltage axis and 18 V/1.2 kΩ or 15 mA on the collector current axis.

Next we locate the **operating point** (or **quiescent point**) from the point of intersection of the $I_B = 30$ μA characteristic and the load line. Having located the operating point we can read off the **quiescent values**, i.e. the no-signal values, of collector-emitter voltage (V_{CQ}) and collector current (I_{CQ}). Hence, $\mathbf{V_{CQ} = 9.2\,V}$ and $\mathbf{I_{CQ} = 7.3\,mA}$

(b) Next we can determine the maximum and minimum values of collector-emitter voltage by locating the appropriate intercept points on Fig. 12.25. Note that the maximum and minimum values of base current will be (30 μA + 20 μA) = 50 μA on positive peaks of the signal and (30 μA − 20 μA) = 10 μA on negative peaks of the signal. The maximum and minimum values of V_{CE} are, respectively, 14.8 V and 3.3 V. Hence,

the output voltage swing = (14.8 V − 3.3 V)

= **11.5 V peak-peak**

Problem 12. An n-p-n transistor has the following characteristics, which may be assumed to be linear between the values of collector voltage stated.

Base current (μA)	Collector current (mA) for collector voltages of: 1 V	5 V
30	1.4	1.6
50	3.0	3.5
70	4.6	5.2

The transistor is used as a common-emitter amplifier with load resistor $R_L = 1.2$ kΩ and a collector supply of 7 V. The signal input resistance is 1 kΩ. If an input current of 20 μA peak varies sinusoidally about a mean bias of 50 μA, estimate (a) the quiescent values of collector voltage and current, (b) the output voltage swing, (c) the voltage gain, (d) the dynamic current gain, and (e) the power gain.

The characteristics are drawn as shown in Fig. 12.26.

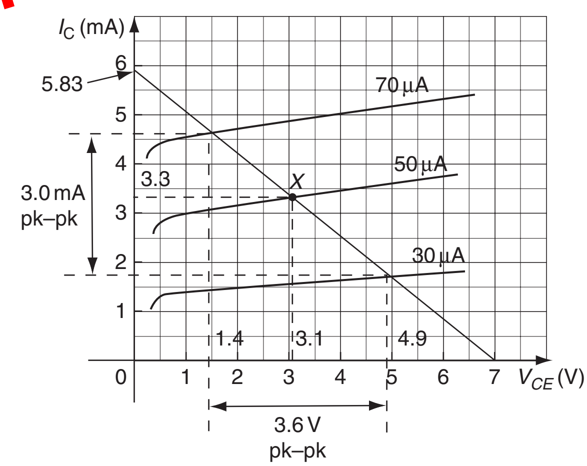


Figure 12.26

The two ends of the load line will correspond to V_{CC}, the 7 V supply, on the collector-emitter voltage axis and 7 V/1.2 kΩ = 5.83 mA on the collector current axis.

(a) The operating point (or quiescent point), X, is located from the point of intersection of the from a 12 V supply with a base bias of 60 μA and a load resistor of 1 kΩ, determine (a) the quiescent values of collector-emitter voltage and collector current, and (b) the peak-peak collector voltage when an 80 μA peak-peak signal current is applied.

[(a) 5 V, 7 mA (b) 8.5 V]

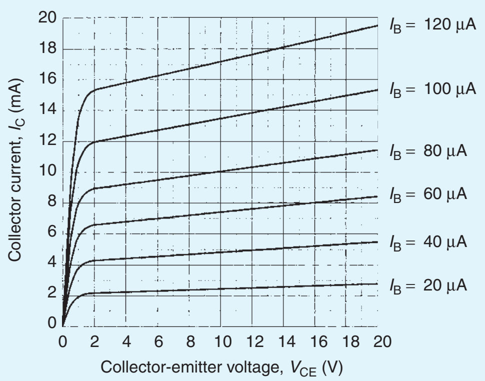


Figure 12.27

7. The output characteristics of a JFET are shown in Fig. 12.28. If this device is used in an amplifier circuit operating from an 18 V supply with a gate-source bias voltage of −3 V and a load resistance of 900 Ω, determine (a) the quiescent values of drain-source voltage and drain current, (b) the peak-peak output voltage when an input voltage of 2 V peak-peak is applied, and (c) the voltage gain of the stage.

[(a) 12.2 V, 6.1 mA (b) 5.5 V (c) 2.75]

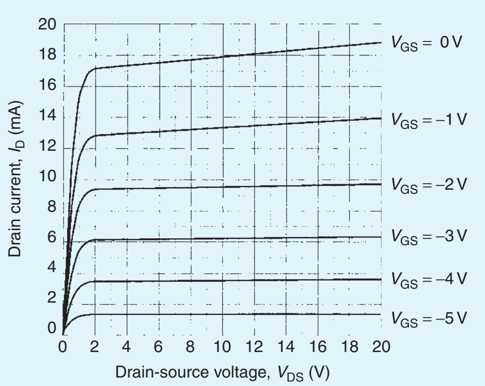


Figure 12.28

8. An amplifier has a current gain of 40 and a voltage gain of 30. Determine the power gain.

[1200]

9. The output characteristics of a transistor in common-emitter mode configuration can be regarded as straight lines connecting the following points.

	$I_B = 20\,\mu A$		$50\,\mu A$		$80\,\mu A$	
V_{CE} (v)	1.0	8.0	1.0	8.0	1.0	8.0
I_C (mA)	1.2	1.4	3.4	4.2	6.1	8.1

Plot the characteristics and superimpose the load line for a 1 kΩ load, given that the supply voltage is 9 V and the d.c. base bias is 50 μA. The signal input resistance is 800 Ω. When a peak input current of 30 μA varies sinusoidally about a mean bias of 50 μA, determine (a) the quiescent values of collector voltage and current, V_{CQ} and I_{CQ} (b) the output voltage swing, (c) the voltage gain, (d) the dynamic current gain, and (e) the power gain.

[(a) 5.2 V, 3.7 mA (b) 5.1 V (c) 106 (d) 87 (e) 9222]

Exercise 66 Short answer questions on transistors

1. In a p-n-p transistor the p-type material regions are called the . . . . . . and . . . . . . , and the n-type material region is called the . . . . . .

2. In an n-p-n transistor, the p-type material region is called the . . . . . . and the n-type material regions are called the . . . . . . and the . . . . . .

3. In a p-n-p transistor, the base-emitter junction is . . . . . .biased and the base-collector junction is . . . . . . biased

4. In an n-p-n transistor, the base-collector junction is . . . . . . biased and the base-emitter junction is . . . . . . biased

5. Majority charge carriers in the emitter of a transistor pass into the base region. Most of

Further Electrical and Electronic Principles

2. For the networks shown in Fig. 13.12, find the values of the currents marked.
[(a) $I_1 = 4$ A, $I_2 = -1$ A, $I_3 = 13$ A
(b) $I_1 = 40$ A, $I_2 = 60$ A, $I_3 = 120$ A
$I_4 = 100$ A, $I_5 = -80$ A]

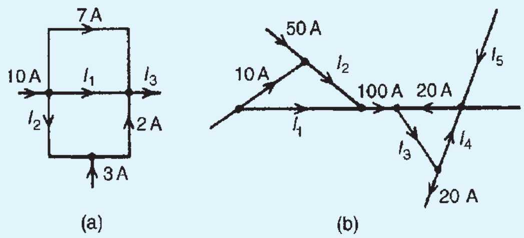


Figure 13.12

3. Calculate the currents I_1 and I_2 in Fig. 13.13.
[$I_1 = 0.8$ A, $I_2 = 0.5$ A]

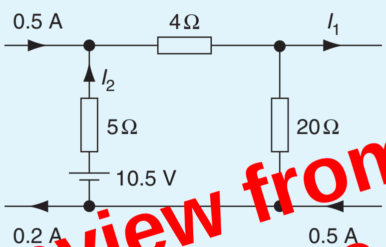


Figure 13.13

4. Use Kirchhoff's laws to find the current flowing in the 6 Ω resistor of Fig. 13.14 and the power dissipated in the 4 Ω resistor.
[2.162 A, 42.07 W]

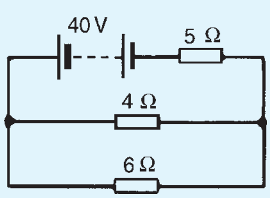


Figure 13.14

5. Find the current flowing in the 3 Ω resistor for the network shown in Fig. 13.15(a). Find also the p.d. across the 10 Ω and 2 Ω resistors.
[2.715 A, 7.410 V, 3.948 V]

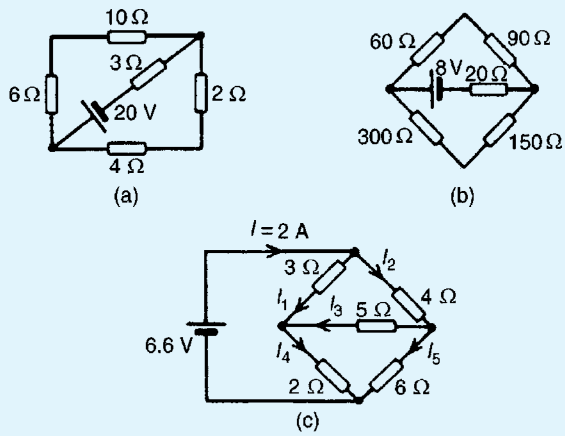


Figure 13.15

6. For the network shown in Fig. 13.15(b) find: (a) the current in the battery, (b) the current in the 300 Ω resistor, (c) the current in the 90 Ω resistor, and (d) the power dissipated in the 150 Ω resistor.
[(a) 60.38 mA (b) 15.10 mA
(c) 45.28 mA (d) 34.20 mW]

7. For the bridge network shown in Fig. 13.15(c), find the currents I_1 to I_5
[$I_1 = 1.26$ A, $I_2 = 0.74$ A, $I_3 = 0.16$ A,
$I_4 = 1.42$ A, $I_5 = 0.58$ A]

13.3 The superposition theorem

The superposition theorem states:

In any network made up of linear resistances and containing more than one source of e.m.f., the resultant current flowing in any branch is the algebraic sum of the currents that would flow in that branch if each source was considered separately, all other sources being replaced at that time by their respective internal resistances.

The superposition theorem is demonstrated in the following worked problems

Problem 5. Figure 13.16 shows a circuit containing two sources of e.m.f., each with their internal resistance. Determine the current in each branch of the network by using the superposition theorem.

If terminals AB in Fig. 13.68 are short-circuited, the short-circuit current $I_{SC} = 10/2 = 5\,A$

The resistance 'looking-in' at terminals AB is 2 Ω. Hence the equivalent Norton network is as shown in Fig. 13.69

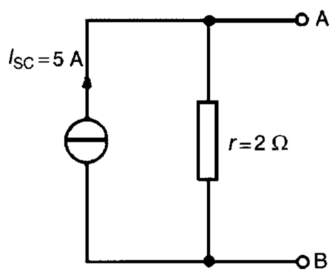


Figure 13.69

Problem 19. Convert the network shown in Fig. 13.70 to an equivalent Thévenin circuit.

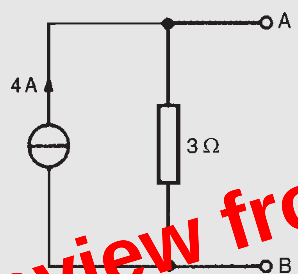


Figure 13.70

The open-circuit voltage E across terminals AB in Fig. 13.70 is given by:

$$E = (I_{SC})(r) = (4)(3) = 12\,V.$$

The resistance 'looking-in' at terminals AB is 3 Ω. Hence the equivalent Thévenin circuit is as shown in Fig. 13.71

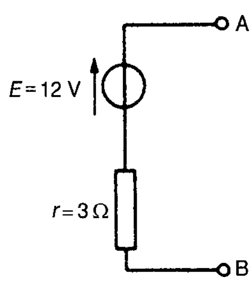


Figure 13.71

Problem 20. (a) Convert the circuit to the left of terminals AB in Fig. 13.72 to an equivalent Thévenin circuit by initially converting to a Norton equivalent circuit. (b) Determine the current flowing in the 1.8 Ω resistor.

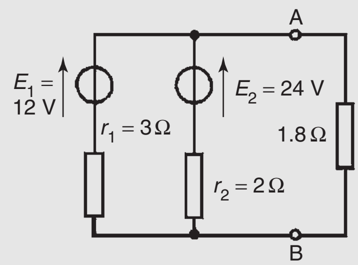


Figure 13.72

(a) For the branch containing the 12 V source, converting to a Norton equivalent circuit gives $I_{SC} = 12/3 = 4\,A$ and $r_1 = 3\,\Omega$. For the branch containing the 24 V source, converting to a Norton equivalent circuit gives $I_{SC2} = 24/2 = 12\,A$ and $r_2 = 2\,\Omega$. Thus Fig. 13.73(a) shows a network equivalent to Fig. 13.72

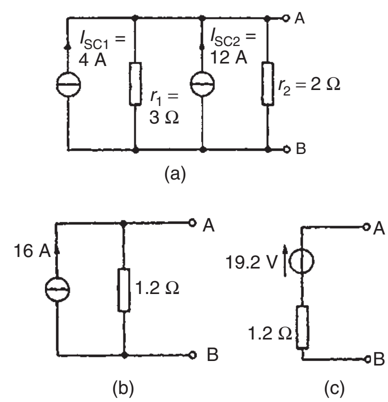


Figure 13.73

From Fig. 13.73(a) the total short-circuit current is $4 + 12 = 16\,A$ and the total resistance is given by $(3 \times 2)/(3 + 2) = \mathbf{1.2\,\Omega}$. Thus Fig. 13.73(a) simplifies to Fig. 13.73(b). The open-circuit voltage across AB of Fig. 13.73(b), $E = (16)(1.2) = \mathbf{19.2\,V}$, and the resistance 'looking-in' at AB is 1.2 Ω. Hence the Thévenin equivalent circuit is as shown in Fig. 13.73(c).

(b) When the 1.8 Ω resistance is connected between terminals A and B of Fig. 13.73(c) the current I flowing is given by

$$\boldsymbol{I} = \left(\frac{19.2}{1.2 + 1.8}\right) = \mathbf{6.4\,A}$$

Problem 21. Determine by successive conversions between Thévenin and Norton equivalent networks a Thévenin equivalent circuit for terminals AB of Fig. 13.74. Hence determine the current flowing in the 200 Ω resistance.

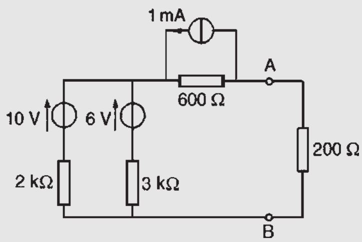


Figure 13.74

For the branch containing the 10 V source, converting to a Norton equivalent network gives
$I_{SC} = 10/2000 = 5$ mA and $r_1 = 2$ kΩ.
For the branch containing the 6 V source, converting to a Norton equivalent network gives
$I_{SC} = 6/3000 = 2$ mA and $r_2 = 3$ kΩ.

Thus the network of Fig. 13.74 converts to Fig. 13.75(a). Combining the 5 mA and 2 mA current sources gives the equivalent network of Fig. 13.75(b) where the short-circuit current for the original two branches considered is 7 mA and the resistance is $(2 \times 3)/(2+3) = 1.2$ kΩ

Both of the Norton equivalent networks shown in Fig. 13.75(b) may be converted to Thévenin equivalent circuits. The open-circuit voltage across CD is $(7 \times 10^{-3})(1.2 \times 10^{3}) = 8.4$ V and the resistance 'looking-in' at CD is 1.2 kΩ. The open-circuit voltage across EF is $(1 \times 10^{-3})(600) = 0.6$ V and the resistance 'looking-in' at EF is 0.6 kΩ. Thus Fig. 13.75(b) converts to Fig. 13.75(c). Combining the two Thévenin circuits gives $E = 8.4 - 0.6 = 7.8$ V and the resistance $r = (1.2 + 0.6)$ kΩ $=$ **1.8 kΩ**

Thus the Thévenin equivalent circuit for terminals AB of Fig. 13.74 is as shown in Fig. 13.75(d)

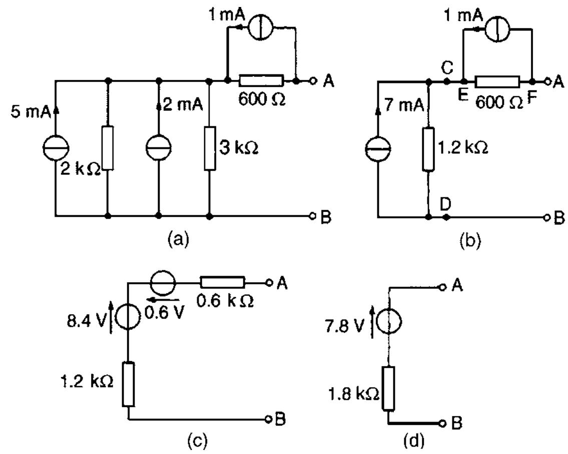


Figure 13.75

Hence the current I flowing in a 200 Ω resistance connected between A and B is given by

$$\mathbf{I} = \frac{7.8}{1800 + 200} = \frac{7.8}{2000} = \mathbf{3.9\,mA}$$

Now try the following exercise

Exercise 72 Further problems on Thévenin and Norton equivalent networks

1. Convert the circuits shown in Fig. 13.76 to Norton equivalent networks.
[(a) $I_{SC} = 25$ A, $r = 2$ Ω
(b) $I_{SC} = 2$ mA, $r = 5$ Ω]

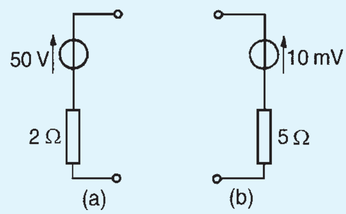


Figure 13.76

2. Convert the networks shown in Fig. 13.77 to Thévenin equivalent circuits
[(a) $E = 20$ V, $r = 4$ Ω
(b) $E = 12$ mV, $r = 3$ Ω]

Section 2

10. For the circuit shown in Fig. 13.94, maximum power transfer from the source is required. For this to be so, which of the following statements is true?
(a) $R_2 = 10\,\Omega$ (b) $R_2 = 30\,\Omega$
(c) $R_2 = 7.5\,\Omega$ (d) $R_2 = 15\,\Omega$

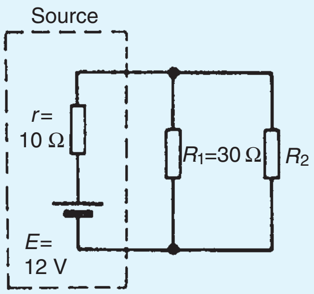


Figure 13.94

11. The open-circuit voltage E across terminals XY of Fig. 13.95 is:
(a) 0 V (b) 20 V
(c) 4 V (d) 16 V

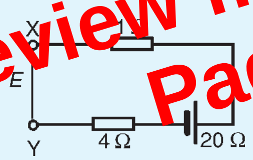


Figure 13.95

12. The maximum power transferred by the source in Fig. 13.96 is:
(a) 5 W (b) 200 W
(c) 40 W (d) 50 W

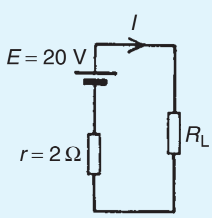


Figure 13.96

13. For the circuit shown in Fig. 13.97, voltage V is:
(a) 0 V (b) 20 V
(c) 4 V (d) 16 V

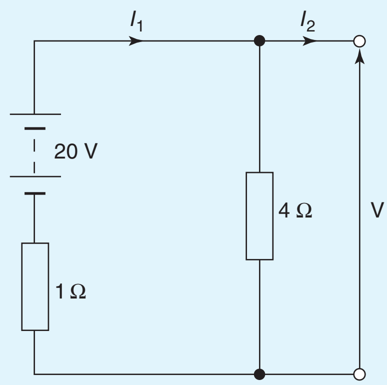


Figure 13.97

14. For the circuit shown in Fig. 13.97, current I_1 is:
(a) 25 A (b) 4 A
(c) 0 A (d) 20 A

15. For the circuit shown in Fig. 13.97, current I_2 is:
(a) 25 A (b) 4 A
(c) 0 A (d) 20 A

16. The current flowing in the branches of a d.c. circuit may be determined using:
(a) Kirchhoff's laws
(b) Lenz's law
(c) Faraday's laws
(d) Fleming's left-hand rule

Section 2

Using the sine rule,

$$\frac{100}{\sin\phi} = \frac{145.5}{\sin 150°}$$

from which $\sin\phi = \dfrac{100\sin 150°}{145.5}$

$$= 0.3436$$

and $\phi = \sin^{-1}0.3436 = 20.096° = 0.35$ radians, and lags v_1. Hence

$$\mathbf{v_R = v_1 + v_2 = 145.5\sin(\omega t - 0.35)\,V}$$

Problem 15. Find a sinusoidal expression for $(i_1 + i_2)$ of Problem 13, (b) by drawing phasors, (b) by calculation.

(a) The relative positions of i_1 and i_2 at time $t = 0$ are shown as phasors in Fig. 14.12(a). The phasor diagram in Fig. 14.12(b) shows the resultant i_R, and i_R is measured as 26 A and angle ϕ as 19° or 0.33 rads leading i_1.

Hence, by drawing, $\mathbf{i_R = 26\sin(\omega t + 0.33)\,A}$

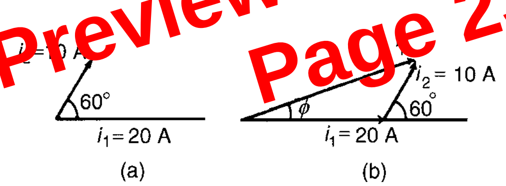


Figure 14.12

(b) From Fig. 14.12(b), by the cosine rule:

$$i_R^2 = 20^2 + 10^2 - 2(20)(10)(\cos 120°)$$

from which $i_R = \mathbf{26.46\,A}$

By the sine rule:

$$\frac{10}{\sin\phi} = \frac{26.46}{\sin 120°}$$

from which $\phi = 19.10°$ (i.e. 0.333 rads)

Hence, by calculation,

$\mathbf{i_R = 26.46\sin(\omega t + 0.333)\,A}$

An alternative method of calculation is to use **complex numbers** (see '*Engineering Mathematics*').

$$\text{Then } i_1 + i_2 = 20\sin\omega t + 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\equiv 20\angle 0 + 10\angle\frac{\pi}{3}\text{rad or}$$

$$20\angle 0° + 10\angle 60°$$

$$= (20 + j0) + (5 + j8.66)$$

$$= (25 + j8.66)$$

$$= 26.46\angle 19.106° \text{ or } 26.46\angle 0.333 \text{ rad}$$

$$\equiv \mathbf{26.46\sin(\omega t + 0.333)\,A}$$

Problem 16. Two alternating voltages are given by $v_1 = 120\sin\omega t$ volts and $v_2 = 200\sin(\omega t - \pi/4)$ volts. Obtain sinusoidal expressions for $v_1 - v_2$ (a) by plotting waveforms, and (b) by resolution of phasors.

(a) $v_1 = 120\sin\omega t$ and $v_2 = 200\sin(\omega t - \pi/4)$ are shown plotted in Fig. 14.13 Care must be taken when subtracting values of ordinates especially when at least one of the ordinates is negative. For example

at 30°, $v_1 - v_2 = 60 - (-52) = 112$ V
at 60°, $v_1 - v_2 = 104 - 52 = 52$ V
at 150°, $v_1 - v_2 = 60 - 193 = -133$ V and so on.

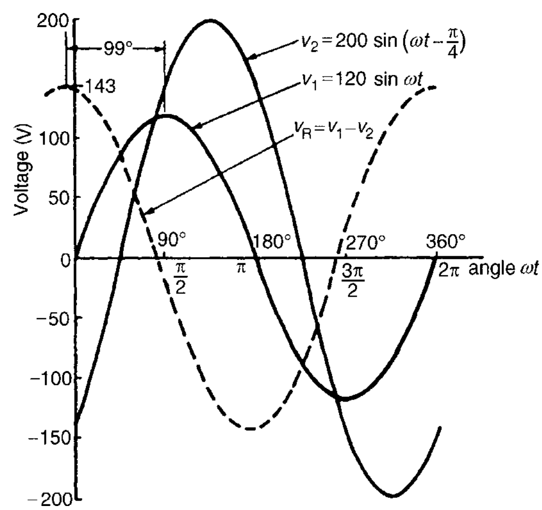


Figure 14.13

The resultant waveform, $v_R = v_1 - v_2$, is shown by the broken line in Fig. 14.13 The maximum

Section 2

Series connected impedances

For series-connected impedances the total circuit impedance can be represented as a single *L–C–R* circuit by combining all values of resistance together, all values of inductance together and all values of capacitance together, (remembering that for series connected capacitors

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots\Big)$$

For example, the circuit of Fig. 15.15(a) showing three impedances has an equivalent circuit of Fig. 15.15(b).

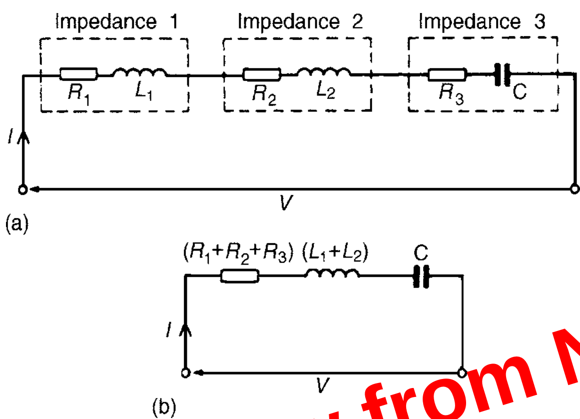


Figure 15.15

Problem 16. The following three impedances are connected in series across a 40 V, 20 kHz supply: (i) a resistance of 8 Ω, (ii) a coil of inductance 130 μH and 5 Ω resistance, and (iii) a 10 Ω resistor in series with a 0.25 μF capacitor. Calculate (a) the circuit current, (b) the circuit phase angle and (c) the voltage drop across each impedance.

The circuit diagram is shown in Fig. 15.16(a). Since the total circuit resistance is 8 + 5 + 10, i.e. 23 Ω, an equivalent circuit diagram may be drawn as shown in Fig. 15.16(b).

Inductive reactance,

$$X_L = 2\pi fL = 2\pi(20\times10^3)(130\times10^{-6}) = 16.34\,\Omega$$

Capacitive reactance,

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(20\times10^3)(0.25\times10^{-6})}$$
$$= 31.83\,\Omega$$

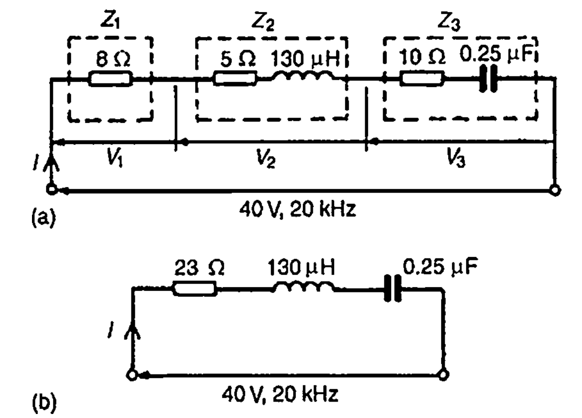


Figure 15.16

Since $X_C > X_L$, the circuit is capacitive (see phasor diagram in Fig. 15.12(c)).

$$X_C - X_L = 31.83 - 16.34 = 15.49\,\Omega$$

(a) Circuit impedance, $Z=\sqrt{R^2+(X_C-X_L)^2}$

$$=\sqrt{23^2+15.49^2}=27.73\,\Omega$$

Circuit current, $I = V/Z = 40/27.73 = \mathbf{1.442\,A}$

From Fig. 15.12(c), circuit phase angle

$$\phi=\tan^{-1}\left(\frac{X_C-X_L}{R}\right)$$

i.e.

$$\phi=\tan^{-1}\left(\frac{15.49}{23}\right)=\mathbf{33.96^\circ\ leading}$$

(b) From Fig. 15.16(a),

$V_1 = IR_1 = (1.442)(8) = \mathbf{11.54\,V}$

$V_2 = IZ_2 = I\sqrt{5^2+16.34^2}$

$= (1.442)(17.09) = \mathbf{24.64\,V}$

$V_3 = IZ_3 = I\sqrt{10^2+31.83^2}$

$= (1.442)(33.36) = \mathbf{48.11\,V}$

The 40 V supply voltage is the phasor sum of V_1, V_2 and V_3

Problem 17. Determine the p.d.'s V_1 and V_2 for the circuit shown in Fig. 15.17 if the frequency of the supply is 5 kHz. Draw the phasor diagram and hence determine the supply voltage V and the circuit phase angle.

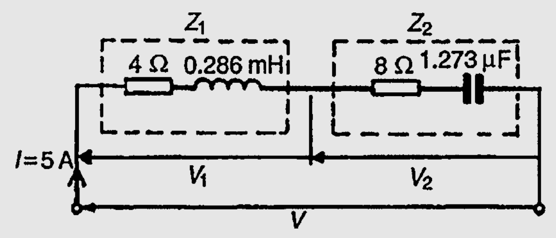


Figure 15.17

For impedance $\mathbf{Z_1}$: $R_1 = 4\,\Omega$ and

$$X_L = 2\pi fL$$
$$= 2\pi(5\times10^3)(0.286\times10^{-3})$$
$$= 8.985\,\Omega$$
$$V_1 = IZ_1 = I\sqrt{R^2 + X_L^2}$$
$$= 5\sqrt{4^2 + 8.985^2} = 49.18\,\text{V}$$

$$\text{Phase angle } \phi_1 = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{8.985}{4}\right)$$
$$= \mathbf{66.0^\circ\ lagging}$$

For impedance $\mathbf{Z_2}$: $R_2 = 8\,\Omega$ and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\times10^3)(1.273\times10^{-6})}$$
$$= 25.0\,\Omega$$
$$V_2 = IZ_2 = I\sqrt{R^2 + X_C^2} = 5\sqrt{8^2 + 25.0^2}$$
$$= 131.2\,\text{V}.$$

$$\text{Phase angle } \phi_2 = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{25.0}{8}\right)$$
$$= 72.26^\circ \ \text{leading}$$

The phasor diagram is shown in Fig. 15.18

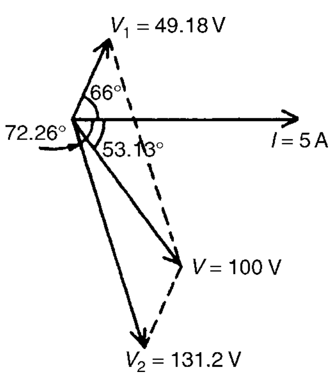


Figure 15.18

The phasor sum of V_1 and V_2 gives the supply voltage V of 100 V at a phase angle of **53.13° leading**. These values may be determined by drawing or by calculation — either by resolving into horizontal and vertical components or by the cosine and sine rules.

Now try the following exercise

Exercise 85 Further problems on R–L–C a.c. circuits

1. A 40 μF capacitor in series with a coil of resistance 8 Ω and inductance 80 mH is connected to a 200 V, 100 Hz supply. Calculate (a) the circuit impedance, (b) the current flowing, (c) the phase angle between voltage and current, (d) the voltage across the coil, and (e) the voltage across the capacitor.
[(a) 13.18 Ω (b) 15.17 A (c) 52.63° lagging (d) 772.1 V (e) 603.6 V]

2. Find the values of resistance R and inductance L in the circuit of Fig. 15.19.
[$R = 131\,\Omega$, $L = 0.545$ H]

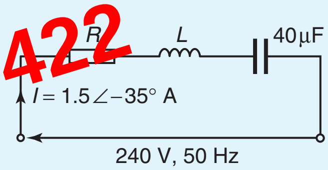


Figure 15.19

3. Three impedances are connected in series across a 100 V, 2 kHz supply. The impedances comprise:
   (i) an inductance of 0.45 mH and 2 Ω resistance,
   (ii) an inductance of 570 μH and 5 Ω resistance, and
   (iii) a capacitor of capacitance 10 μF and resistance 3 Ω

   Assuming no mutual inductive effects between the two inductances calculate (a) the circuit impedance, (b) the circuit current, (c) the circuit phase angle and (d) the voltage across each impedance. Draw the phasor diagram.
[(a) 11.12 Ω (b) 8.99 A (c) 25.92° lagging (d) 53.92 V, 78.53 V, 76.46 V]

4. For the circuit shown in Fig. 15.20 determine the voltages V_1 and V_2 if the supply frequency

Section 2

At resonance,

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{60 \times 10^{-3}}{30 \times 10^{-6}}}$$

$$= \frac{1}{2}\sqrt{\frac{60 \times 10^{6}}{30 \times 10^{3}}}$$

$$= \frac{1}{2}\sqrt{2000} = \mathbf{22.36}$$

Problem 22. A coil of negligible resistance and inductance 100 mH is connected in series with a capacitance of 2 µF and a resistance of 10 Ω across a 50 V, variable frequency supply. Determine (a) the resonant frequency, (b) the current at resonance, (c) the voltages across the coil and the capacitor at resonance, and (d) the Q-factor of the circuit.

(a) Resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(\frac{100}{10^3}\right)\left(\frac{2}{10^6}\right)}}$$

$$= \frac{1}{2\pi\sqrt{\frac{20}{10^8}}} = \frac{1}{\frac{2\pi\sqrt{20}}{10^4}}$$

$$= \frac{10^4}{2\pi\sqrt{20}} = \mathbf{355.9\ Hz}$$

(b) Current at resonance $I = V/R = 50/10 = \mathbf{5\ A}$

(c) Voltage across coil at resonance,

$$V_L = IX_L = I(2\pi f_r L)$$

$$= (5)(2\pi \times 355.9 \times 100 \times 10^{-3}) = \mathbf{1118\ V}$$

Voltage across capacitance at resonance,

$$V_C = IX_C = \frac{I}{2\pi f_r C}$$

$$= \frac{5}{2\pi(355.9)(2 \times 10^{-6})} = \mathbf{1118\ V}$$

(d) Q-factor (i.e. voltage magnification at resonance)

$$= \frac{V_L}{V} \text{ or } \frac{V_C}{V} = \frac{1118}{50} = \mathbf{22.36}$$

Q-factor may also have been determined by

$$\frac{2\pi f_r L}{R} \quad \text{or} \quad \frac{1}{2\pi f_r CR} \quad \text{or} \quad \frac{1}{R}\sqrt{\frac{L}{C}}$$

Now try the following exercise

Exercise 86 Further problems on series resonance and Q-factor

1. Find the resonant frequency of a series a.c. circuit consisting of a coil of resistance 10 Ω and inductance 50 mH and capacitance 0.05 µF. Find also the current flowing at resonance if the supply voltage is 100 V. [3.183 kHz, 10 A]

2. The current at resonance in a series L–C–R circuit is 0.2 mA. If the applied voltage is 250 mV at a frequency of 100 kHz and the circuit capacitance is 0.04 µF, find the circuit resistance and inductance. [1.25 kΩ, 63.3 µH]

3. A coil of resistance 25 Ω and inductance 100 mH is connected in series with a capacitance of 0.12 µF across a 200 V, variable frequency supply. Calculate (a) the resonant frequency, (b) the current at resonance and (c) the factor by which the voltage across the reactance is greater than the supply voltage. [(a) 1.453 kHz (b) 8 A (c) 36.51]

4. A coil of 0.5 H inductance and 8 Ω resistance is connected in series with a capacitor across a 200 V, 50 Hz supply. If the current is in phase with the supply voltage, determine the capacitance of the capacitor and the p.d. across its terminals. [20.26 µF, 3.928 kV]

5. Calculate the inductance which must be connected in series with a 1000 pF capacitor to give a resonant frequency of 400 kHz. [0.158 mH]

6. A series circuit comprises a coil of resistance 20 Ω and inductance 2 mH and a 500 pF capacitor. Determine the Q-factor of the circuit at resonance. If the supply voltage is 1.5 V, what is the voltage across the capacitor? [100, 150 V]

Section 2

9. Which of the following statements is false?
   (a) Impedance is at a minimum at resonance in an a.c. circuit
   (b) The product of r.m.s. current and voltage gives the apparent power in an a.c. circuit
   (c) Current is at a maximum at resonance in an a.c. circuit
   (d) $\dfrac{\text{Apparent power}}{\text{True power}}$ gives power factor

10. The impedance of a coil, which has a resistance of X ohms and an inductance of Y henrys, connected across a supply of frequency K Hz, is
    (a) $2\pi KY$ (b) $X+Y$
    (c) $\sqrt{X^2+Y^2}$ (d) $\sqrt{X^2+(2\pi KY)^2}$

11. In question 10, the phase angle between the current and the applied voltage is given by
    (a) $\tan^{-1}\dfrac{Y}{X}$ (b) $\tan^{-1}\dfrac{2\pi KY}{X}$
    (c) $\tan^{-1}\dfrac{X}{2\pi KY}$ (d) $\tan\left(\dfrac{2\pi KY}{X}\right)$

12. When a capacitor is connected to an a.c. supply the current
    (a) leads the voltage by 180°
    (b) is in phase with the voltage
    (c) leads the voltage by $\pi/2$ rad
    (d) lags the voltage by 90°

13. When the frequency of an a.c. circuit containing resistance and capacitance is increased the impedance
    (a) increases (b) decreases
    (c) stays the same

14. In an R–L–C series a.c. circuit a current of 5 A flows when the supply voltage is 100 V. The phase angle between current and voltage is 60° lagging. Which of the following statements is false?
    (a) The circuit is effectively inductive
    (b) The apparent power is 500 VA
    (c) The equivalent circuit reactance is 20 Ω
    (d) The true power is 250 W

15. A series a.c. circuit comprising a coil of inductance 100 mH and resistance 1 Ω and a 10 μF capacitor is connected across a 10 V supply. At resonance the p.d. across the capacitor is
    (a) 10 kV (b) 1 kV
    (c) 100 V (d) 10 V

16. The amplitude of the current I flowing in the circuit of Fig. 15.26 is:
    (a) 21 A (b) 16.8 A
    (c) 28 A (d) 12 A

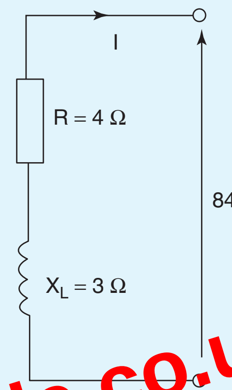


Figure 15.26

17. If the supply frequency is increased at resonance in a series R–L–C circuit and the values of L, C and R are constant, the circuit will become:
    (a) capacitive (b) resistive
    (c) inductive (d) resonant

18. For the circuit shown in Fig. 15.27, the value of Q-factor is:
    (a) 50 (b) 100
    (c) 5×10^{-4} (d) 40

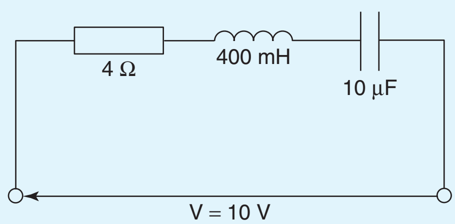


Figure 15.27

19. A series R–L–C circuit has a resistance of 8 Ω, an inductance of 100 mH and a capacitance of 5 μF. If the current flowing is 2 A, the impedance at resonance is:
    (a) 160 Ω (b) 16 Ω
    (c) 8 mΩ (d) 8 Ω

(by trigonometric ratios)

Circuit impedance, $\boldsymbol{Z = \frac{V}{I}}$

Problem 2. A 30 μF capacitor is connected in parallel with an 80 Ω resistor across a 240 V, 50 Hz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power dissipated, and (f) the apparent power

The circuit and phasor diagrams are as shown in Fig. 16.2

(a) Current in resistor,

$$I_R = \frac{V}{R} = \frac{240}{80} = \mathbf{3\,A}$$

Current in capacitor,

$$I_C = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{2\pi fC}\right)} = 2\pi fCV$$

$$= 2\pi(50)(30 \times 10^{6})(240) = \mathbf{2.262\,A}$$

(b) Supply current,

$$I = \sqrt{I_R^2 + I_C^2} = \sqrt{3^2 + 2.262^2}$$

$$= \mathbf{3.757\,A}$$

(c) Circuit phase angle,

$$\alpha = \tan^{-1}\frac{I_C}{I_R} = \tan^{-1}\frac{2.262}{3}$$

$$= \mathbf{37.02^\circ\ leading}$$

(d) Circuit impedance,

$$Z = \frac{V}{I} = \frac{240}{3.757} = \mathbf{63.88\,\Omega}$$

(e) True or active power dissipated,

$$P = VI\cos\alpha = (240)(3.757)\cos 37.02^\circ$$

$$= \mathbf{720\,W}$$

(Alternatively, true power

$$P = I_R^2 R = (3)^2(80) = 720\,\text{W})$$

(f) Apparent power,

$$S = VI = (240)(3.757) = \mathbf{901.7\,VA}$$

Problem 3. A capacitor C is connected in parallel with a resistor R across a 120 V, 200 Hz supply. The supply current is 2 A at a power factor of 0.6 leading. Determine the values of C and R

The circuit diagram is shown in Fig. 16.3(a).

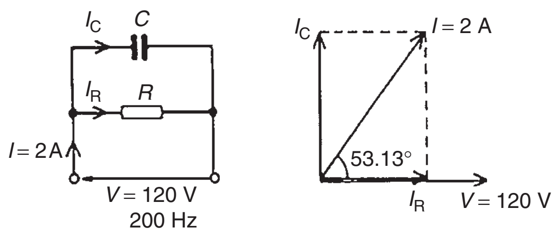


Figure 16.3

Power factor = cos ϕ = 0.6 leading, hence $\phi = \cos^{-1} 0.6 = 53.13^\circ$ leading.

From the phasor diagram shown in Fig. 16.3(b),

$$I_R = I\cos 53.13^\circ = (2)(0.6)$$

$$= \mathbf{1.2\,A}$$

and $$I_C = I\sin 53.13^\circ = (2)(0.8)$$

$$= \mathbf{1.6\,A}$$

(Alternatively, I_R and I_C can be measured from the scaled phasor diagram).

From the circuit diagram,

$$I_R = \frac{V}{R} \text{ from which}$$

$$R = \frac{V}{I_R}$$

$$= \frac{120}{1.2} = \mathbf{100\,\Omega}$$

and $$I_C = \frac{V}{X_C}$$

$$= 2\pi fCV \text{ from which}$$

$$C = \frac{I_C}{2\pi f V}$$

$$= \frac{1.6}{2\pi(200)(120)}$$

$$= \mathbf{10.61\,\mu F}$$

Alternatively the current I_{LR} and I_C may be resolved into their horizontal (or 'in-phase') and vertical (or 'quadrature') components.
The horizontal component of I_{LR} is:
$I_{LR} \cos 51.34° = 3.748 \cos 51.34° = 2.341$ A.
The horizontal component of I_C is

$$I_C \cos 90° = 0$$

Thus the total horizontal component,

$$I_H = \mathbf{2.341\,A}$$

The vertical component of I_{LR}

$$= -I_{LR} \sin 51.34° = -3.748 \sin 51.34° = -2.927\text{ A}$$

The vertical component of I_C

$$= I_C \sin 90° = 2.262 \sin 90° = 2.262\text{ A}$$

Thus the total vertical component,

$$I_V = -2.927 + 2.262 = \mathbf{-0.665\,A}$$

I_H and I_V are shown in Fig. 16.7, from which,

$$I = \sqrt{2.341^2 + (-0.665)^2} = 2.434\text{ A}$$

$$\text{Angle } \phi = \tan^{-1}\frac{0.665}{2.341} = 15.86° \text{ lagging}$$

Hence **the supply current I = 2.434 A lagging V by 15.86°**

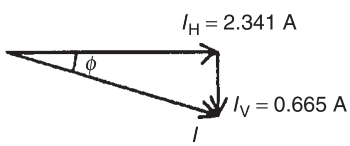


Figure 16.7

(d) Circuit impedance,

$$Z = \frac{V}{I} = \frac{240}{2.434} = \mathbf{98.60\,\Omega}$$

(e) Power consumed,

$$P = VI \cos\phi = (240)(2.434)\cos 15.86° = \mathbf{562\,W}$$

(Alternatively, $P = I_R^2 R = I_{LR}^2 R$ (in this case)

$$= (3.748)^2(40) = \mathbf{562\,W})$$

(f) Apparent power,

$$S = VI = (240)(2.434) = \mathbf{584.2\,VA}$$

(g) Reactive power,

$$Q = VI \sin\phi = (240)(2.434)(\sin 15.86°) = \mathbf{159.6\,var}$$

Problem 7. A coil of inductance 0.12 H and resistance 3 kΩ is connected in parallel with a 0.02 μF capacitor and is supplied at 40 V at a frequency of 5 kHz. Determine (a) the current in the coil, and (b) the current in the capacitor. (c) Draw to scale the phasor diagram and measure the supply current and its phase angle; check the answer by calculation. Determine (d) the circuit impedance and (e) the power consumed.

The circuit diagram is shown in Fig. 16.8(a).

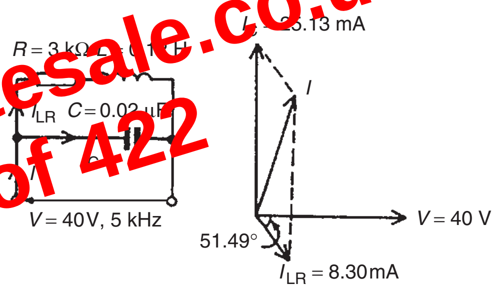


Figure 16.8

(a) Inductive reactance,

$$X_L = 2\pi fL = 2\pi(5000)(0.12) = 3770\,\Omega$$

Impedance of coil,

$$Z_1 = \sqrt{R^2 + X_L} = \sqrt{3000^2 + 3770^2} = 4818\,\Omega$$

Current in coil,

$$I_{LR} = \frac{V}{Z_1} = \frac{40}{4818} = \mathbf{8.30\,mA}$$

Branch phase angle

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{3770}{3000} = \mathbf{51.49°\ lagging}$$

and **inductance** $L = \dfrac{R_0}{4\pi f_c}$ (8)

Problem 6. A filter section is required to pass all frequencies above 25 kHz and to have a nominal impedance of 600 Ω. Design (a) a high-pass T-section filter, and (b) a high-pass π-section filter to meet these requirements.

Cut-off frequency $f_c = 25\,\text{kHz} = 25 \times 10^3$ Hz, and nominal impedance, $R_0 = 600\,\Omega$.

From equation (7), capacitance,

$$C = \frac{1}{4\pi R_0 f_c} = \frac{1}{4\pi(600)(25 \times 10^3)}\ \text{F}$$

$$= \frac{10^{12}}{4\pi(600)(25 \times 10^3)}\ \text{pF}$$

$$= 5305\ \text{pF} \quad \text{or} \quad 5.305\ \text{nF}$$

From equation (8), inductance,

$$L = \frac{R_0}{4\pi f_c} = \frac{600}{4\pi(25 \times 10^3)}$$

$$= 0.00191\ \text{H} = 1.91\ \text{mH}$$

(a) A high-pass T-section filter is shown in Fig. 17.19(a), where the series arm capacitances are each $2C$ (see Fig. 17.16(a)), i.e. $2 \times 5.305 = 10.61$ nF

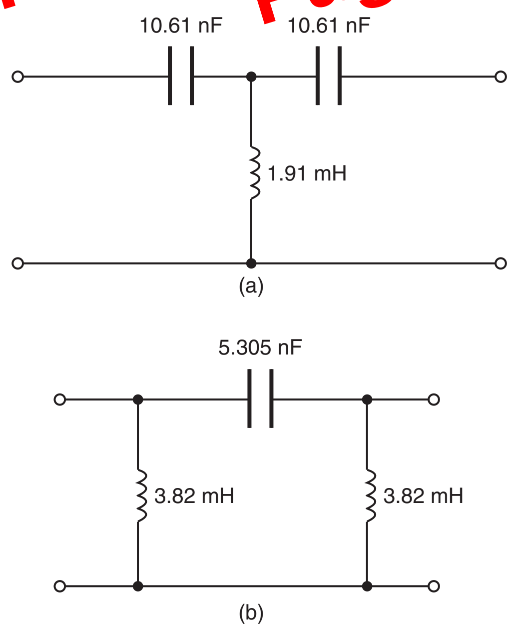


Figure 17.19

(b) A high-pass π-section filter is shown in Fig. 17.19(b), where the shunt arm inductances are each $2L$ (see Fig. 17.6(b)), i.e. $2 \times 1.91 = 3.82$ mH.

Now try the following exercise

Exercise 99 Further problems on high-pass filter sections

1. Determine the cut-off frequency and the nominal impedance of each of the high-pass filter sections shown in Fig. 17.20.
[(a) 22.51 kHz; 14.14 kΩ
(b) 281.3 Hz; 1414 Ω]

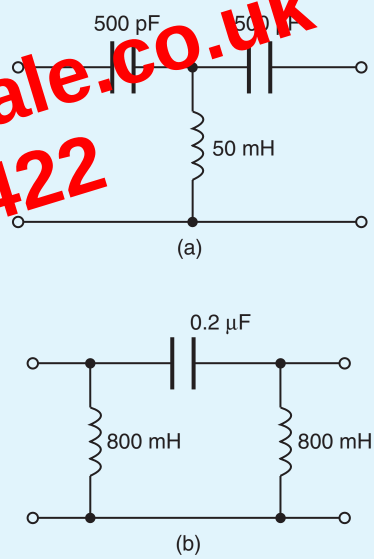


Figure 17.20

2. A filter section is required to pass all frequencies above 4 kHz and to have a nominal impedance 750 Ω. Design (a) an appropriate high-pass T section filter, and (b) an appropriate high-pass π-section filter to meet these requirements.
[(a) Each series arm = 53.06 nF, shunt arm = 14.92 mH
(b) Series arm = 26.53 nF, each shunt arm = 29.84 mH]

R is small, $\tau = CR$ is small and an output waveform such as that shown in Fig. 18.18(a) is obtained. As the value of R is increased, the waveform changes to that shown in Fig. 18.18(b). When R is large, the waveform is as shown in Fig. 18.18(c), the circuit then being described as an **integrator circuit**.

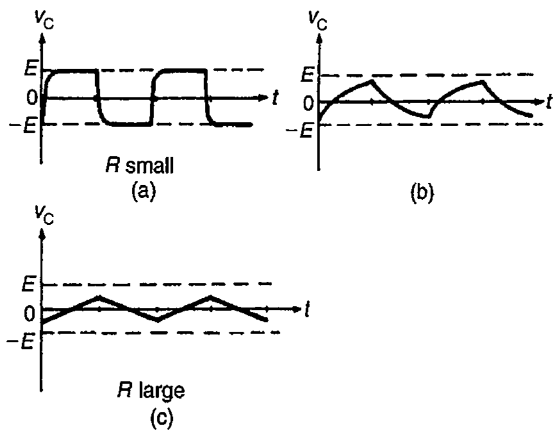


Figure 18.18

Differentiator circuit

If a rectangular waveform varying from $+E$ to $-E$ is applied to a series connected $C-R$ circuit and the waveform of the voltage drop across the resistor is observed, as shown in Fig. 18.19, the output waveform alters as R is varied due to the time constant, ($\tau = CR$), altering.

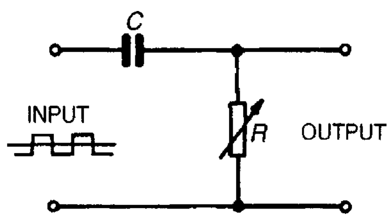


Figure 18.19

When R is small, the waveform is as shown in Fig. 18.20(a), the voltage being generated across R by the capacitor discharging fairly quickly. Since the change in capacitor voltage is from $+E$ to $-E$, the change in discharge current is $2E/R$, resulting in a change in voltage across the resistor of $2E$. This circuit is called a **differentiator circuit**. When R is large, the waveform is as shown in Fig. 18.20(b).

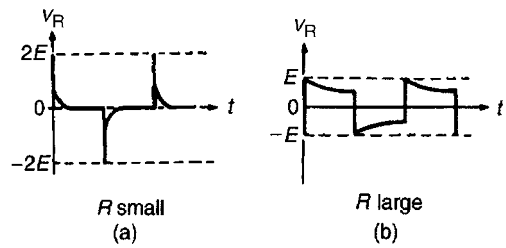


Figure 18.20

Now try the following exercises

Exercise 105 Short answer questions on d.c. transients

A capacitor of capacitance C farads is connected in series with a resistor of R ohms and is switched across a constant voltage d.c. supply of V volts. After a time of t seconds, the current flowing is i amperes. Use this data to answer questions 1 to 10.

1. The voltage drop across the resistor at time t seconds is $v_R = \ldots\ldots$
2. The capacitor voltage at time t seconds is $v_C = \ldots\ldots$
3. The voltage equation for the circuit is $V = \ldots\ldots$
4. The time constant for the circuit is $\tau = \ldots\ldots$
5. The final value of the current flowing is ......
6. The initial value of the current flowing is $I = \ldots\ldots$
7. The final value of capacitor voltage is ......
8. The initial value of capacitor voltage is ......
9. The final value of the voltage drop across the resistor is ......
10. The initial value of the voltage drop across the resistor is ......

A capacitor charged to V volts is disconnected from the supply and discharged through a resistor of R ohms. Use this data to answer questions 11 to 15

11. The initial value of current flowing is $I = \ldots\ldots$

Section 2

5. Determine the initial voltage across the capacitor

6. Determine the initial current flowing in the circuit

7. Determine the final current flowing in the circuit

In questions 8 and 9, a series connected $C-R$ circuit is suddenly connected to a d.c. source of V volts. Which of the statements is false ?

8. (a) The initial current flowing is given by V/R
(b) The time constant of the circuit is given by CR
(c) The current grows exponentially
(d) The final value of the current is zero

9. (a) The capacitor voltage is equal to the voltage drop across the resistor
(b) The voltage drop across the resistor decays exponentially
(c) The initial capacitor voltage is zero
(d) The initial voltage drop across the resistor is IR, where I is the steady-state current

10. A capacitor which is charged to V volts is discharged through a resistor of R ohms. Which of the following statements is false ?
(a) The initial current flowing is V/R amperes
(b) The voltage drop across the resistor is equal to the capacitor voltage
(c) The time constant of the circuit is CR seconds
(d) The current grows exponentially to a final value of V/R amperes

An inductor of inductance 0.1 H and negligible resistance is connected in series with a 50 Ω resistor to a 20 V d.c. supply. In questions 11 to 15, use this data to determine the value required, selecting your answer from those given below:

(a) 5 ms	(b) 12.6 V	(c) 0.4 A
(d) 500 ms	(e) 7.4 V	(f) 2.5 A
(g) 2 ms	(h) 0 V	(i) 0 A
(j) 20 V		

11. The value of the time constant of the circuit

12. The approximate value of the voltage across the resistor after a time equal to the time constant

13. The final value of the current flowing in the circuit

14. The initial value of the voltage across the inductor

15. The final value of the steady-state voltage across the inductor

16. The time constant for a circuit containing a capacitance of 100 nF in series with a 5 Ω resistance is:
(a) 0.5 μs (b) 20 ns (c) 5 μs (d) 50 μs

17. The time constant for a circuit containing an inductance of 100 mH in series with a resistance of 4 Ω is:
(a) 25 ms (b) 400 s (c) 0.4 s (d) 40 s

18. The graph shown in Fig. 18.21 represents the growth of current in an $L-R$ series circuit connected to a d.c. voltage V volts. The equation for the graph is:
(a) $i=I(1-e^{-Rt/L})$ (b) $i=Ie^{-Li/t}$
(c) $i=Ie^{-Rt/L}$ (d) $i=I(1-e^{RL/t})$

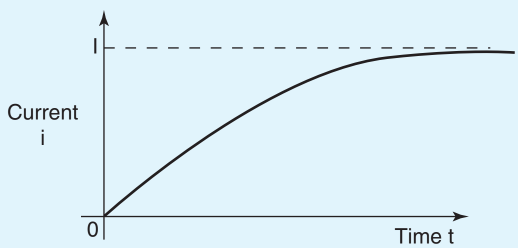


Figure 18.21

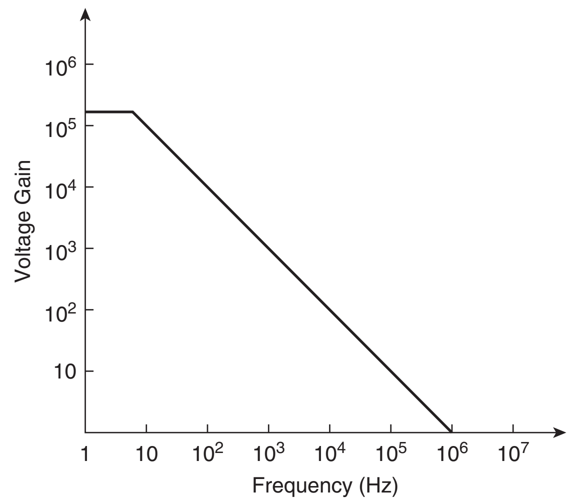


Figure 19.3

frequency at which the open-loop gain has fallen to unity is called the transition frequency f_T.

$$f_T = \text{closed-loop voltage gain} \times \text{bandwidth} \quad (2)$$

In Fig. 19.3, $f_T = 10^6$ Hz or 1 MHz; a gain of 20 dB (i.e. $20\log_{10} 10$) gives a 100 kHz bandwidth, whilst a gain of 80 dB (i.e. $20\log_{10} 10^4$) restricts the bandwidth to 100 Hz.

19.2 Some op amp parameters

Input bias current

The input bias current, I_B, is the average of the currents into the two input terminals with the output at zero volts, which is typically around 80 nA (i.e. 80×10^{-9} A) for a 741 op amp. The input bias current causes a volt drop across the equivalent source impedance seen by the op amp input.

Input offset current

The input offset current, I_{os}, of an op amp is the difference between the two input currents with the output at zero volts. In a 741 op amp, I_{os} is typically 20 nA.

Input offset voltage

In the ideal op amp, with both inputs at zero there should be zero output. Due to imbalances within the amplifier this is not always the case and a small output voltage results. The effect can be nullified by applying a small offset voltage, V_{os}, to the amplifier. In a 741 op amp, V_{os} is typically 1 mV.

Common-mode rejection ratio

The output voltage of an op amp is proportional to the difference between the voltages applied to its two input terminals. Ideally, when the two voltages are equal, the output voltages should be zero. A signal applied to both input terminals is called a common-mode signal and it is usually an unwanted noise voltage. The ability of an op amp to suppress common-mode signals is expressed in terms of its common-mode rejection ratio (CMRR), which is defined by:

$$\mathbf{CMRR} = \mathbf{20\log_{10}}\left(\frac{\textbf{differential voltage gain}}{\textbf{common mode gain}}\right)\ \mathbf{dB} \quad (3)$$

In a 741 op amp, the CMRR is typically 90 dB. The common-mode gain, A_{com}, is defined as:

$$\mathbf{A_{com}} = \frac{\mathbf{V_o}}{\mathbf{V_{com}}} \quad (4)$$

where V_{com} is the common input signal.

Problem 2. Determine the common-mode gain of an op amp that has a differential voltage gain of 150×10^3 and a CMRR of 90 dB.

From equation (3),

$$\text{CMRR} = 20\log_{10}\left(\frac{\text{differential voltage gain}}{\text{common mode gain}}\right) \text{dB}$$

Hence $$90 = 20\log_{10}\left(\frac{150 \times 10^3}{\text{common mode gain}}\right)$$

from which

$$4.5 = \log_{10}\left(\frac{150 \times 10^3}{\text{common mode gain}}\right)$$

and $$10^{4.5} = \frac{150 \times 10^3}{\text{common mode gain}}$$

Hence, $$\textbf{common-mode gain} = \frac{150 \times 10^3}{10^{4.5}} = \mathbf{4.74}$$

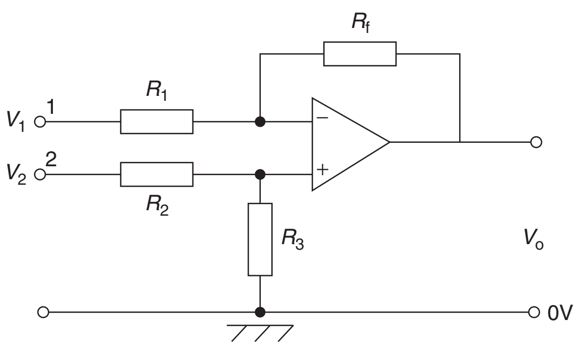


Figure 19.17

op amp inputs is practically zero and hence the inverting terminal must be at zero potential. Then $I_1 = V_1/R_1$. Since the op amp input resistance is high, this current flows through the feedback resistor R_f. The volt drop across R_f, which is the output voltage

$$V_o = \frac{V_1}{R_1} R_f$$

hence, the closed loop voltage gain A is given by:

$$A = \frac{V_o}{V_1} = -\frac{R_f}{R_1} \quad (10)$$

(ii) By similar reasoning, if V_2 is applied to terminal 2 and 0 V to terminal 1, then the voltage appearing at the non-inverting terminal will be

$$\left(\frac{R_3}{R_2 + R_3}\right) V_2 \text{ volts.}$$

This voltage will also appear at the inverting (−) terminal and thus the voltage across R_1 is equal to

$$-\left(\frac{R_3}{R_2 + R_3}\right) V_2 \text{ volts.}$$

Now the output voltage,

$$V_o = \left(\frac{R_3}{R_2 + R_3}\right) V_2 + \left[-\left(\frac{R_3}{R_2 + R_3}\right) V_2\right]\left(\frac{-R_f}{R_1}\right)$$

and the voltage gain,

$$A = \frac{V_o}{V_2} = \left(\frac{R_3}{R_2 + R_3}\right) + \left[-\left(\frac{R_3}{R_2 + R_3}\right)\right]\left(-\frac{R_f}{R_1}\right)$$

i.e. $$A = \frac{V_o}{V_2} = \left(\frac{R_3}{R_2 + R_3}\right)\left(1 + \frac{R_f}{R_1}\right) \quad (11)$$

(iii) Finally, if the voltages applied to terminals 1 and 2 are V_1 and V_2 respectively, then the difference between the two voltages will be amplified.

If $V_1 > V_2$, then:

$$V_o = (V_1 - V_2)\left(-\frac{R_f}{R_1}\right) \quad (12)$$

If $V_2 > V_1$, then:

$$V_o = (V_2 - V_1)\left(\frac{R_3}{R_2 + R_3}\right)\left(1 + \frac{R_f}{R_1}\right) \quad (13)$$

Problem 11. In the differential amplifier shown in Fig. 19.17, $R_1 = 10\,k\Omega$, $R_2 = 10\,k\Omega$, $R_3 = 100\,k\Omega$ and $R_f = 100\,k\Omega$. Determine the output voltage V_o if:
(a) $V_1 = 5$ mV and $V_2 = 0$
(b) $V_1 = 0$ and $V_2 = 5$ mV
(c) $V_1 = 50$ mV and $V_2 = 25$ mV
(d) $V_1 = 25$ mV and $V_2 = 50$ mV.

(a) From equation (10),

$$V_o = -\frac{R_f}{R_1} V_1 = -\left(\frac{100 \times 10^3}{10 \times 10^3}\right)(5)\,\text{mV}$$
$$= \mathbf{-50\,mV}$$

(b) From equation (11),

$$V_o = \left(\frac{R_3}{R_2 + R_3}\right)\left(1 + \frac{R_f}{R_1}\right) V_2$$
$$= \left(\frac{100}{110}\right)\left(1 + \frac{100}{10}\right)(5)\,\text{mV} = \mathbf{+50\,mV}$$

(c) $V_1 > V_2$ hence from equation (12),

$$V_o = (V_1 - V_2)\left(-\frac{R_f}{R_1}\right)$$
$$= (50 - 25)\left(-\frac{100}{10}\right)\text{mV} = \mathbf{-250\,mV}$$

(d) $V_2 > V_1$ hence from equation (13),

$$V_o = (V_2 - V_1)\left(\frac{R_3}{R_2 + R_3}\right)\left(1 + \frac{R_f}{R_1}\right)$$
$$= (50 - 25)\left(\frac{100}{100 + 10}\right)\left(1 + \frac{100}{10}\right)\text{mV}$$
$$= (25)\left(\frac{100}{110}\right)(11) = \mathbf{+250\,mV}$$

Section 2

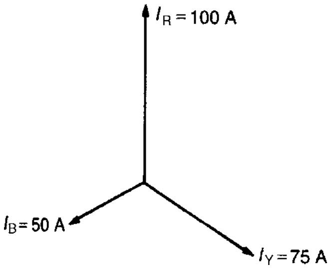


Figure 20.8

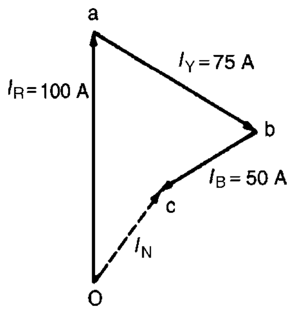


Figure 20.9

magnitude and direction. From the nose of ab, bc is drawn representing I_B in magnitude and direction. oc represents the resultant, I_N By measurement, I_N = **43 A**.

Alternatively, by calculation, considering I_R at 90°, I_B at 210° and I_Y at 330°: Total horizontal component = 100 cos 90° + 75 cos 330° + 50 cos 210° = 21.65. Total vertical component = 100 sin 90° + 75 sin 330° + 50 sin 210° = 37.50. Hence magnitude of $I_N = \sqrt{21.65^2 + 37.50^2} = \mathbf{43.3\,A}$.

Now try the following exercise

Exercise 111 Further problems on star connections

1. Three loads, each of resistance 50 Ω are connected in star to a 400 V, 3-phase supply. Determine (a) the phase voltage, (b) the phase current and (c) the line current.
[(a) 231 V (b) 4.62 A (c) 4.62 A]

2. A star-connected load consists of three identical coils, each of inductance 159.2 mH and resistance 50 Ω. If the supply frequency is 50 Hz and the line current is 3 A determine (a) the phase voltage and (b) the line voltage.
[(a) 212 V (b) 367 V]

3. Three identical capacitors are connected in star to a 400 V, 50 Hz 3-phase supply. If the line current is 12 A determine the capacitance of each of the capacitors. [165.4 μF]

4. Three coils each having resistance 6 Ω and inductance L H are connected in star to a 415 V, 50 Hz, 3-phase supply. If the line current is 30 A, find the value of L. [16.78 mH]

5. A 400 V, 3-phase, 4 wire, star-connected system supplies three resistive loads of 15 kW, 20 kW and 25 kW in the red, yellow and blue phases respectively. Determine the current flowing in each of the four conductors.
[I_R = 64.95 A, I_Y = 86.60 A, I_B = 108.25 A, I_N = 37.50 A]

20.4 Delta connection

(i) A **delta (or mesh) connected load** is shown in Fig. 20.10 where the end of one load is connected to the start of the next load.

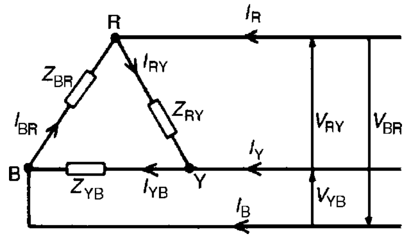


Figure 20.10

(ii) From Fig. 20.10, it can be seen that the line voltages V_{RY}, V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

(iii) Using Kirchhoff's current law in Fig. 20.10, $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$ From the phasor diagram shown in Fig. 20.11, by trigonometry or by measurement, $I_R = \sqrt{3}\,I_{RY}$, i.e. for a delta connection:

$$I_L = \sqrt{3}\,I_p$$

It is possible, depending on the load power factor, for one wattmeter to have to be 'reversed' to obtain a reading. In this case it is taken as a negative reading (see Problem 17).

(iii) **Three-wattmeter method for a three-phase, 4-wire system for balanced and unbalanced loads** (see Fig. 20.14).

$$\textbf{Total power} = P_1 + P_2 + P_3$$

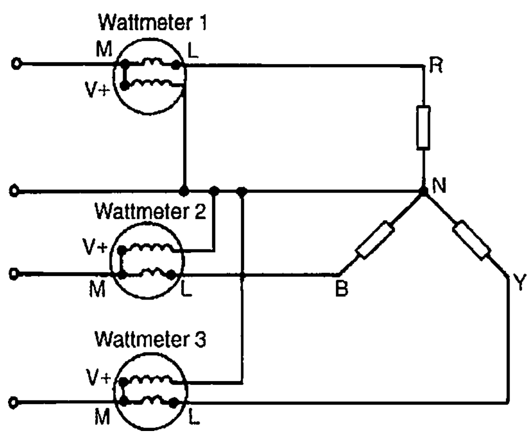


Figure 20.14

Problem 12. (a) Show that the total power in a 3-phase, 3-wire system using the two-wattmeter method of measurement is given by the sum of the wattmeter readings. Draw a connection diagram. (b) Draw a phasor diagram for the two-wattmeter method for a balanced load. (c) Use the phasor diagram of part (b) to derive a formula from which the power factor of a 3-phase system may be determined using only the wattmeter readings.

(a) A connection diagram for the two-wattmeter method of a power measurement is shown in Fig. 20.15 for a star-connected load.

Total instantaneous power, $p = e_R i_R + e_Y i_Y + e_B i_B$ and in any 3-phase system $i_R + i_Y + i_B = 0$; hence $i_B = -i_R - i_Y$ Thus,

$$p = e_R i_R + e_Y i_Y + e_B(-i_R - i_Y)$$
$$= (e_R - e_B)i_R + (e_Y - e_B)i_Y$$

However, $(e_R - e_B)$ is the p.d. across wattmeter 1 in Fig. 20.15 and $(e_Y - e_B)$ is the p.d. across wattmeter 2 Hence total instantaneous power,

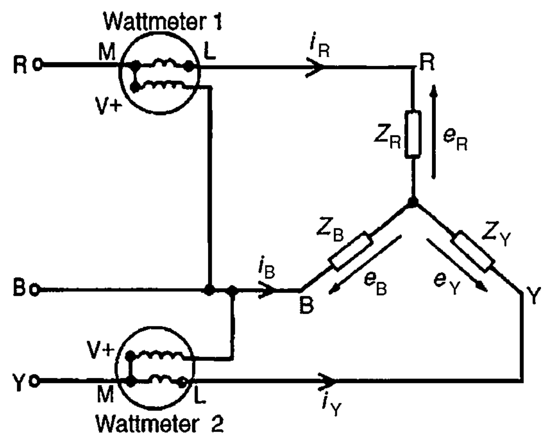


Figure 20.15

$$p = \textbf{(wattmeter 1 reading)} + \textbf{(wattmeter 2 reading)} = p_1 + p_2$$

The moving systems of the wattmeters are unable to follow the variations which take place at normal frequencies and they indicate the mean power taken over a cycle. Hence the total power, $P = P_1 + P_2$ for balanced or unbalanced loads.

(b) The phasor diagram for the two-wattmeter method for a balanced load having a lagging current is shown in Fig. 20.16, where $V_{RB} = V_R - V_B$ and $V_{YB} = V_Y - V_B$ (phasorially).

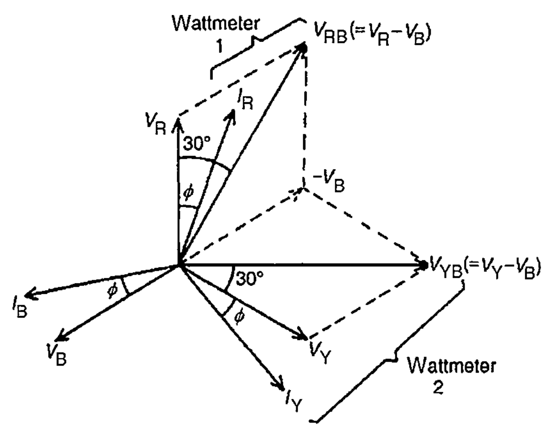


Figure 20.16

(c) Wattmeter 1 reads $V_{RB}I_R \cos(30° - \phi) = P_1$

Wattmeter 2 reads $V_{YB}I_Y \cos(30° + \phi) = P_2$

$$\frac{P_1}{P_2} = \frac{V_{RB}I_R \cos(30° - \phi)}{V_{YB}I_Y \cos(30° + \phi)} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

Since the reversing switch on the wattmeter had to be operated the 3 kW reading is taken as −3 kW

(a) Total input power,

$$P = P_1 + P_2 = 10 + (-3) = \mathbf{7\,kW}$$

(b) $$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) = \sqrt{3}\left(\frac{10-(-3)}{10+(-3)}\right) = \sqrt{3}\left(\frac{13}{7}\right) = 3.2167$$

Angle $\phi = \tan^{-1}3.2167 = 72.73°$

Power factor $= \cos\phi = \cos 72.73° = \mathbf{0.297}$

Problem 18. Three similar coils, each having a resistance of 8 Ω and an inductive reactance of 8 Ω are connected (a) in star and (b) in delta, across a 415 V, 3-phase supply. Calculate for each connection the readings on each of two wattmeters connected to measure the power by the two-wattmeter method.

(a) **Star connection:** $V_L = \sqrt{3}\,V_p$ and $I_L = I_p$

Phase voltage, $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{415}{\sqrt{3}}$

and phase impedance,

$$Z_p = \sqrt{R_p^2 + X_L^2} = \sqrt{8^2 + 8^2} = 11.31\,\Omega$$

Hence phase current,

$$I_p = \frac{V_p}{Z_p} = \frac{\frac{415}{\sqrt{3}}}{11.31} = 21.18\,A$$

Total power,

$$P = 3I_p^2 R_p = 3(21.18)^2(8) = 10\,766\,W$$

If wattmeter readings are P_1 and P_2 then:

$$P_1 + P_2 = 10\,766 \qquad (1)$$

Since $R_p = 8\,\Omega$ and $X_L = 8\,\Omega$, then phase angle $\phi = 45°$ (from impedance triangle).

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

hence $$\tan 45° = \frac{\sqrt{3}(P_1 - P_2)}{10\,766}$$

from which

$$P_1 - P_2 = \frac{(10\,766)(1)}{\sqrt{3}} = 6216\,W \qquad (2)$$

Adding Equations (1) and (2) gives:

$2P_1 = 10\,766 + 6216 = 16\,982\,W$

Hence $P_1 = 8491\,W$

From Equation (1), $P_2 = 10\,766 - 8491 = 2275\,W$.

When the coils are star-connected the wattmeter readings are thus 8.491 kW and 2.275 kW

(b) **Delta connection:** $V_L = V_p$ and $I_L = \sqrt{3}\,I_p$

Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{415}{11.31} = 36.69\,A.$

Total power,

$P = 3I_p^2 R_p = 3(36.69)^2(8) = 32\,310\,W$

Hence $P_1 + P_2 = 32\,310\,W \qquad (3)$

$\tan\phi = \sqrt{3}\left(\dfrac{P_1 - P_2}{P_1 + P_2}\right)$ thus $1 = \dfrac{\sqrt{3}(P_1 - P_2)}{32\,310}$

from which,

$$P_1 - P_2 = \frac{32\,310}{\sqrt{3}} = 18\,650\,W \qquad (4)$$

Adding Equations (3) and (4) gives:

$2P_1 = 50\,960$ from which $P_1 = 25\,480\,W$.

From Equation (3), $P_2 = 32\,310 - 25\,480 = 6830\,W$

When the coils are delta-connected the wattmeter readings are thus 25.48 kW and 6.83 kW

Now try the following exercise

Exercise 114 Further problems on the measurement of power in 3-phase circuits

1. Two wattmeters are connected to measure the input power to a balanced three-phase load. If the wattmeter readings are 9.3 kW and 5.4 kW determine (a) the total output power, and (b) the load power factor
[(a) 14.7 kW (b) 0.909]

Problem 3. An ideal transformer has a turns ratio of 8:1 and the primary current is 3 A when it is supplied at 240 V. Calculate the secondary voltage and current.

A turns ratio of 8:1 means $(N_1/N_2) = (1/8)$ i.e. a step-down transformer.

$$\left(\frac{N_1}{N_2}\right) = \left(\frac{V_1}{V_2}\right) \text{ or secondary voltage}$$

$$\boldsymbol{V_2} = V_1\left(\frac{N_1}{N_2}\right) = 240\left(\frac{1}{8}\right) = \textbf{30 volts}$$

Also, $\left(\frac{N_1}{N_2}\right) = \left(\frac{I_2}{I_1}\right)$ hence secondary current

$$\boldsymbol{I_2} = I_1\left(\frac{N_1}{N_2}\right) = 3\left(\frac{8}{1}\right) = \textbf{24 A}$$

Problem 4. An ideal transformer, connected to a 240 V mains, supplies a 12 V, 150 W lamp. Calculate the transformer turns ratio and the current taken from the supply.

$V_1 = 240\,\text{V}$, $V_2 = 12\,\text{V}$, $I_2 = (P/V_2) = (150/12) = 12.5\,\text{A}$.

$$\textbf{Turns ratio} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{240}{12} = \textbf{20}$$

$\left(\frac{V_1}{V_2}\right) = \left(\frac{I_2}{I_1}\right)$, from which,

$$I_1 = I_2\left(\frac{V_2}{V_1}\right) = 12.5\left(\frac{12}{240}\right)$$

Hence current taken from the supply,

$$\boldsymbol{I_1} = \frac{12.5}{20} = \textbf{0.625 A}$$

Problem 5. A 12 Ω resistor is connected across the secondary winding of an ideal transformer whose secondary voltage is 120 V. Determine the primary voltage if the supply current is 4 A.

Secondary current $I_2 = (V_2/R_2) = (120/12) = 10\,\text{A}$. $(V_1/V_2) = (I_2/I_1)$, from which the primary voltage

$$\boldsymbol{V_1} = V_2\left(\frac{I_2}{I_1}\right) = 120\left(\frac{10}{4}\right) = \textbf{300 volts}$$

Problem 6. A 5 kVA single-phase transformer has a turns ratio of 10:1 and is fed from a 2.5 kV supply. Neglecting losses, determine (a) the full-load secondary current, (b) the minimum load resistance which can be connected across the secondary winding to give full load kVA, (c) the primary current at full load kVA.

(a) $N_1/N_2 = 10/1$ and $V_1 = 2.5\,\text{kV} = 2500\,\text{V}$.

Since $\left(\frac{N_1}{N_2}\right) = \left(\frac{V_1}{V_2}\right)$, secondary voltage

$$V_2 = V_1\left(\frac{N_2}{N_1}\right) = 2500\left(\frac{1}{10}\right) = 250\,\text{V}$$

The transformer rating in volt-amperes $= V_2I_2$ (at full load) i.e. $5000 = 250I_2$
Hence full load secondary current,
$I_2 = (5000/\ 250) = \textbf{20 A}$.

(b) Minimum value of load resistance,

$$R_L = \left(\frac{V_2}{I_2}\right) = \left(\frac{250}{20}\right) = \textbf{12.5 Ω}.$$

(c) $\left(\frac{N_1}{N_2}\right) = \left(\frac{I_2}{I_1}\right)$ from which primary current

$$I_1 = I_2\left(\frac{N_1}{N_2}\right) = 20\left(\frac{1}{10}\right) = \textbf{2 A}$$

Now try the following exercise

Exercise 117 Further problems on the transformer principle of operation

1. A transformer has 600 primary turns connected to a 1.5 kV supply. Determine the number of secondary turns for a 240 V output voltage, assuming no losses. [96]

2. An ideal transformer with a turns ratio of 2:9 is fed from a 220 V supply. Determine its output voltage. [990 V]

3. A transformer has 800 primary turns and 2000 secondary turns. If the primary voltage is 160 V, determine the secondary voltage assuming an ideal transformer. [400 V]

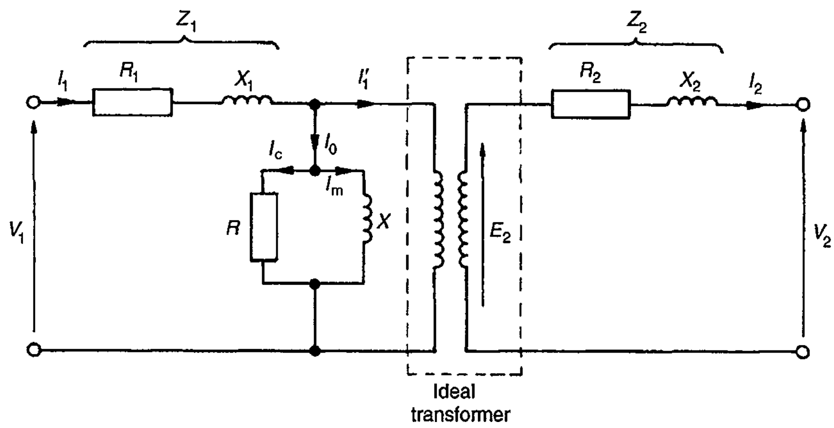


Figure 21.7

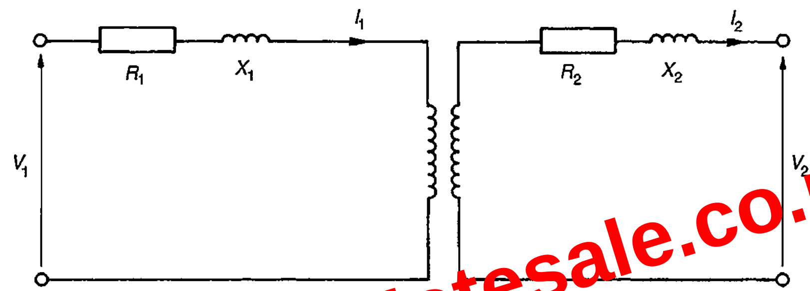


Figure 21.8

by inserting an additional resistance R'_2 in the primary circuit such that the power absorbed in R'_2 when carrying the primary current is equal to that in R_2 due to the secondary current, i.e.

$$I_1^2 R'_2 = I_2^2 R_2$$

from which, $$R'_2 = R_2\left(\frac{I_2}{I_1}\right)^2 = R_2\left(\frac{V_1}{V_2}\right)^2$$

Then the total equivalent resistance in the primary circuit R_e is equal to the primary and secondary resistances of the actual transformer.
Hence $R_e = R_1 + R'_2$

i.e. $$R_e = R_1 + R_2\left(\frac{V_1}{V_2}\right)^2 \qquad (6)$$

By similar reasoning, the equivalent reactance in the primary circuit is given by $X_e = X_1 + X'_2$

i.e. $$X_e = X_1 + X_2\left(\frac{V_1}{V_2}\right)^2 \qquad (7)$$

The equivalent impedance Z_e of the primary and secondary windings referred to the primary is given by

$$Z_e = \sqrt{R_e^2 + X_e^2} \qquad (8)$$

If ϕ_e is the phase angle between I_1 and the volt drop $I_1 Z_e$ then

$$\cos\phi_e = \frac{R_e}{Z_e} \qquad (9)$$

The simplified equivalent circuit of a transformer is shown in Fig. 21.9.

Problem 14. A transformer has 600 primary turns and 150 secondary turns. The primary and secondary resistances are 0.25 Ω and 0.01 Ω respectively and the corresponding leakage reactances are 1.0 Ω and 0.04 Ω respectively. Determine (a) the equivalent resistance referred to the primary winding, (b) the equivalent reactance referred to the primary winding, (c) the equivalent impedance referred to the primary winding, and (d) the phase angle of the impedance.

Section 3

21.10 Resistance matching

Varying a load resistance to be equal, or almost equal, to the source internal resistance is called **matching**. Examples where resistance matching is important include coupling an aerial to a transmitter or receiver, or in coupling a loudspeaker to an amplifier, where coupling transformers may be used to give maximum power transfer.

With d.c. generators or secondary cells, the internal resistance is usually very small. In such cases, if an attempt is made to make the load resistance as small as the source internal resistance, overloading of the source results.

A method of achieving maximum power transfer between a source and a load (see Section 13.9, page 200), is to adjust the value of the load resistance to 'match' the source internal resistance. A transformer may be used as a **resistance matching device** by connecting it between the load and the source.

The reason why a transformer can be used for this is shown below. With reference to Fig. 21.10:

$$R_L = \frac{V_2}{I_2} \quad \text{and} \quad R_1 = \frac{V_1}{I_1}$$

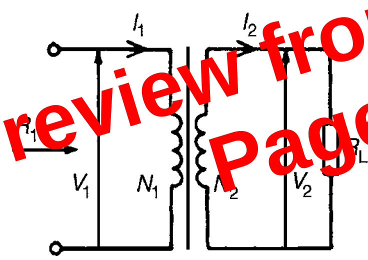


Figure 21.10

For an ideal transformer,

$$V_1 = \left(\frac{N_1}{N_2}\right) V_2$$

and $$I_1 = \left(\frac{N_2}{N_1}\right) I_2$$

Thus the equivalent input resistance R_1 of the transformer is given by:

$$R_1 = \frac{V_1}{I_1} = \frac{\left(\frac{N_1}{N_2}\right) V_2}{\left(\frac{N_2}{N_1}\right) I_2}$$

$$= \left(\frac{N_1}{N_2}\right)^2 \left(\frac{V_2}{I_2}\right) = \left(\frac{N_1}{N_2}\right)^2 R_L$$

i.e. $$\boldsymbol{R_1 = \left(\frac{N_1}{N_2}\right)^2 R_L}$$

Hence by varying the value of the turns ratio, the equivalent input resistance of a transformer can be 'matched' to the internal resistance of a load to achieve maximum power transfer.

Problem 21. A transformer having a turns ratio of 4:1 supplies a load of resistance 100 Ω. Determine the equivalent input resistance of the transformer.

From above, the equivalent input resistance,

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_L$$

$$= \left(\frac{4}{1}\right)^2 (100) = \mathbf{1600\ \Omega}$$

Problem 22. The output stage of an amplifier has an output resistance of 112 Ω. Calculate the optimum turns ratio of a transformer which would match a load resistance of 7 Ω to the output resistance of the amplifier.

The circuit is shown in Fig. 21.11.

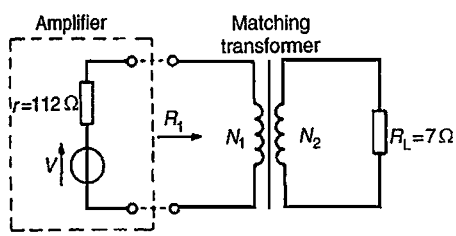


Figure 21.11

The equivalent input resistance, R_1 of the transformer needs to be 112 Ω for maximum power transfer.

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_L$$

Hence $$\left(\frac{N_1}{N_2}\right)^2 = \frac{R_1}{R_L} = \frac{112}{7} = 16$$

i.e. $$\frac{N_1}{N_2} = \sqrt{16} = 4$$

Hence the optimum turns ratio is 4:1

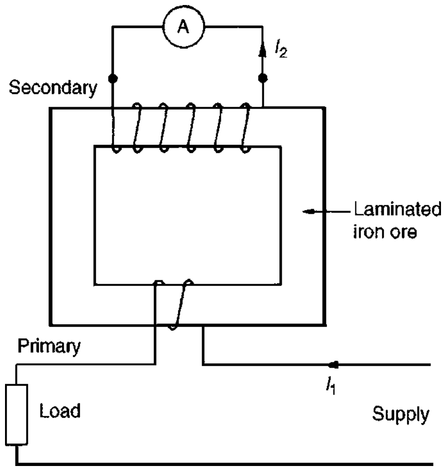


Figure 21.19

cannot be used since the proportion of the current which flows in the meter will depend on its impedance, which varies with frequency.

In a double-wound transformer:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

from which,

$$\textbf{secondary current } I_2 = I_1\left(\frac{N_1}{N_2}\right)$$

In current transformers the primary usually consists of one or two turns whilst the secondary can have several hundred turns. A typical arrangement is shown in Fig. 21.19.

If, for example, the primary has 2 turns and the secondary 200 turns, then if the primary current is 500 A,

$$\text{secondary current, } I_2 = I_1\left(\frac{N_2}{N_1}\right) = (500)\left(\frac{2}{200}\right)$$
$$= 5\text{ A}$$

Current transformers isolate the ammeter from the main circuit and allow the use of a standard range of ammeters giving full-scale deflections of 1 A, 2 A or 5 A.

For very large currents the transformer core can be mounted around the conductor or bus-bar. Thus the primary then has just one turn.

It is very important to short-circuit the secondary winding before removing the ammeter. This is because if current is flowing in the primary, dangerously high voltages could be induced in the secondary should it be open-circuited.

Current transformer circuit diagram symbols are shown in Fig. 21.20.

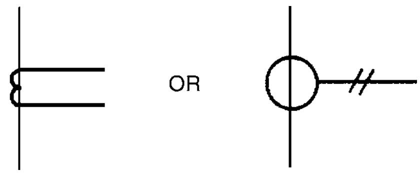


Figure 21.20

Problem 29. A current transformer has a single turn on the primary winding and a secondary winding of 60 turns. The secondary winding is connected to an ammeter with a resistance of 0.15 Ω. The resistance of the secondary winding is 0.25 Ω. If the current in the primary winding is 300 A, determine (a) the reading on the ammeter, (b) the potential difference across the ammeter and (c) the total load (in VA) on the secondary.

(a) Reading on the ammeter,

$$I_2 = I_1\left(\frac{N_1}{N_2}\right) = 300\left(\frac{1}{60}\right) = \mathbf{5\,A}.$$

(b) P.d. across the ammeter $= I_2R_A$, (where R_A is the ammeter resistance) $= (5)(0.15) = \mathbf{0.75\ volts}$.

(c) Total resistance of secondary circuit $= 0.15 + 0.25 = 0.40\ \Omega$.
Induced e.m.f. in secondary $= (5)(0.40) = 2.0$ V.
Total load on secondary $= (2.0)(5) = \mathbf{10\ VA}$.

Now try the following exercises

Exercise 127 A further problem on the current transformer

1. A current transformer has two turns on the primary winding and a secondary winding of 260 turns. The secondary winding is connected to an ammeter with a resistance of 0.2 Ω. The resistance of the secondary winding is 0.3 Ω. If the current in the primary winding is 650 A, determine (a) the reading on the ammeter, (b) the potential difference across the ammeter, and (c) the total load in VA on the secondary.
[(a) 5 A (b) 1 V (c) 7.5 VA]

27. In what applications are auto transformers used?
28. What is an isolating transformer? Give two applications
29. Describe briefly the construction of a three-phase transformer
30. For what reason are current transformers used?
31. Describe how a current transformer operates
32. For what reason are voltage transformers used?
33. Describe how a voltage transformer operates

Exercise 129 Multi-choice questions on transformers

(Answers on page 399)

1. The e.m.f. equation of a transformer of secondary turns N_2, magnetic flux density B_m, magnetic area of core a, and operating at frequency f is given by:
   (a) $E_2 = 4.44 N_2 B_m a f$ volts
   (b) $E_2 = 4.44 \dfrac{N_2 B_m f}{a}$ volts
   (c) $E_2 = \dfrac{N_2 B_m f}{a}$ volts
   (d) $E_2 = 1.11 N_2 B_m a f$ volts

2. In the auto-transformer shown in Fig. 21.22, the current in section PQ is:
   (a) 3.3 A (b) 1.7 A (c) 5 A (d) 1.6 A

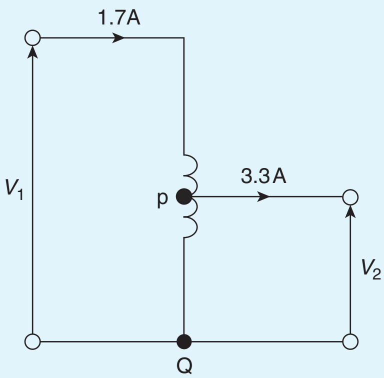


Figure 21.22

3. A step-up transformer has a turns ratio of 10. If the output current is 5 A, the input current is:
   (a) 50 A (b) 5 A
   (c) 2.5 A (d) 0.5 A

4. A 440 V/110 V transformer has 1000 turns on the primary winding. The number of turns on the secondary is:
   (a) 550 (b) 250
   (c) 4000 (d) 25

5. An advantage of an auto-transformer is that:
   (a) it gives a high step-up ratio
   (b) iron losses are reduced
   (c) copper loss is reduced
   (d) it reduces capacitance between turns

6. A 1 kV/250 V transformer has 500 turns on the secondary winding. The number of turns on the primary is:
   (a) 2000 (b) 125
   (c) 1000 (d) 250

7. The core of a transformer is laminated to:
   (a) limit hysteresis loss
   (b) reduce the inductance of the windings
   (c) reduce the effects of eddy current loss
   (d) prevent eddy currents from occurring

8. The power input to a mains transformer is 200 W. If the primary current is 2.5 A, the secondary voltage is 2 V and assuming no losses in the transformer, the turns ratio is:
   (a) 40:1 step down (b) 40:1 step up
   (c) 80:1 step down (d) 80:1 step up

9. A transformer has 800 primary turns and 100 secondary turns. To obtain 40 V from the secondary winding the voltage applied to the primary winding must be:
   (a) 5 V (b) 320 V
   (c) 2.5 V (d) 20 V

A 100 kVA, 250 V/10 kV, single-phase transformer has a full-load copper loss of 800 W and an iron loss of 500 W. The primary winding contains 120 turns. For the statements in

Revision Test 6

This revision test covers the material contained in Chapters 20 to 21. *The marks for each question are shown in brackets at the end of each question.*

1. Three identical coils each of resistance 40 Ω and inductive reactance 30 Ω are connected (i) in star, and (ii) in delta to a 400 V, three-phase supply. Calculate for each connection (a) the line and phase voltages, (b) the phase and line currents, and (c) the total power dissipated. (12)

2. Two wattmeters are connected to measure the input power to a balanced three-phase load by the two-wattmeter method. If the instrument readings are 10 kW and 6 kW, determine (a) the total power input, and (b) the load power factor. (5)

3. An ideal transformer connected to a 250 V mains, supplies a 25 V, 200 W lamp. Calculate the transformer turns ratio and the current taken from the supply. (5)

4. A 200 kVA, 8000 V/320 V, 50 Hz single phase transformer has 120 secondary turns. Determine (a) the primary and secondary currents, (b) the number of primary turns, and (c) the maximum value of flux. (9)

5. Determine the percentage regulation of an 8 kVA, 100 V/200 V, single phase transformer when its secondary terminal voltage is 194 V when loaded. (3)

6. A 500 kVA rated transformer has a full-load copper loss of 4 kW and an iron loss of 3 kW. Determine the transformer efficiency (a) at full load and 0.80 power factor, and (b) at half full load and 0.80 power factor. (10)

7. Determine the optimum value of load resistance for maximum power transfer if the load is connected to an amplifier of output resistance 288 Ω through a transformer with a turns ratio 6:1 (3)

8. A single-phase auto transformer has a voltage ratio of 250 V:200 V and supplies a load of 15 kVA at 200 V. Assuming an ideal transformer, determine the current in each section of the winding. (3)

Chapter 22

D.C. machines

At the end of this chapter you should be able to:

- distinguish between the function of a motor and a generator
- describe the action of a commutator
- describe the construction of a d.c. machine
- distinguish between wave and lap windings
- understand shunt, series and compound windings of d.c. machines
- understand armature reaction
- calculate generated e.m.f. in an armature winding using $E = 2p\Phi nZ/c$
- describe types of d.c. generator and their characteristics
- calculate generated e.m.f. for a generator using $E = V + I_a R_a$
- state typical applications of d.c. generators
- list d.c. machine losses and calculate efficiency
- calculate back e.m.f. for a d.c. motor using $E = V - I_a R_a$
- calculate the torque of a d.c. motor using $T = EI_a/2\pi n$ and $T = p\Phi ZI_a/\pi c$
- describe types of d.c. motor and their characteristics
- state typical applications of d.c. motors
- describe a d.c. motor starter
- describe methods of speed control of d.c. motors
- list types of enclosure for d.c. motors

22.1 Introduction

When the input to an electrical machine is electrical energy, (seen as applying a voltage to the electrical terminals of the machine), and the output is mechanical energy, (seen as a rotating shaft), the machine is called an electric **motor**. Thus an electric motor converts electrical energy into mechanical energy.

The principle of operation of a motor is explained in Section 8.4, page 91. When the input to an electrical machine is mechanical energy, (seen as, say, a diesel motor, coupled to the machine by a shaft), and the output is electrical energy, (seen as a voltage appearing at the electrical terminals of the machine), the machine is called a **generator**. Thus, a generator converts mechanical energy to electrical energy.

The number of conductors in series in each path $= Z/c$

The total e.m.f. between

$$\text{brushes} = \text{(average e.m.f./conductor) (number of conductors in series per path)}$$
$$= 2p\Phi nZ/c$$

i.e. **generated e.m.f.** $\boldsymbol{E = \dfrac{2p\Phi nZ}{c}}$ **volts** (1)

Since Z, p and c are constant for a given machine, then $E \propto \Phi n$. However $2\pi n$ is the angular velocity ω in radians per second, hence the generated e.m.f. is proportional to Φ and ω,

i.e. **generated e.m.f.** $\boldsymbol{E \propto \Phi\omega}$ (2)

Problem 1. An 8-pole, wave-connected armature has 600 conductors and is driven at 625 rev/min. If the flux per pole is 20 mWb, determine the generated e.m.f.

$Z = 600$, $c = 2$ (for a wave winding), $p = 4$ pairs, $n = 625/60$ rev/s and $\Phi = 20 \times 10^{-3}$ Wb.

Generated e.m.f.

$$E = \frac{2p\Phi nZ}{c} = \frac{2(4)(20 \times 10^{-3})\left(\frac{625}{60}\right)(600)}{2}$$
$$= \textbf{500 volts}$$

Problem 2. A 4-pole generator has a lap-wound armature with 50 slots with 16 conductors per slot. The useful flux per pole is 30 mWb. Determine the speed at which the machine must be driven to generate an e.m.f. of 240 V.

$E = 240$ V, $c = 2p$ (for a lap winding), $Z = 50 \times 16 = 800$ and $\Phi = 30 \times 10^{-3}$ Wb.

Generated e.m.f.

$$E = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2p} = \Phi nZ$$

Rearranging gives, speed,

$$n = \frac{E}{\Phi Z} = \frac{240}{(30 \times 10^{-3})(800)}$$
$$= \textbf{10 rev/s or 600 rev/min}$$

Problem 3. An 8-pole, lap-wound armature has 1200 conductors and a flux per pole of 0.03 Wb. Determine the e.m.f. generated when running at 500 rev/min.

Generated e.m.f.,

$$E = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2p} \text{ for a lap-wound machine,}$$

i.e. $E = \Phi nZ$

$$= (0.03)\left(\frac{500}{60}\right)(1200)$$
$$= \textbf{300 volts}$$

Problem 4. Determine the generated e.m.f. in Problem 3 if the armature is wave-wound.

Generated e.m.f.

$$E = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2} \text{ (since } c = 2 \text{ for wave-wound)}$$
$$= p\Phi nZ = (4)(\Phi nZ)$$
$$= (4)(300) \text{ from Problem 3}$$
$$= \textbf{1200 volts}$$

Problem 5. A d.c. shunt-wound generator running at constant speed generates a voltage of 150 V at a certain value of field current. Determine the change in the generated voltage when the field current is reduced by 20 per cent, assuming the flux is proportional to the field current.

The generated e.m.f. E of a generator is proportional to $\Phi\omega$, i.e. is proportional to Φn, where Φ is the flux and n is the speed of rotation. It follows that $E = k\Phi n$, where k is a constant.

At speed n_1 and flux Φ_1, $E_1 = k\Phi_1 n_1$

At speed n_2 and flux Φ_2, $E_2 = k\Phi_2 n_2$

Thus, by division:

$$\frac{E_1}{E_2} = \frac{k\Phi_1 n_1}{k\Phi_2 n_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$$

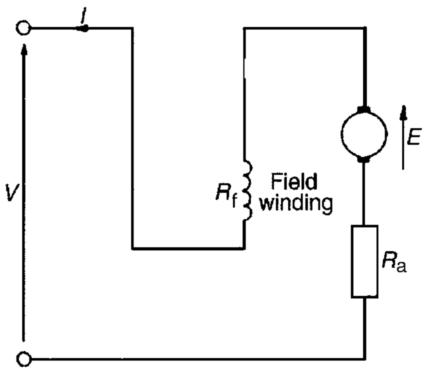


Figure 22.10

Characteristic

The load characteristic is the terminal voltage/current characteristic. The generated e.m.f. E, is proportional to $\Phi\omega$ and at constant speed $\omega(=2\pi n)$ is a constant. Thus E is proportional to Φ. For values of current below magnetic saturation of the yoke, poles, air gaps and armature core, the flux Φ is proportional to the current, hence $E \propto I$. For values of current above those required for magnetic saturation, the generated e.m.f. is approximately constant. The values of field resistance and armature resistance in a series wound machine are small, hence the terminal voltage V is very nearly equal to E. A typical load characteristic for a series generator is shown in Fig. 22.11.

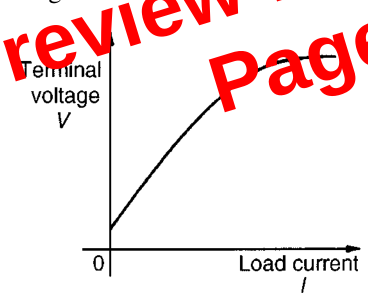


Figure 22.11

In a series-wound generator, the field winding is in series with the armature and it is not possible to have a value of field current when the terminals are open circuited, thus it is not possible to obtain an open-circuit characteristic.

Series-wound generators are rarely used in practise, but can be used as a 'booster' on d.c. transmission lines.

(d) Compound-wound generator

In the compound-wound generator two methods of connection are used, both having a mixture of shunt and series windings, designed to combine the advantages of each. Fig. 22.12(a) shows what is termed a **long-shunt** compound generator, and Fig. 22.12(b) shows a **short-shunt** compound generator. The latter is the most generally used form of d.c. generator.

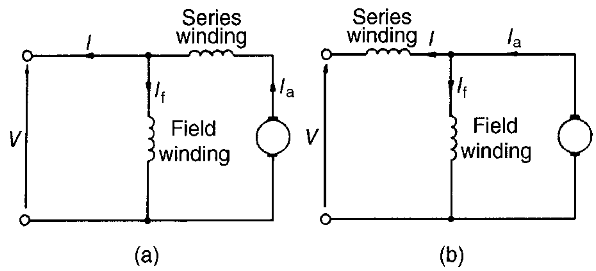


Figure 22.12

Problem 11. A short-shunt compound generator supplies 80 A at 200 V. If the field resistance, $R_f = 40\ \Omega$, the series resistance, $R_{Se} = 0.02\ \Omega$ and the armature resistance, $R_a = 0.04\ \Omega$, determine the e.m.f. generated.

The circuit is shown in Fig. 22.13.

Volt drop in series winding $= IR_{Se} = (80)(0.02)$
$= 1.6\ \text{V}$.

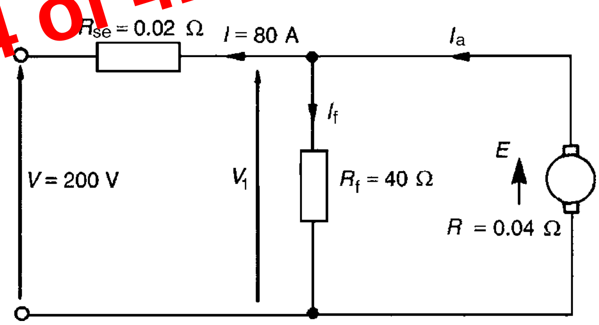


Figure 22.13

P.d. across the field winding = p.d. across armature $= V_1 = 200 + 1.6 = 201.6\ \text{V}$

$$\text{Field current } I_f = \frac{V_1}{R_f} = \frac{201.6}{40} = 5.04\ \text{A}$$

Armature current, $I_a = I + I_f = 80 + 5.04 = 85.04\ \text{A}$

$$\textbf{Generated e.m.f., } E = V_1 + I_a R_a$$
$$= 201.6 + (85.04)(0.04)$$
$$= 201.6 + 3.4016$$
$$= \mathbf{205\ volts}$$

For a shunt motor, V, Φ and R_a are constants, hence as armature current I_a increases, I_aR_a increases and $V - I_aR_a$ decreases, and the speed is proportional to a quantity which is decreasing and is as shown in Fig. 22.18 As the load on the shaft of the motor increases, I_a increases and the speed drops slightly. In practice, the speed falls by about 10 per cent between no-load and full-load on many d.c. shunt-wound motors. Due to this relatively small drop in speed, the d.c. shunt-wound motor is taken as basically being a constant-speed machine and may be used for driving lathes, lines of shafts, fans, conveyor belts, pumps, compressors, drilling machines and so on.

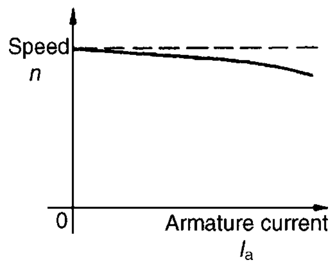


Figure 22.18

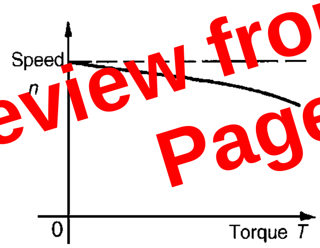


Figure 22.19

(iii) Since torque is proportional to armature current, (see (i) above), the theoretical speed/torque characteristic is as shown in Fig. 22.19.

Problem 21. A 200 V, d.c. shunt-wound motor has an armature resistance of 0.4 Ω and at a certain load has an armature current of 30 A and runs at 1350 rev/min. If the load on the shaft of the motor is increased so that the armature current increases to 45 A, determine the speed of the motor, assuming the flux remains constant.

The relationship $E \propto \Phi n$ applies to both generators and motors. For a motor, $E = V - I_aR_a$, (see equation (5))

Hence $E_1 = 200 - 30 \times 0.4 = 188\,\text{V}$

and $E_2 = 200 - 45 \times 0.4 = 182\,\text{V}$

The relationship

$$\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$$

applies to both generators and motors. Since the flux is constant, $\Phi_1 = \Phi_2$. Hence

$$\frac{188}{182} = \frac{\Phi_1 \times \left(\frac{1350}{60}\right)}{\Phi_1 \times n_2}$$

i.e. $$n_2 = \frac{22.5 \times 182}{188} = 21.78\,\text{rev/s}$$

Thus **the speed of the motor when the armature current is 45 A** is 21.78 × 60 rev/min i.e. **1307 rev/min**.

Problem 22. A 220 V, d.c. shunt-wound motor runs at 800 rev/min and the armature current is 30 A. The armature circuit resistance is 0.4 Ω. Determine (a) the maximum value of armature current if the flux is suddenly reduced by 10 per cent and (b) the steady state value of the armature current at the new value of flux, assuming the shaft torque of the motor remains constant.

(a) For a d.c. shunt-wound motor, $E = V - I_aR_a$. Hence initial generated e.m.f., $E_1 = 220 = 30 \times 0.4 = 208\,\text{V}$. The generated e.m.f. is also such that $E \propto \Phi n$, so at the instant the flux is reduced, the speed has not had time to change, and $E = 208 \times 90/100 - 187.2\,\text{V}$ Hence, the voltage drop due to the armature resistance is 220 − 187.2 i.e. 32.8 V. The **instantaneous value of the current** = 32.8/0.4 = **82 A**. This increase in current is about three times the initial value and causes an increase in torque, ($T \propto \Phi I_a$). The motor accelerates because of the larger torque value until steady state conditions are reached.

(b) $T \propto \Phi I_a$ and, since the torque is constant, $\Phi_1 I_{a1} = \Phi_2 I_{a2}$. The flux Φ is reduced by 10 per cent, hence $\Phi_2 = 0.9\Phi_1$. Thus, $\Phi_1 \times 30 = 0.9\Phi_1 \times I_{a2}$ i.e. the steady state value of armature current, $I_{a2} = 30/0.9 =$ **33.33 A**.

(b) Series-wound motor

In the series-wound motor the field winding is in series with the armature across the supply as shown in Fig. 22.20.

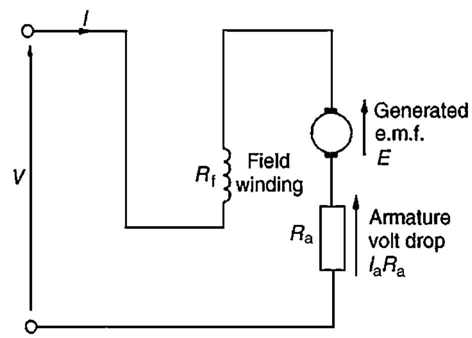


Figure 22.20

For the series motor shown in Fig. 22.20,

Supply voltage $V = E + I(R_a + R_f)$

or generated e.m.f. $E = V - I(R_a + R_f)$

Characteristics

In a series motor, the armature current flows in the field winding and is equal to the supply current, I.

(i) **The torque/current characteristic**
It is shown in Section 22.11 that torque $T \propto \Phi I_a$. Since the armature and field currents are the same current, I, in a series machine, then $T \propto \Phi I$ over a limited range, before magnetic saturation of the magnetic circuit of the motor is reached, (i.e. the linear portion of the B–H curve for the yoke, poles, air gap, brushes and armature in series). Thus $\Phi \propto I$ and $T \propto I^2$. After magnetic saturation, Φ almost becomes a constant and $T \propto I$. Thus the theoretical torque/current characteristic is as shown in Fig. 22.21.

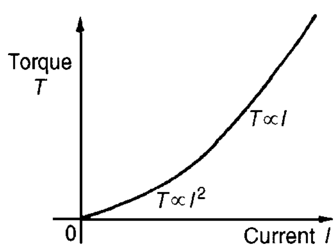


Figure 22.21

(ii) **The speed/current characteristic**
It is shown in equation (9) that

$$n \propto \frac{V - I_aR_a}{\Phi}$$

In a series motor, $I_a = I$ and below the magnetic saturation level, $\Phi \propto I$. Thus $n \propto (V - IR)/I$ where R is the combined resistance of the series field and armature circuit. Since IR is small compared with V, then an approximate relationship for the speed is $n \propto V/I \propto 1/I$ since V is constant. Hence the theoretical speed/current characteristic is as shown in Fig. 22.22. The high speed at small values of current indicate that this type of motor must not be run on very light loads and invariably, such motors are permanently coupled to their loads.

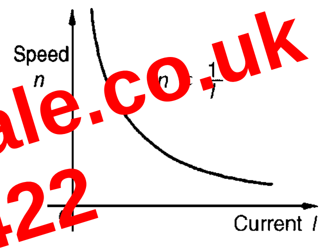


Figure 22.22

(iii) The theoretical **speed/torque characteristic** may be derived from (i) and (ii) above by obtaining the torque and speed for various values of current and plotting the co-ordinates on the speed/torque characteristics. A typical speed/torque characteristic is shown in Fig. 22.23.

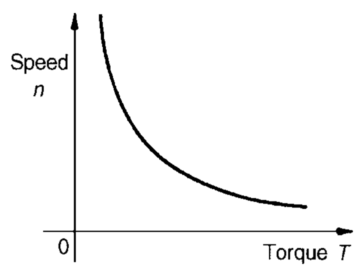


Figure 22.23

A d.c. series motor takes a large current on starting and the characteristic shown in Fig. 22.21 shows that the series-wound motor has a large torque when the current is large. Hence these its efficiency at this load is 88 per cent, find the current taken from the supply. [30.94 A]

6. In a test on a d.c. motor, the following data was obtained. Supply voltage: 500 V, current taken from the supply: 42.4 A, speed: 850 rev/min, shaft torque: 187 Nm. Determine the efficiency of the motor correct to the nearest 0.5 per cent. [78.5 per cent]

7. A 300 V series motor draws a current of 50 A. The field resistance is 40 mΩ and the armature resistance is 0.2 Ω. Determine the maximum efficiency of the motor. [92 per cent]

8. A series motor drives a load at 1500 rev/min and takes a current of 20 A when the supply voltage is 250 V. If the total resistance of the motor is 1.5 Ω and the iron, friction and windage losses amount to 400 W, determine the efficiency of the motor. [80 per cent]

9. A series-wound motor is connected to a d.c. supply and develops full-load torque when the current is 30 A and speed is 1000 rev/min. If the flux per pole is proportional to the current flowing, find the current and speed at half full-load torque, when connected to the same supply. [21.2 A, 1415 rev/min]

22.14 D.C. motor starter

If a d.c. motor whose armature is stationary is switched directly to its supply voltage, it is likely that the fuses protecting the motor will burn out. This is because the armature resistance is small, frequently being less than one ohm. Thus, additional resistance must be added to the armature circuit at the instant of closing the switch to start the motor.

As the speed of the motor increases, the armature conductors are cutting flux and a generated voltage, acting in opposition to the applied voltage, is produced, which limits the flow of armature current. Thus the value of the additional armature resistance can then be reduced.

When at normal running speed, the generated e.m.f. is such that no additional resistance is required in the armature circuit. To achieve this varying resistance in the armature circuit on starting, a d.c. motor starter is used, as shown in Fig. 22.28.

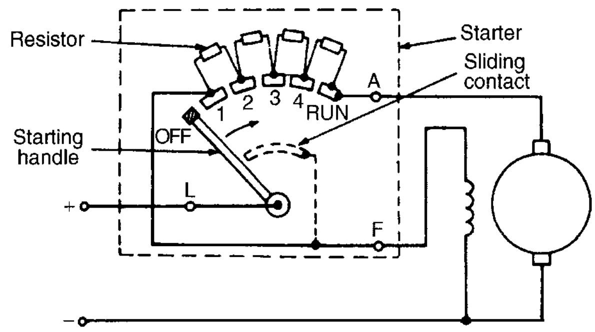


Figure 22.28

The starting handle is moved **slowly** in a clockwise direction to start the motor. For a shunt-wound motor, the field winding is connected to stud 1 or to L via a sliding contact on the starting handle, to give maximum field current, hence maximum flux, hence maximum torque on starting, since $T \propto \Phi I_a$. A similar arrangement without the field connection is used for series motors.

22.15 Speed control of d.c. motors

Shunt-wound motors

The speed of a shunt-wound d.c. motor, n, is proportional to

$$\frac{V - I_a R_a}{\Phi}$$

(see equation (9)). The speed is varied either by varying the value of flux, Φ, or by varying the value of R_a. The former is achieved by using a variable resistor in series with the field winding, as shown in Fig. 22.29(a) and such a resistor is called the **shunt field regulator**.

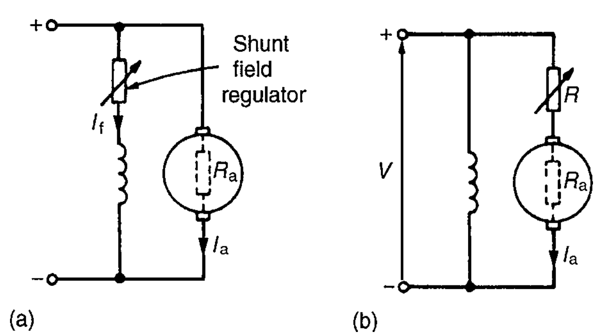


Figure 22.29

As the value of resistance of the shunt field regulator is increased, the value of the field current, I_f, is decreased.

12. Sketch a typical open-circuit characteristic for (a) a separately excited generator (b) a shunt generator (c) a series generator
13. Sketch a typical load characteristic for (a) a separately excited generator (b) a shunt generator
14. State one application for (a) a shunt generator (b) a series generator (c) a compound generator
15. State the principle losses in d.c. machines
16. The efficiency of a d.c. machine is given by the ratio (. . . . . .) per cent
17. The equation relating the generated e.m.f., E, terminal voltage, armature current and armature resistance for a d.c. motor is $E =$ . . . . . .
18. The torque T of a d.c. motor is given by $T = p\Phi Z I_a/\pi c$ newton metres. State what p, Φ, Z, I and c represent
19. Complete the following. In a d.c. machine
    (a) generated e.m.f. $\propto$ . . . . . . $\times$ . . . . . .
    (b) torque $\propto$ . . . . . . $\times$ . . . . . .
20. Sketch typical characteristics of torque/armature current for
    (a) a shunt motor
    (b) a series motor
    (c) a compound motor
21. Sketch typical speed/torque characteristics for a shunt and series motor
22. State two applications for each of the following motors:
    (a) shunt (b) series (c) compound

    In questions 23 to 26, an electrical machine runs at n rev/s, has a shaft torque of T, and takes a current of I from a supply voltage V
23. The power input to a generator is . . . . . . watts
24. The power input to a motor is . . . . . . watts
25. The power output from a generator is . . . . . . watts
26. The power output from a motor is . . . . . . watts
27. The generated e.m.f. of a d.c machine is proportional to . . . . . . volts
28. The torque produced by a d.c. motor is proportional to . . . . . . Nm
29. A starter is necessary for a d.c. motor because the generated e.m.f. is . . . . . . at low speeds
30. The speed of a d.c. shunt-wound motor will . . . . . . if the value of resistance of the shunt field regulator is increased
31. The speed of a d.c. motor will . . . . . . if the value of resistance in the armature circuit is increased
32. The value of the speed of a d.c. shunt-wound motor . . . . . . as the value of the armature current increases
33. At a large value of torque, the speed of a d.c. series-wound motor is . . . . . .
34. At a large value of field current, the generated e.m.f. of a d.c. shunt-wound generator is approximately . . . . . .
35. In a series-wound generator, the terminal voltage increases as the load current . . . . . .
36. One type of d.c. motor uses resistance in series with the field winding to obtain speed variations and another type uses resistance in parallel with the field winding for the same purpose. Explain briefly why these two distinct methods are used and why the field current plays a significant part in controlling the speed of a d.c. motor.
37. Name three types of motor enclosure

Exercise 138 Multi-choice questions on d.c. machines
(Answers on page 399)

1. Which of the following statements is false?
   (a) A d.c. motor converts electrical energy to mechanical energy
   (b) The efficiency of a d.c. motor is the ratio input power to output power
   (c) A d.c. generator converts mechanical power to electrical power
   (d) The efficiency of a d.c. generator is the ratio output power to input power

A shunt-wound d.c. machine is running at n rev/s and has a shaft torque of T Nm. The supply current is IA when connected to d.c. bus-bars of voltage V volts. The armature resistance of the machine is R_a ohms, the armature current is I_aA and the generated voltage is E volts. Use this data to find the formulae of the quantities stated in questions 2 to 9, selecting the correct answer from the following list:

(a) $V - I_aR_a$ (b) $E + I_aR_a$
(c) VI (d) $E - I_aR_a$
(e) $T(2\pi n)$ (f) $V + I_aR_a$

2. The input power when running as a generator
3. The output power when running as a motor
4. The input power when running as a motor
5. The output power when running as a generator
6. The generated voltage when running as a motor
7. The terminal voltage when running as a generator
8. The generated voltage when running as a generator
9. The terminal voltage when running as a motor
10. Which of the following statements is false?
    (a) A commutator is necessary as part of a d.c. motor to keep the armature rotating in the same direction
    (b) A commutator is necessary as part of a d.c. generator to produce unidirectional voltage at the terminals of the generator
    (c) The field winding of a d.c. machine is housed in slots on the armature
    (d) The brushes of a d.c. machine are usually made of carbon and do not rotate with the armature
11. If the speed of a d.c. machine is doubled and the flux remains constant, the generated e.m.f. (a) remains the same (b) is doubled (c) is halved
12. If the flux per pole of a shunt-wound d.c. generator is increased, and all other variables are kept the same, the speed (a) decreases (b) stays the same (c) increases
13. If the flux per pole of a shunt-wound d.c. generator is halved, the generated e.m.f. at constant speed (a) is doubled (b) is halved (c) remains the same
14. In a series-wound generator running at constant speed, as the load current increases, the terminal voltage
    (a) increases (b) decreases
    (c) stays the same
15. Which of the following statements is false for a series-wound d.c. motor?
    (a) The speed decreases with increase of resistance in the armature circuit
    (b) The speed increases as the flux decreases
    (c) The speed can be controlled by a diverter
    (d) The speed can be controlled by a shunt field regulator
16. Which of the following statements is false?
    (a) A series-wound motor has a large starting torque
    (b) A shunt-wound motor must be permanently connected to its load
    (c) The speed of a series-wound motor drops considerably when load is applied
    (d) A shunt-wound motor is essentially a constant-speed machine
17. The speed of a d.c. motor may be increased by
    (a) increasing the armature current
    (b) decreasing the field current
    (c) decreasing the applied voltage
    (d) increasing the field current
18. The armature resistance of a d.c. motor is 0.5 Ω, the supply voltage is 200 V and the back e.m.f. is 196 V at full speed. The armature current is:
    (a) 4 A (b) 8 A
    (c) 400 A (d) 392 A
19. In d.c. generators iron losses are made up of:
    (a) hysteresis and friction losses
    (b) hysteresis, eddy current and brush contact losses
    (c) hysteresis and eddy current losses

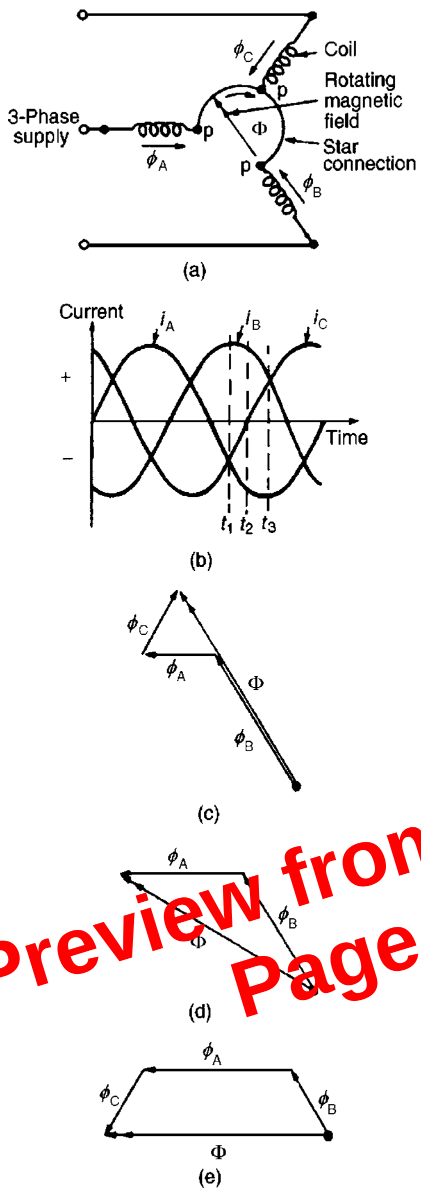


Figure 23.2

in Fig. 23.2(c). The resultant flux is the phasor sum of ϕ_A, ϕ_B and ϕ_C, shown as Φ in Fig. 23.2(c). At time t_2, the currents flowing are:

i_B, 0.866 × maximum positive value, i_C, zero, and i_A, 0.866 × maximum negative value.

The magnetic fluxes and the resultant magnetic flux are as shown in Fig. 23.2(d).

At time t_3,

i_B is 0.5 × maximum value and is positive

i_A is a maximum negative value, and

i_C is 0.5 × maximum value and is positive.

The magnetic fluxes and the resultant magnetic flux are as shown in Fig. 23.2(e)

Inspection of Fig. 23.2(c), (d) and (e) shows that the magnitude of the resultant magnetic flux, Φ, in each case is constant and is $1\frac{1}{2}$ × the maximum value of ϕ_A, ϕ_B or ϕ_C, but that its direction is changing. The process of determining the resultant flux may be repeated for all values of time and shows that the magnitude of the resultant flux is constant for all values of time and also that it rotates at constant speed, making one revolution for each cycle of the supply voltage.

23.3 Synchronous speed

The rotating magnetic field produced by three-phase windings could have been produced by rotating a permanent magnet's north and south pole at synchronous speed, (shown as N and S at the ends of the flux phasors in Fig. 23.1(b), (c) and (d)). For this reason, it is called a 2-pole system and an induction motor using three phase windings only is called a 2-pole induction motor. If six windings displaced from one another by 60° are used, as shown in Fig. 23.3(a), by drawing the current and resultant magnetic field diagrams at various time values, it may be shown that one cycle of the supply current to

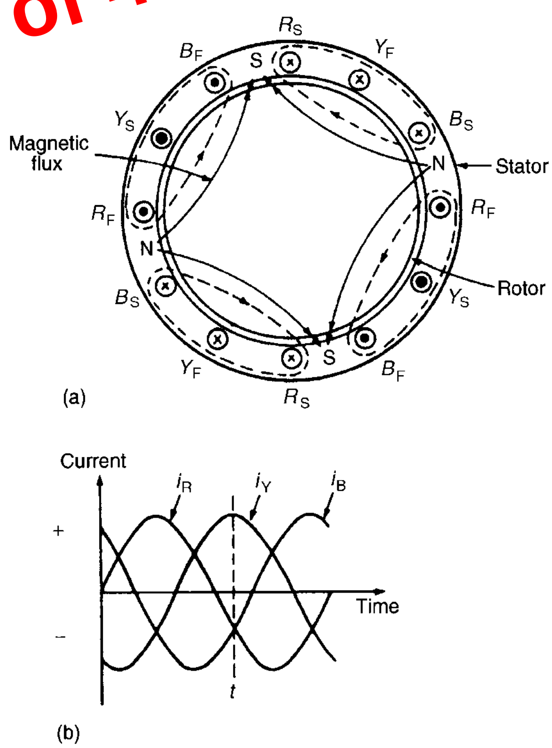


Figure 23.3

the stator windings causes the magnetic field to move through half a revolution. The current distribution in the stator windings are shown in Fig. 23.3(a), for the time t shown in Fig. 23.3(b).

It can be seen that for six windings on the stator, the magnetic flux produced is the same as that produced by rotating two permanent magnet north poles and two permanent magnet south poles at synchronous speed. This is called a 4-pole system and an induction motor using six phase windings is called a 4-pole induction motor. By increasing the number of phase windings the number of poles can be increased to any even number.

In general, if f is the frequency of the currents in the stator windings and the stator is wound to be equivalent to p **pairs** of poles, the speed of revolution of the rotating magnetic field, i.e. the synchronous speed, n_s is given by:

$$n_s = \frac{f}{p} \text{ rev/s}$$

Problem 1. A three-phase two-pole induction motor is connected to a 50 Hz supply. Determine the synchronous speed of the motor in rev/min.

From above, $n_s = (f/p)$ rev/s, where n_s is the synchronous speed, f is the frequency in hertz of the supply to the stator and p is the number of **pairs** of poles. Since the motor is connected to a 50 hertz supply, $f = 50$.

The motor has a two-pole system, hence p, the number of pairs of poles, is 1. Thus, synchronous speed, $\boldsymbol{n_s} = (50/1) = 50$ rev/s $= 50 \times 60$ rev/min $=$ **3000 rev/min**.

Problem 2. A stator winding supplied from a three-phase 60 Hz system is required to produce a magnetic flux rotating at 900 rev/min. Determine the number of poles.

Synchronous speed,

$$n_s = 900 \text{ rev/min} = \frac{900}{60} \text{ rev/s} = 15 \text{ rev/s}$$

Since

$$n_s = \left(\frac{f}{p}\right) \text{ then } p = \left(\frac{f}{n_s}\right) = \left(\frac{60}{15}\right) = 4$$

Hence **the number of pole pairs is 4** and thus **the number of poles is 8**

Problem 3. A three-phase 2-pole motor is to have a synchronous speed of 6000 rev/min. Calculate the frequency of the supply voltage.

Since $n_s = \left(\frac{f}{p}\right)$ then

$$\textbf{frequency, } f = (n_s)(p) = \left(\frac{6000}{60}\right)\left(\frac{2}{2}\right) = \textbf{100 Hz}$$

Now try the following exercise

Exercise 139 Further problems on synchronous speed

1. The synchronous speed of a 3-phase, 4-pole induction motor is 60 rev/s. Determine the frequency of the supply to the stator windings. [120 Hz]
2. The synchronous speed of a 3-phase induction motor is 25 rev/s and the frequency of the supply to the stator is 50 Hz. Calculate the equivalent number of pairs of poles of the motor. [2]
3. A 6-pole, 3-phase induction motor is connected to a 300 Hz supply. Determine the speed of rotation of the magnetic field produced by the stator. [100 rev/s]

23.4 Construction of a three-phase induction motor

The stator of a three-phase induction motor is the stationary part corresponding to the yoke of a d.c. machine. It is wound to give a 2-pole, 4-pole, 6-pole, ... rotating magnetic field, depending on the rotor speed required. The rotor, corresponding to the armature of a d.c. machine, is built up of laminated iron, to reduce eddy currents.

In the type most widely used, known as a **squirrel-cage rotor**, copper or aluminium bars are placed in slots cut in the laminated iron, the ends of the bars being welded or brazed into a heavy conducting ring, (see Fig. 23.4(a)). A cross-sectional view of a three-phase induction motor is shown in Fig. 23.4(b).