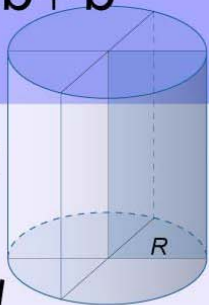
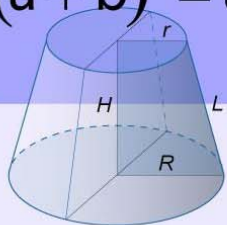
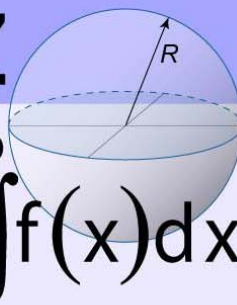


$$(a + b)^2 = a^2 + 2ab + b^2$$



$$\sqrt[n]{z}$$



$$y = \cos x$$

$$\Sigma$$

$$\int_a^b f(x) dx$$

1300

Math

Formulas

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Page 1 of 338

Alex Svirin, Ph.D.

60. Quotient in Polar Representation

$$\frac{z_1}{z_2} = \frac{r_1(\cos \varphi_1 + i \sin \varphi_1)}{r_2(\cos \varphi_2 + i \sin \varphi_2)} = \frac{r_1}{r_2} [\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)]$$

61. Power of a Complex Number

$$z^n = [r(\cos \varphi + i \sin \varphi)]^n = r^n [\cos(n\varphi) + i \sin(n\varphi)]$$

62. Formula “De Moivre”

$$(\cos \varphi + i \sin \varphi)^n = \cos(n\varphi) + i \sin(n\varphi)$$

63. Nth Root of a Complex Number

$$\sqrt[n]{z} = \sqrt[n]{r(\cos \varphi + i \sin \varphi)} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2\pi k}{n} + i \sin \frac{\varphi + 2\pi k}{n} \right)$$

where

$$k = 0, 1, 2, \dots, n-1.$$

64. Euler’s Formula

$$e^{ix} = \cos x + i \sin x$$

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Page 21 of 338

2.4 Roots

Bases: a, b Powers (rational numbers): n, m $a, b \geq 0$ for even roots ($n = 2k, k \in \mathbb{N}$)

$$91. \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$92. \quad \sqrt[n]{a} \sqrt[m]{b} = \sqrt[nm]{a^m b^n}$$

$$93. \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad b \neq 0$$

$$94. \quad \frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \frac{\sqrt[nm]{a^m}}{\sqrt[nm]{b^n}} = \sqrt[nm]{\frac{a^m}{b^n}}, \quad b \neq 0.$$

$$95. \quad \left(\sqrt[n]{a^m}\right)^n = \sqrt[n]{a^m}^n = a^m$$

$$96. \quad \left(\sqrt[n]{a}\right)^n = a$$

$$97. \quad \sqrt[n]{a^m} = \sqrt[np]{a^{mp}}$$

$$98. \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$99. \quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$100. \quad \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

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Page 25 of 338

165. $h^2 = fg$,
where h is the altitude from the right angle.

166. $m_a^2 = b^2 - \frac{a^2}{4}$, $m_b^2 = a^2 - \frac{b^2}{4}$,
where m_a and m_b are the medians to the legs a and b .

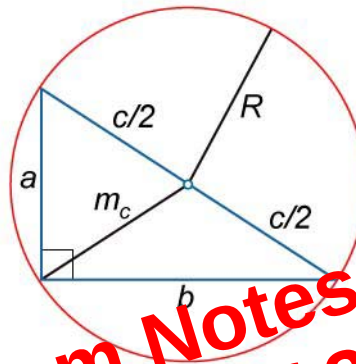


Figure 10.

167. $m_c = \frac{c}{2}$
where m_c is the median to the hypotenuse c .

168. $R = \frac{c}{2} = m_c$

169. $r = \frac{a + b - c}{2} = \frac{ab}{a + b + c}$

170. $ab = ch$

$$171. \quad S = \frac{ab}{2} = \frac{ch}{2}$$

3.2 Isosceles Triangle

Base: a

Legs: b

Base angle: β

Vertex angle: α

Altitude to the base: h

Perimeter: L

Area: S

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Page 37 of 338

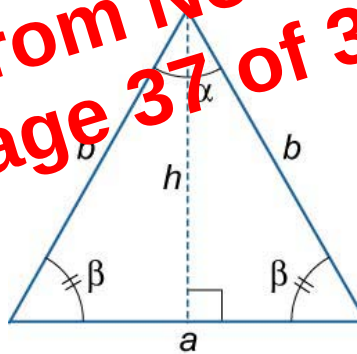


Figure 11.

$$172. \quad \beta = 90^\circ - \frac{\alpha}{2}$$

$$173. \quad h^2 = b^2 - \frac{a^2}{4}$$

207. $h = b \sin \alpha = b \sin \beta$

208. $L = 2(a + b)$

209. $S = ah = ab \sin \alpha$,
 $S = \frac{1}{2} d_1 d_2 \sin \varphi$.

3.8 Rhombus

Side of a rhombus: a

Diagonals: d_1, d_2

Consecutive angles: α, β

Altitude: H

Radius of inscribed circle: r

Perimeter: L

Area: S

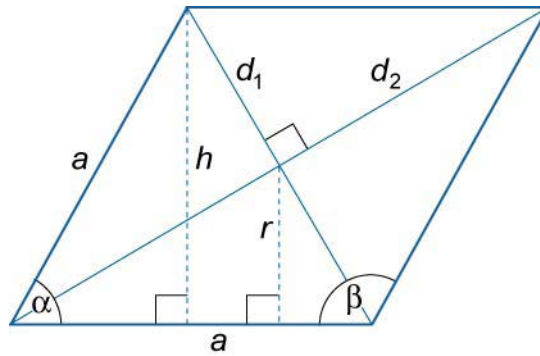


Figure 19.

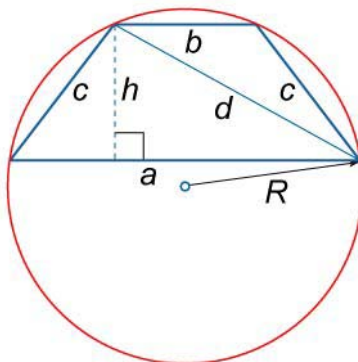


Figure 21.

$$218. \quad q = \frac{a+b}{2}$$

$$219. \quad d = \sqrt{ab + c^2}$$

$$220. \quad h = \sqrt{c^2 - \frac{1}{4}(b-a)^2}$$

$$221. \quad R = \frac{c\sqrt{ab + c^2}}{\sqrt{(2c - a + b)(2c + a - b)}}$$

$$222. \quad S = \frac{a+b}{2} \cdot h = qh$$

251. $R = a$

252. $L = 6a$

253. $S = pr = \frac{a^2 3\sqrt{3}}{2},$
 where $p = \frac{L}{2}.$

3.18 Regular Polygon

Side: a

Number of sides: n

Internal angle: α

Slant height: n

Radius of inscribed circle: r

Radius of circumscribed circle: R

Perimeter: L

Semiperimeter: p

Area: S

3.21 Segment of a Circle

Radius of a circle: R

Arc length: s

Chord: a

Central angle (in radians): x

Central angle (in degrees): α

Height of the segment: h

Perimeter: L

Area: S

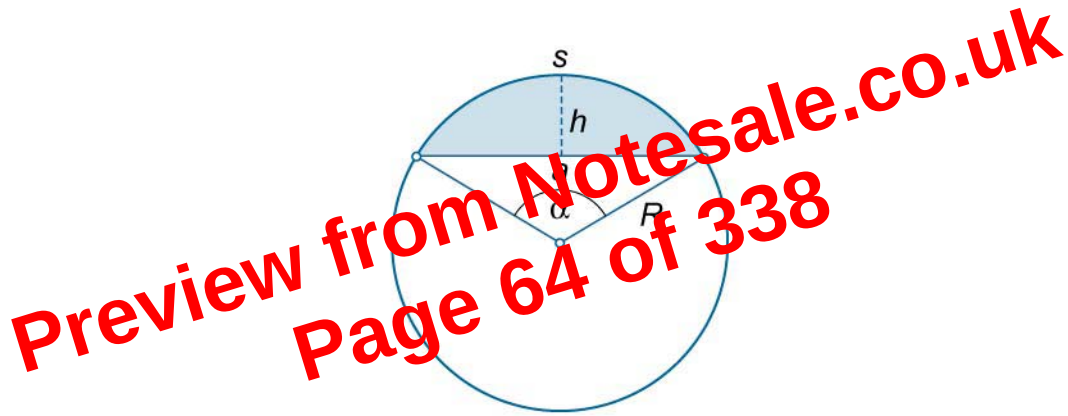


Figure 36.

$$271. \quad a = 2\sqrt{2hR - h^2}$$

$$272. \quad h = R - \frac{1}{2}\sqrt{4R^2 - a^2}, \quad h < R$$

$$273. \quad L = s + a$$

$$289. \quad S_B = \frac{\sqrt{3}a^2}{4}$$

$$290. \quad S = \sqrt{3}a^2$$

$$291. \quad V = \frac{1}{3}S_B h = \frac{a^3}{6\sqrt{2}}.$$

3.26 Regular Pyramid

Side of base: a

Lateral edge: b

Height: h

Slant height: m

Number of sides: n

Perimeter of base: p

Radius of inscribed sphere of base: r

Area of base: S_B

Lateral surface area: S_L

Total surface area: S

Volume: V

$$328. \quad H = \sqrt{m^2 - R^2}$$

$$329. \quad S_L = \pi R m = \frac{\pi m d}{2}$$

$$330. \quad S_B = \pi R^2$$

$$331. \quad S = S_L + S_B = \pi R(m + R) = \frac{1}{2} \pi d \left(m + \frac{d}{2} \right)$$

$$332. \quad V = \frac{1}{3} S_B H = \frac{1}{3} \pi R^2 H$$

3.33 Frustum of a Right Circular Cone

Radius of bases: R

Height: H

Slant height: m

Scale factor: k

Area of bases: S_1, S_2

Lateral surface area: S_L

Total surface area: S

Volume: V

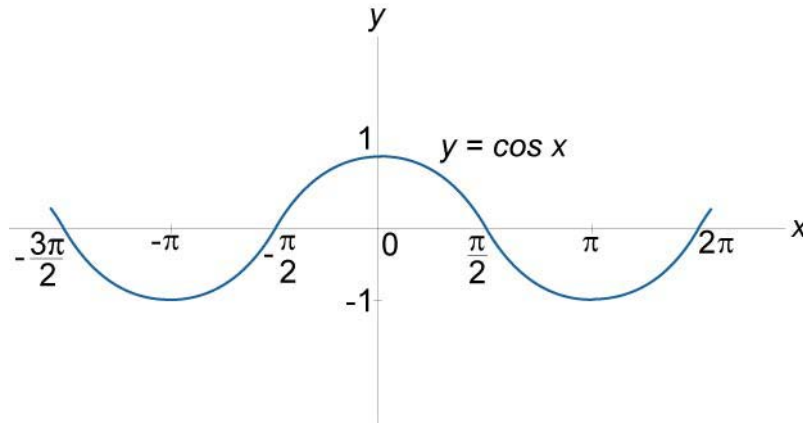


Figure 60.

375. Tangent Function

$$y = \tan x, \quad x \neq (2k+1)\frac{\pi}{2}, \quad -\infty < \tan x < \infty.$$

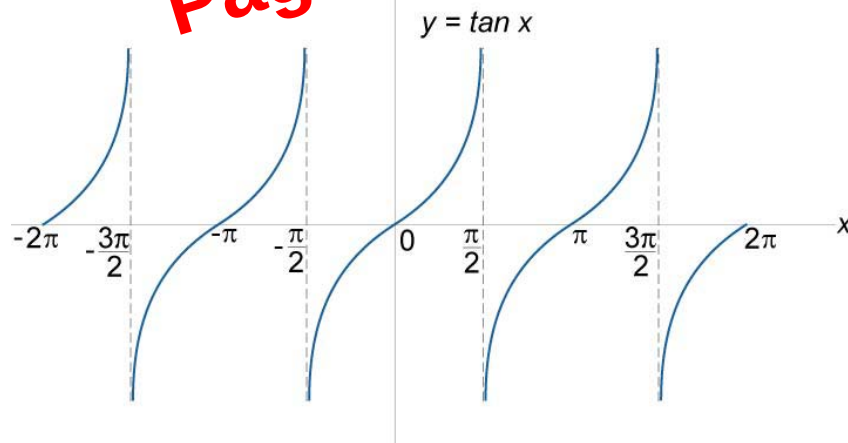


Figure 61.

4.3. Signs of Trigonometric Functions

379.

Quadrant	Sin α	Cos α	Tan α	Cot α	Sec α	Cosec α
I	+	+	+	+	+	+
II	+					+
III			+	+		
IV		+			+	

380.

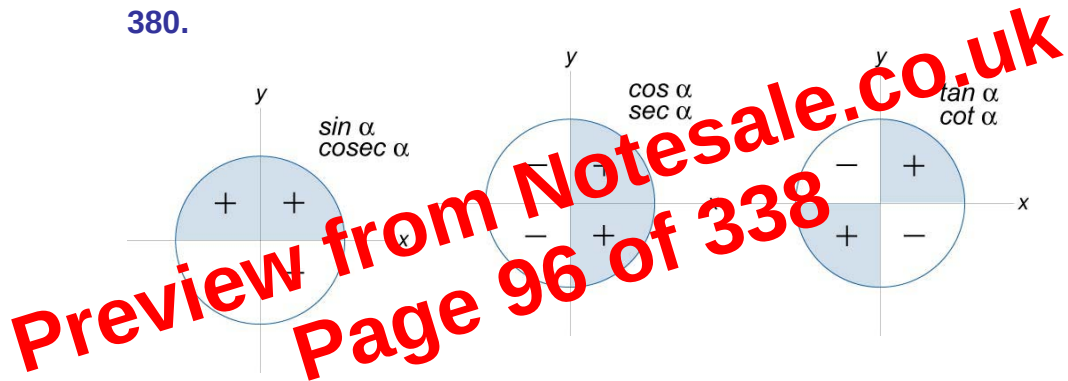


Figure 65.

$$425. \cot 5\alpha = \frac{1 - 10\tan^2 \alpha + 5\tan^4 \alpha}{\tan^5 \alpha - 10\tan^3 \alpha + 5\tan \alpha}$$

4.12 Half Angle Formulas

$$426. \sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$427. \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$428. \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha} = \csc \alpha - \cot \alpha$$

$$429. \cot \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha} = \csc \alpha + \cot \alpha$$

4.13 Half Angle Tangent Identities

$$430. \sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$448. \quad 1 + \sin \alpha = 2 \cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$449. \quad 1 - \sin \alpha = 2 \sin^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

4.15 Transforming of Trigonometric Expressions to Sum

$$450. \quad \sin \alpha \cdot \sin \beta = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$451. \quad \cos \alpha \cdot \cos \beta = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$$

$$452. \quad \sin \alpha \cdot \cos \beta = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2}$$

$$453. \quad \tan \alpha \cdot \tan \beta = \frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta}$$

$$454. \quad \cot \alpha \cdot \cot \beta = \frac{\cot \alpha + \cot \beta}{\tan \alpha + \tan \beta}$$

$$455. \quad \tan \alpha \cdot \cot \beta = \frac{\tan \alpha + \cot \beta}{\cot \alpha + \tan \beta}$$

4.16 Powers of Trigonometric Functions

$$456. \quad \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$457. \quad \sin^3 \alpha = \frac{3 \sin \alpha - \sin 3\alpha}{4}$$

$$458. \quad \sin^4 \alpha = \frac{\cos 4\alpha - 4 \cos 2\alpha + 3}{8}$$

$$459. \quad \sin^5 \alpha = \frac{10 \sin \alpha - 5 \sin 3\alpha + \sin 5\alpha}{16}$$

$$460. \quad \sin^6 \alpha = \frac{10 - 15 \cos 2\alpha + 6 \cos 4\alpha - \cos 6\alpha}{32}$$

$$461. \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$462. \quad \cos^3 \alpha = \frac{3 \cos \alpha + \cos 3\alpha}{4}$$

$$463. \quad \cos^4 \alpha = \frac{\cos 4\alpha + 4 \cos 2\alpha + 3}{8}$$

$$464. \quad \cos^5 \alpha = \frac{10 \cos \alpha + 5 \sin 3\alpha + \cos 5\alpha}{16}$$

$$465. \quad \cos^6 \alpha = \frac{10 + 15 \cos 2\alpha + 6 \cos 4\alpha + \cos 6\alpha}{32}$$

469. Inverse Cotangent Function

$$y = \operatorname{arccot} x, \quad -\infty \leq x \leq \infty, \quad 0 < \operatorname{arccot} x < \pi.$$

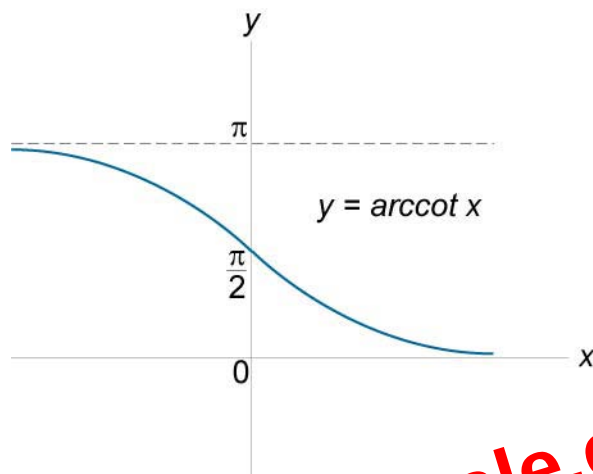


Figure 69.

470. Inverse Secant Function

$$y = \operatorname{arcsec} x, \quad x \in (-\infty, -1] \cup [1, \infty), \quad \operatorname{arcsec} x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right].$$

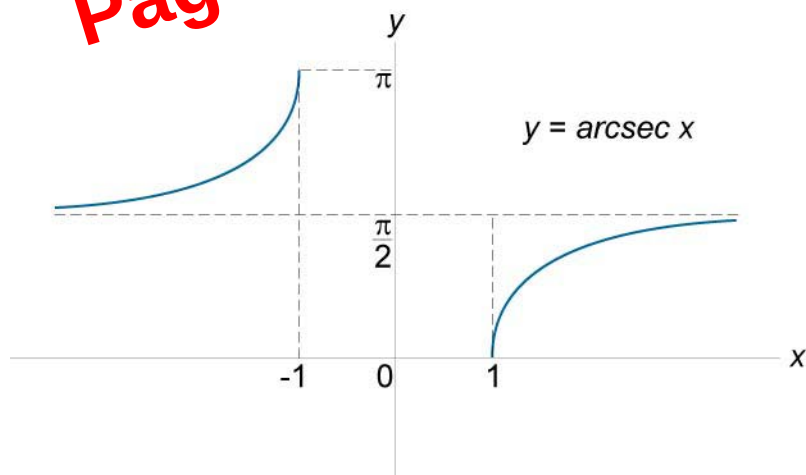


Figure 70.

471. Inverse Cosecant Function

$$y = \operatorname{arccsc} x, \quad x \in (-\infty, -1] \cup [1, \infty), \quad \operatorname{arccsc} x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right].$$

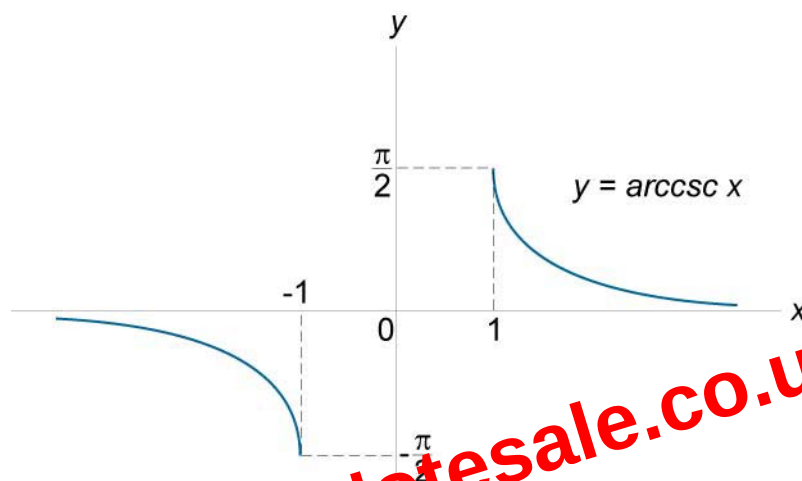


Figure 71.

4.18 Principal Values of Inverse Trigonometric Functions

472.

x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arcsin x$	0°	30°	45°	60°	90°
$\arccos x$	90°	60°	45°	30°	0°
x	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	
$\arcsin x$	-30°	-45°	-60°	-90°	
$\arccos x$	120°	135°	150°	180°	

$$493. \arctan x = \frac{\pi}{2} - \arctan \frac{1}{x}, x > 0.$$

$$494. \arctan x = -\frac{\pi}{2} - \arctan \frac{1}{x}, x < 0.$$

$$495. \arctan x = \operatorname{arccot} \frac{1}{x}, x > 0.$$

$$496. \arctan x = \operatorname{arccot} \frac{1}{x} - \pi, x < 0.$$

$$497. \operatorname{arccot}(-x) = \pi - \operatorname{arccot} x$$

$$498. \operatorname{arccot} x = \frac{\pi}{2} - \arctan x$$

$$499. \operatorname{arccot} x = \arcsin \frac{1}{\sqrt{1+x^2}}, x > 0.$$

$$500. \operatorname{arccot} x = \pi - \arcsin \frac{1}{\sqrt{1+x^2}}, x < 0.$$

$$501. \operatorname{arccot} x = \arccos \frac{x}{\sqrt{1+x^2}}$$

$$502. \operatorname{arccot} x = \arctan \frac{1}{x}, x > 0.$$

$$503. \operatorname{arccot} x = \pi + \arctan \frac{1}{x}, x < 0.$$

609. Midpoint of a Line Segment

$$x_0 = \frac{x_1 + x_2}{2}, \lambda = 1.$$

7.2 Two-Dimensional Coordinate System

Point coordinates: $x_0, x_1, x_2, y_0, y_1, y_2$

Polar coordinates: r, φ

Real number: λ

Positive real numbers: $a, b, c,$

Distance between two points: d

Area: S

610. Distance Between Two Points

$$d = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

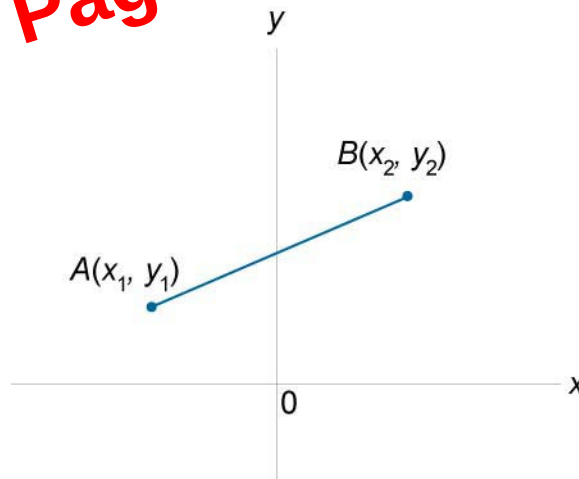


Figure 88.

7.3 Straight Line in Plane

Point coordinates: $X, Y, x, x_0, x_1, y_0, y_1, a_1, a_2, \dots$

Real numbers: $k, a, b, p, t, A, B, C, A_1, A_2, \dots$

Angles: α, β

Angle between two lines: φ

Normal vector: $\vec{n}$

Position vectors: $\vec{r}, \vec{a}, \vec{b}$

622. General Equation of a Straight Line

$$Ax + By + C = 0$$

623. Normal Vector to a Straight Line

The vector $\vec{n}(A, B)$ is normal to the line $Ax + By + C = 0$.

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Page 149 of 338

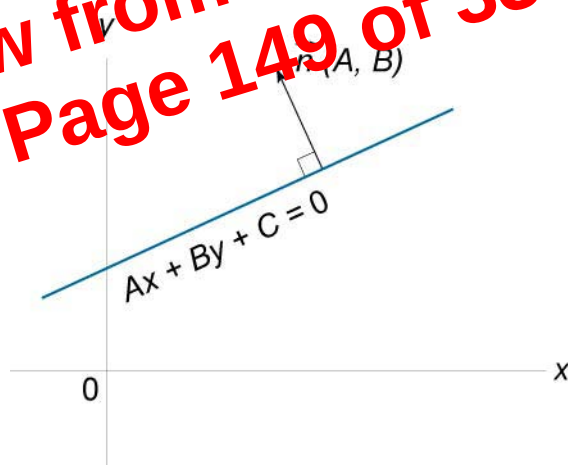


Figure 98.

624. Explicit Equation of a Straight Line (Slope-Intercept Form)

$$y = kx + b.$$

The gradient of the line is $k = \tan \alpha$.

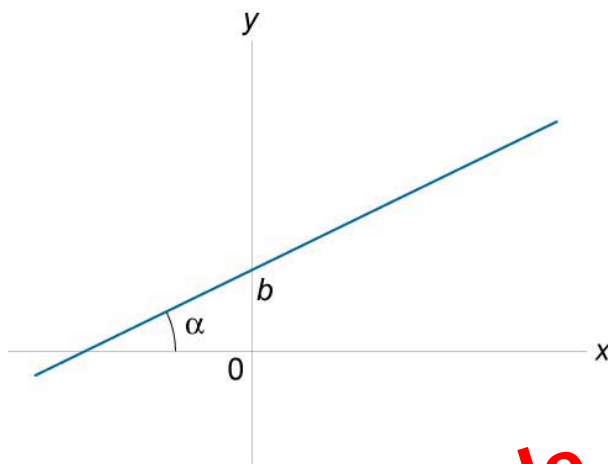


Figure 99.

625. Gradient of a Line

$$k = \tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

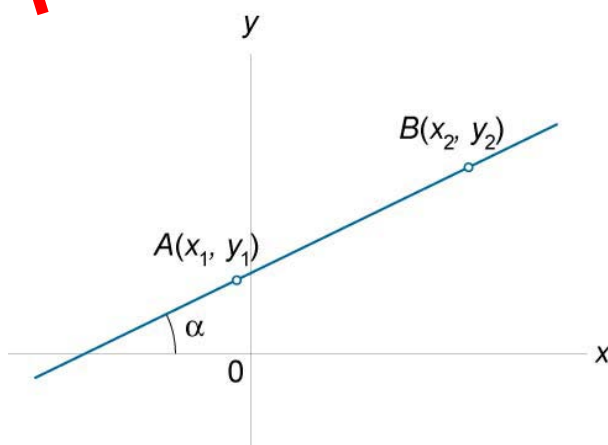


Figure 100.

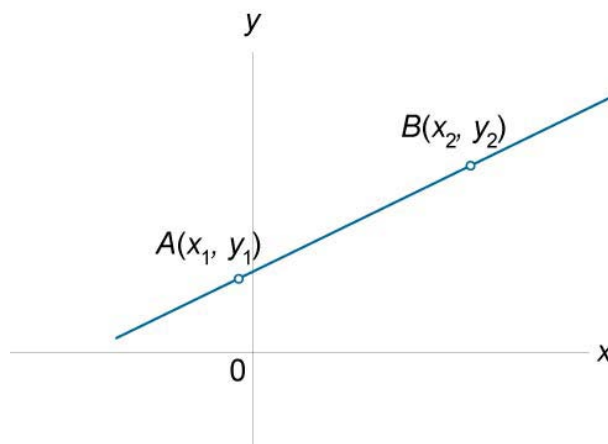


Figure 102.

628. Intercept Form

$$\frac{x}{a} + \frac{y}{b} = 1$$

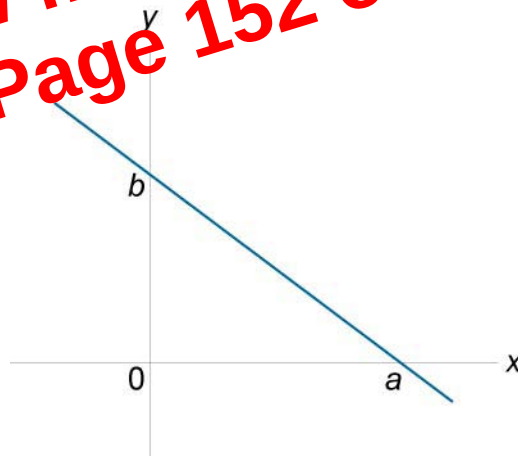


Figure 103.

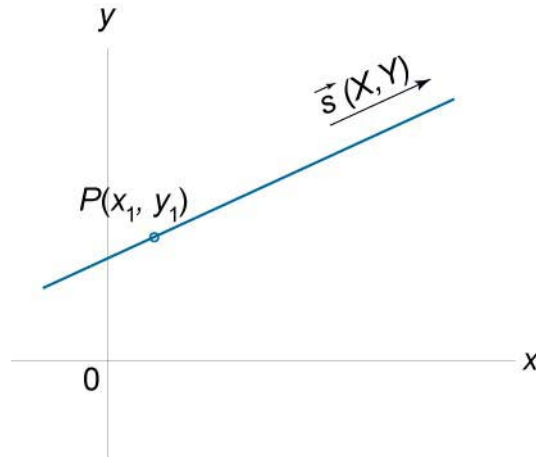


Figure 105.

631. Vertical Line
 $x = a$

632. Horizontal Line
 $y = b$

633. Vector Equation of a Straight Line

$$\vec{r} = \vec{a} + t\vec{b},$$

where

O is the origin of the coordinates,

X is any variable point on the line,

$\vec{a}$ is the position vector of a known point A on the line,

$\vec{b}$ is a known vector of direction, parallel to the line,

t is a parameter,

$\vec{r} = \vec{OX}$ is the position vector of any point X on the line.

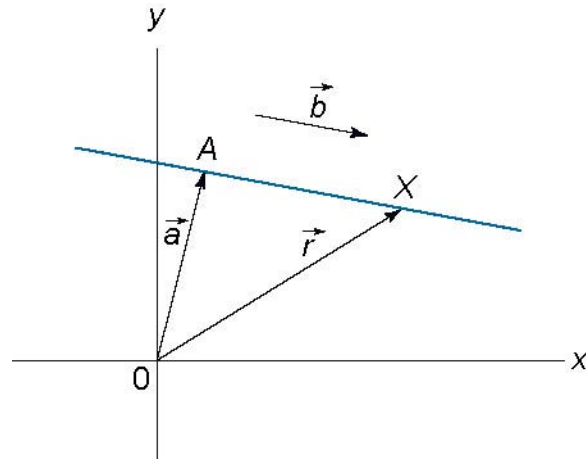


Figure 106.

634. Straight Line in Parametric Form

$$\begin{cases} x = a_1 + tb_1 \\ y = a_2 + tb_2 \end{cases}$$

where

(x, y) are the coordinates of any unknown point on the line,

(a_1, a_2) are the coordinates of a known point on the line,

(b_1, b_2) are the coordinates of a vector parallel to the line,

t is a parameter.

$$R = \sqrt{\frac{D^2 + E^2 - 4AF}{2|A|}}.$$

7.5 Ellipse

Semimajor axis: a

Semiminor axis: b

Foci: $F_1(-c, 0)$, $F_2(c, 0)$

Distance between the foci: $2c$

Eccentricity: e

Real numbers: A, B, C, D, E, F, t

Perimeter: L

Area: S

645 Equation of an Ellipse (Standard Form)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

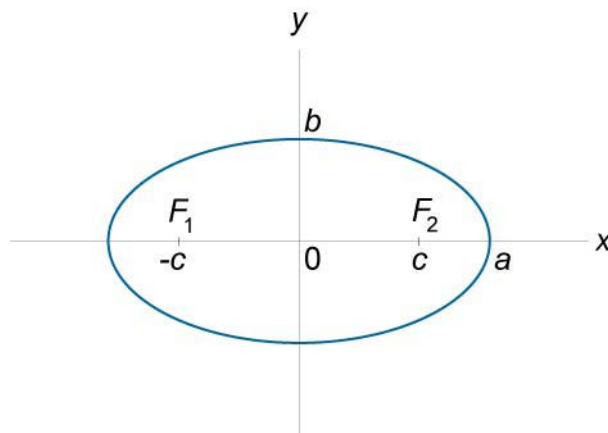


Figure 115.

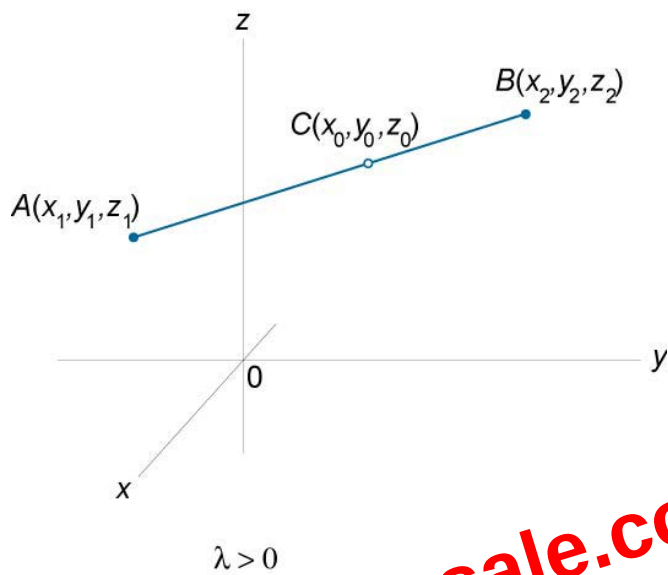


Figure 124.

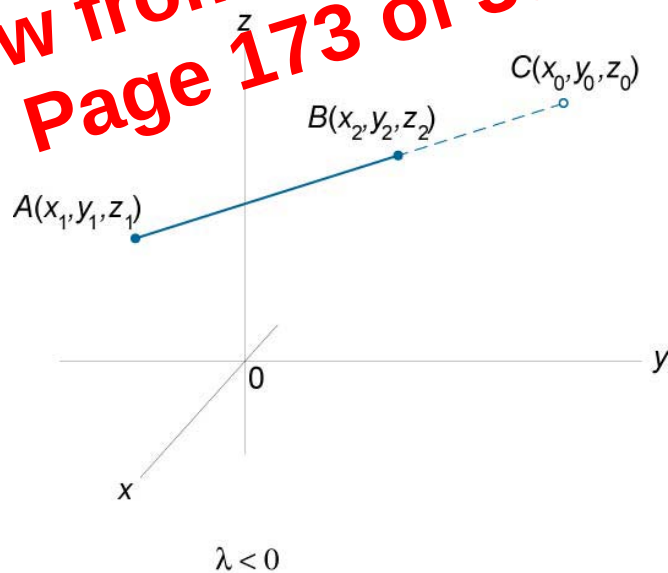


Figure 125.

672. Midpoint of a Line Segment

$$x_0 = \frac{x_1 + x_2}{2}, y_0 = \frac{y_1 + y_2}{2}, z_0 = \frac{z_1 + z_2}{2}, \lambda = 1.$$

673. Area of a Triangle

The area of a triangle with vertices $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$ is given by

$$S = \frac{1}{2} \sqrt{\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2}.$$

674. Volume of a Tetrahedron

The volume of a tetrahedron with vertices $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$, and $P_4(x_4, y_4, z_4)$ is given by

$$V = \pm \frac{1}{6} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

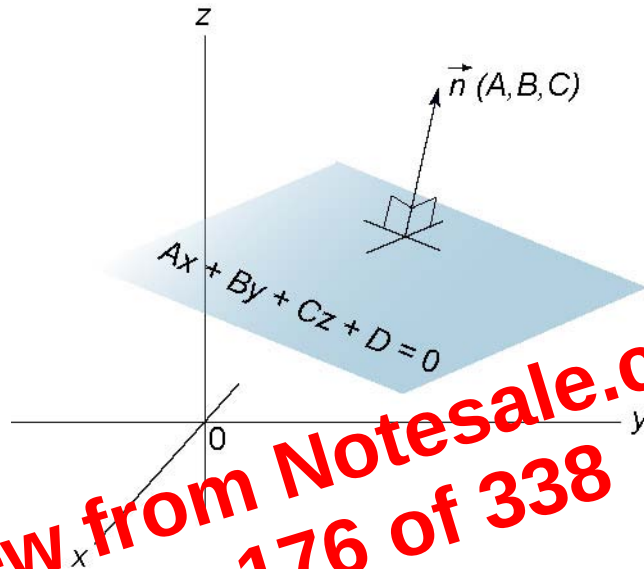
or

$$V = \pm \frac{1}{6} \begin{vmatrix} x_1 - x_4 & y_1 - y_4 & z_1 - z_4 \\ x_2 - x_4 & y_2 - y_4 & z_2 - z_4 \\ x_3 - x_4 & y_3 - y_4 & z_3 - z_4 \end{vmatrix}.$$

Note: We choose the sign (+) or (-) so that to get a positive answer for volume.

676. Normal Vector to a Plane

The vector $\vec{n}(A, B, C)$ is normal to the plane
 $Ax + By + Cz + D = 0$.



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 Page 176 of 338

Figure 127.

677. Particular Cases of the Equation of a Plane

$$Ax + By + Cz + D = 0$$

If $A = 0$, the plane is parallel to the x -axis.

If $B = 0$, the plane is parallel to the y -axis.

If $C = 0$, the plane is parallel to the z -axis.

If $D = 0$, the plane lies on the origin.

If $A = B = 0$, the plane is parallel to the xy -plane.

If $B = C = 0$, the plane is parallel to the yz -plane.

If $A = C = 0$, the plane is parallel to the xz -plane.

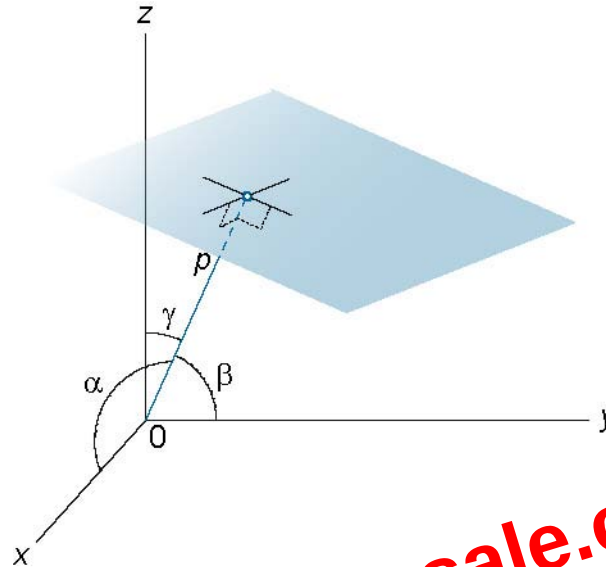


Figure 12.1.

682 Parametric Form

$$\begin{cases} x = x_1 + a_1s + a_2t, \\ y = y_1 + b_1s + b_2t, \\ z = z_1 + c_1s + c_2t \end{cases}$$

where (x, y, z) are the coordinates of any unknown point on the line, the point $P(x_1, y_1, z_1)$ lies in the plane, the vectors (a_1, b_1, c_1) and (a_2, b_2, c_2) are parallel to the plane.

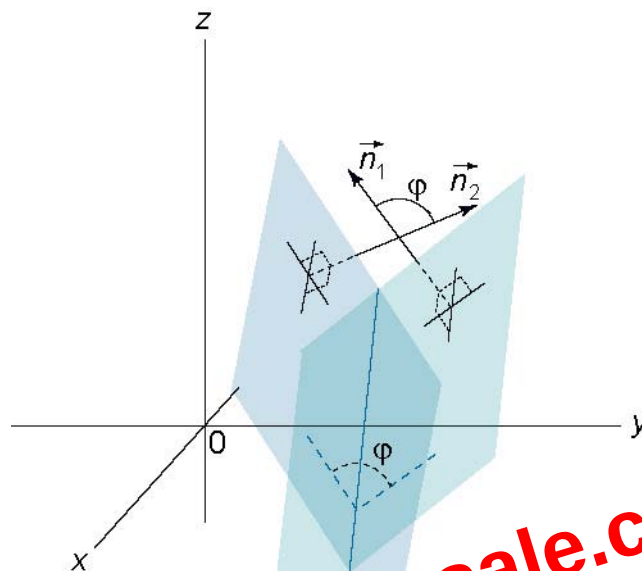


Figure 133.

684. Parallel Planes

Two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ are parallel if

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}.$$

685. Perpendicular Planes

Two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ are perpendicular if $A_1A_2 + B_1B_2 + C_1C_2 = 0$.

686. Equation of a Plane Through $P(x_1, y_1, z_1)$ and Parallel To the Vectors (a_1, b_1, c_1) and (a_2, b_2, c_2) (Fig.132)

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \text{ intersect if}$$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

697. Parallel Line and Plane

The straight line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ and the plane

$Ax + By + Cz + D = 0$ are parallel if

$$\vec{n} \cdot \vec{s} = 0,$$

or

$$Aa + Bb + Cc = 0.$$

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Page 189 of 338

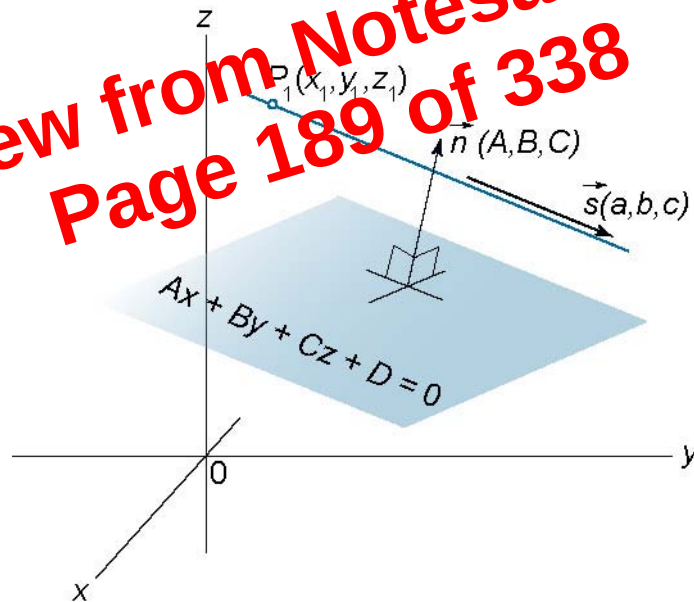


Figure 140.

699. General Quadratic Equation

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy + 2Px + 2Qy + 2Rz + D = 0$$

700. Classification of Quadric Surfaces

Case	Rank(e)	Rank(E)	Δ	k signs	Type of Surface
1	3	4	< 0	Same	Real Ellipsoid
2	3	4	> 0	Same	Imaginary Ellipsoid
3	3	4	> 0	Different	Hyperboloid of 1 Sheet
4	3	4	< 0	Different	Hyperboloid of 2 Sheets
5	3	3		Different	Real Quadric Cone
6	3	3		Same	Imaginary Quadric Cone
7	2	4	< 0	Same	Elliptic Paraboloid
8	2	4	> 0	Different	Hyperbolic Paraboloid
9	2	3		Same	Real Elliptic Cylinder
10	2	3		Same	Imaginary Elliptic Cylinder
11	2	3		Different	Hyperbolic Cylinder
12	2	2		Different	Real Intersecting Planes
13	2	2		Same	Imaginary Intersecting Planes
14	1	3			Parabolic Cylinder
15	1	2			Real Parallel Planes
16	1	2			Imaginary Parallel Planes
17	1	1			Coincident Planes

Here

$$e = \begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}, E = \begin{pmatrix} A & H & Q & P \\ H & B & F & Q \\ G & F & C & R \\ P & Q & R & D \end{pmatrix}, \Delta = \det(E),$$

k_1, k_2, k_3 are the roots of the equation,

$$\begin{vmatrix} A-x & H & G \\ H & B-x & F \\ G & F & C-x \end{vmatrix} = 0.$$

709. Real Elliptic Cylinder (Case 9)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

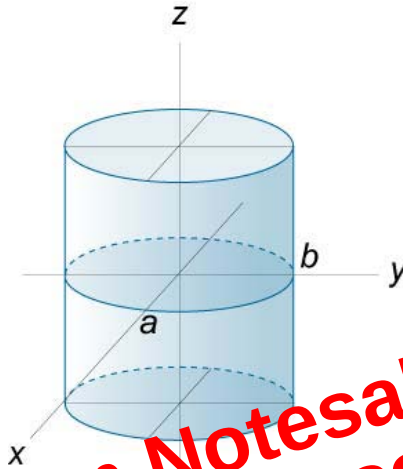


Figure 148.

710. Imaginary Elliptic Cylinder (Case 10)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$$

711. Hyperbolic Cylinder (Case 11)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

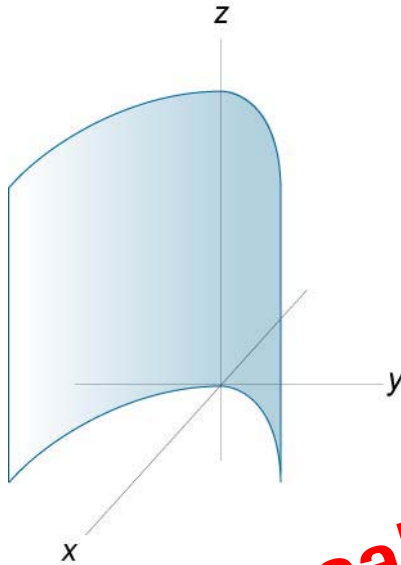


Figure 150.

715. Real Parallel Planes (Case 16)

$$\frac{x^2}{a^2} = 1$$

716. Imaginary Parallel Planes (Case 16)

$$\frac{x^2}{a^2} = -1$$

717. Coincident Planes (Case 17)

$$x^2 = 0$$

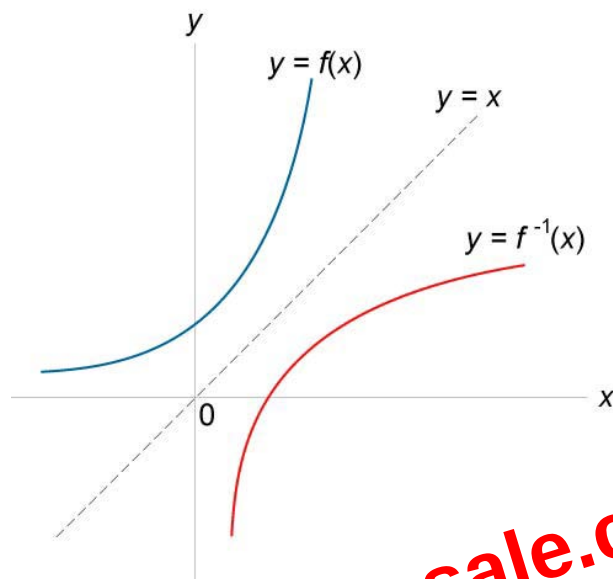


Figure 152.

727. Composite Function

$y = f(u)$, $u = g(x)$, $y = f(g(x))$ is a composite function.

728. Linear Function

$y = ax + b$, $x \in \mathbb{R}$, $a = \tan \alpha$ is the slope of the line, b is the y-intercept.

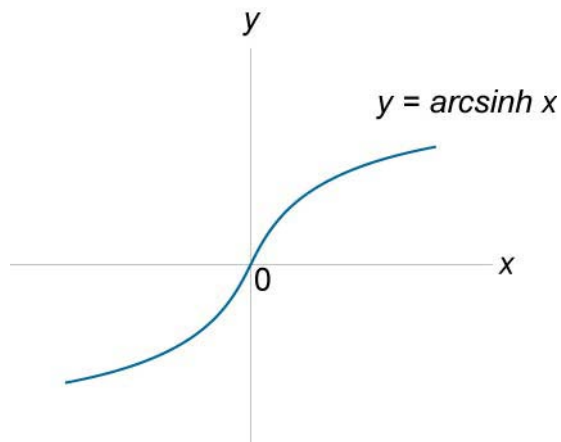


Figure 169.

- 744.** Inverse Hyperbolic Cosine Function
 $y = \operatorname{arccosh} x$, $x \in [1, \infty)$

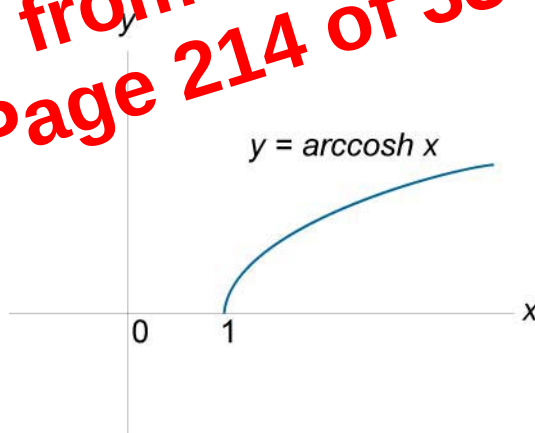


Figure 170.

- 745.** Inverse Hyperbolic Tangent Function
 $y = \operatorname{arctanh} x$, $x \in (-1, 1)$.

$$759. \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$$

$$760. \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$761. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$762. \lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k$$

$$763. \lim_{x \rightarrow 0} a^x = 1$$

8.3 Definition and Properties of the Derivative

Functions: f, g, y, u, v

Independent variable: x

Real constant: k

Angle: α

$$764. y'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$775. \frac{d}{dx}(C) = 0$$

$$776. \frac{d}{dx}(x) = 1$$

$$777. \frac{d}{dx}(ax + b) = a$$

$$778. \frac{d}{dx}(ax^2 + bx + c) = ax + b$$

$$779. \frac{d}{dx}(x^n) = nx^{n-1}$$

$$780. \frac{d}{dx}(x^{-n}) = -\frac{n}{x^{n+1}}$$

$$781. \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$782. \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$783. \frac{d}{dx}(\sqrt[n]{x}) = \frac{1}{n\sqrt[n]{x^{n-1}}}$$

$$784. \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$785. \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, \quad a > 0, \quad a \neq 1.$$

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Page 222 of 338

$$797. \frac{d}{dx}(\operatorname{arccot} x) = -\frac{1}{1+x^2}$$

$$798. \frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$799. \frac{d}{dx}(\operatorname{arccsc} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$800. \frac{d}{dx}(\sinh x) = \cosh x$$

$$801. \frac{d}{dx}(\cosh x) = \sinh x$$

$$802. \frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$803. \frac{d}{dx}(\coth x) = \frac{-1}{\sinh^2 x} = -\operatorname{csch}^2 x$$

$$804. \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$$

$$805. \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \cdot \coth x$$

$$806. \frac{d}{dx}(\operatorname{arcsinh} x) = \frac{1}{\sqrt{x^2+1}}$$

$$807. \frac{d}{dx}(\operatorname{arccosh} x) = \frac{1}{\sqrt{x^2-1}}$$

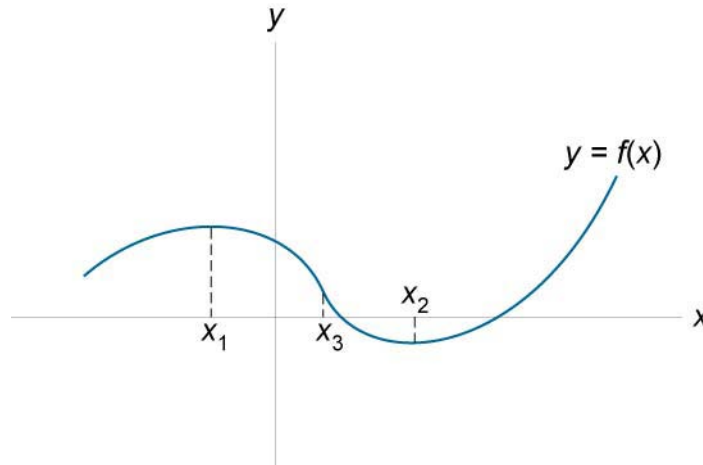


Figure 177.

829. Local extrema

A function $f(x)$ has a **local maximum** at x_1 if and only if there exists some interval containing x_1 such that $f(x_1) \geq f(x)$ for all x in the interval (Fig.177).

A function $f(x)$ has a **local minimum** at x_2 if and only if there exists some interval containing x_2 such that $f(x_2) \leq f(x)$ for all x in the interval (Fig.177).

830. Critical Points

A critical point on $f(x)$ occurs at x_0 if and only if either $f'(x_0)$ is zero or the derivative doesn't exist.

831. First Derivative Test for Local Extrema.

If $f(x)$ is increasing ($f'(x) > 0$) for all x in some interval $(a, x_1]$ and $f(x)$ is decreasing ($f'(x) < 0$) for all x in some interval $[x_1, b)$, then $f(x)$ has a local maximum at x_1 (Fig.177).

8.7 Differential

Functions: f, u, v

Independent variable: x

Derivative of a function: $y'(x), f'(x)$

Real constant: C

Differential of function $y = f(x)$: dy

Differential of x : dx

Small change in x : Δx

Small change in y : Δy

838. $dy = y' dx$

839. $f(x + \Delta x) = f(x) + f'(x)\Delta x$

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Page 231 of 338

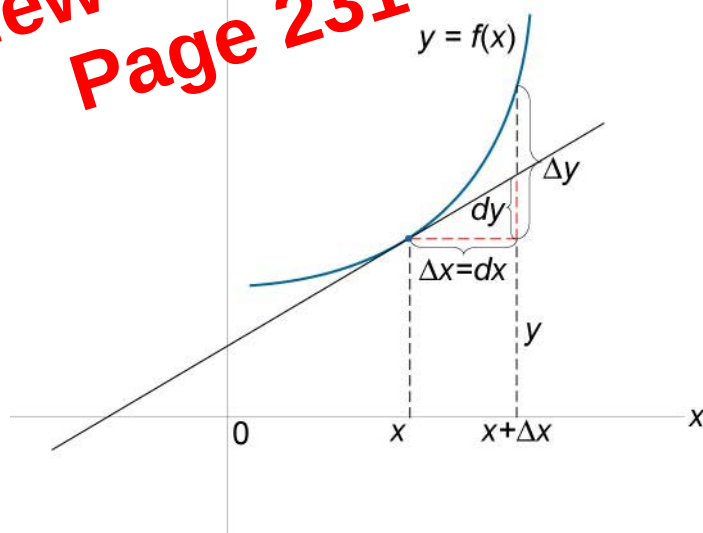


Figure 178.

$$880. \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, n \neq -1.$$

$$881. \int \frac{dx}{x} = \ln|x| + C$$

$$882. \int \frac{dx}{ax + b} = \frac{1}{a} \ln|ax + b| + C$$

$$883. \int \frac{ax + b}{cx + d} dx = \frac{a}{c}x + \frac{bc - ad}{c^2} \ln|cx + d| + C$$

$$884. \int \frac{dx}{(x + a)(x + b)} = \frac{1}{a - b} \ln \left| \frac{x + b}{x + a} \right| + C, a \neq b.$$

$$885. \int \frac{xdx}{a + bx} = \frac{1}{b} \left(\frac{1}{2} (a + bx)^2 - a \ln|a + bx| \right) + C$$

$$886. \int \frac{x^2 dx}{a + bx} = \frac{1}{b^3} \left[\frac{1}{2} (a + bx)^2 - 2a(a + bx) + a^2 \ln|a + bx| \right] + C$$

$$887. \int \frac{dx}{x(a + bx)} = \frac{1}{a} \ln \left| \frac{a + bx}{x} \right| + C$$

$$888. \int \frac{dx}{x^2(a + bx)} = -\frac{1}{ax} + \frac{b}{a^2} \ln \left| \frac{a + bx}{x} \right| + C$$

$$889. \int \frac{xdx}{(a + bx)^2} = \frac{1}{b^2} \left(\ln|a + bx| + \frac{a}{a + bx} \right) + C$$

$$945. \int (x^2 - a^2)^{3/2} dx = -\frac{x}{8}(2x^2 - 5a^2)\sqrt{x^2 - a^2} + \frac{3a^4}{8} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$946. \int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$947. \int x\sqrt{a^2 - x^2} dx = -\frac{1}{3}(a^2 - x^2)^{3/2} + C$$

$$948. \int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} + C$$

$$949. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} \ln \left| \frac{x}{a + \sqrt{a^2 - x^2}} \right| + C$$

$$950. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{a} + C$$

$$951. \int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$952. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin \frac{x}{a} + C$$

$$953. \int \frac{xdx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C$$

$$954. \int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$975. \int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x + C$$

$$976. \int \sin x \cos^2 x \, dx = -\frac{1}{3} \cos^3 x + C$$

$$977. \int \sin^2 x \cos^2 x \, dx = \frac{x}{8} - \frac{1}{32} \sin 4x + C$$

$$978. \int \tan x \, dx = -\ln|\cos x| + C$$

$$979. \int \frac{\sin x}{\cos^2 x} \, dx = \frac{1}{\cos x} + C = \sec x + C$$

$$980. \int \frac{\sin^2 x}{\cos x} \, dx = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| - \sin x + C$$

$$981. \int \tan^2 x \, dx = \tan x - x + C$$

$$982. \int \cot x \, dx = \ln|\sin x| + C$$

$$983. \int \frac{\cos x}{\sin^2 x} \, dx = -\frac{1}{\sin x} + C = -\csc x + C$$

$$984. \int \frac{\cos^2 x}{\sin x} \, dx = \ln \left| \tan \frac{x}{2} \right| + \cos x + C$$

$$985. \int \cot^2 x \, dx = -\cot x - x + C$$

$$986. \int \frac{dx}{\cos x \sin x} = \ln|\tan x| + C$$

$$1008. \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

9.6 Integrals of Exponential and Logarithmic Functions

$$1009. \int e^x \, dx = e^x + C$$

$$1010. \int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$1011. \int e^{ax} \, dx = \frac{e^{ax}}{a} + C$$

$$1012. \int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) + C$$

$$1013. \int \ln x \, dx = x \ln x - x + C$$

$$1014. \int \frac{dx}{x \ln x} = \ln |\ln x| + C$$

$$1015. \int x^n \ln x \, dx = x^{n+1} \left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right] + C$$

$$1016. \int e^{ax} \sin bx \, dx = \frac{a \sin bx - b \cos bx}{a^2 + b^2} e^{ax} + C$$

$$-\frac{n-1}{n+m} \int \sinh^{n-2} x \cosh^m x dx$$

$$1026. \int \tanh^n x dx = -\frac{1}{n-1} \tanh^{n-1} x + \int \tanh^{n-2} x dx, n \neq 1.$$

$$1027. \int \coth^n x dx = -\frac{1}{n-1} \coth^{n-1} x + \int \coth^{n-2} x dx, n \neq 1.$$

$$1028. \int \operatorname{sech}^n x dx = \frac{\operatorname{sech}^{n-2} x \tanh x}{n-1} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} x dx, n \neq 1.$$

$$1029. \int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$1030. \int \frac{dx}{\sin^n x} = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}, n \neq 1.$$

$$1031. \int \cos^n x dx = \frac{1}{n} \sin x \cos^{n-1} x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$1032. \int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}, n \neq 1.$$

$$1033. \int \sin^n x \cos^m x dx = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m} \int \sin^n x \cos^{m-2} x dx$$

$$1034. \int \sin^n x \cos^m x dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m}$$

1063. Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \text{ if } F'(x) = f(x).$$

1064. Method of Substitution

If $x = g(t)$, then

$$\int_a^b f(x)dx = \int_c^d f(g(t))g'(t)dt,$$

where

$$c = g^{-1}(a), d = g^{-1}(b).$$

1065. Integration by Parts

$$\int_a^b u dv = (uv)\Big|_a^b - \int_a^b v du$$

1066. Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n} \left[f(x_0) + f(x_n) + 2 \sum_{i=1}^{n-1} f(x_i) \right]$$

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Page 259 of 338

- If $\int_a^{\infty} f(x)dx$ is convergent, then $\int_a^{\infty} g(x)dx$ is also convergent,
- If $\int_a^{\infty} g(x)dx$ is divergent, then $\int_a^{\infty} f(x)dx$ is also divergent.

1075. Absolute Convergence

If $\int_a^{\infty} |f(x)|dx$ is convergent, then the integral $\int_a^{\infty} f(x)dx$ is absolutely convergent.

1076. Discontinuous Integrand

Let $f(x)$ be a function which is continuous on the interval $[a, b)$ but is discontinuous at $x = b$. Then

$$\int_a^b f(x)dx = \lim_{\epsilon \rightarrow 0^+} \int_a^{b-\epsilon} f(x)dx$$

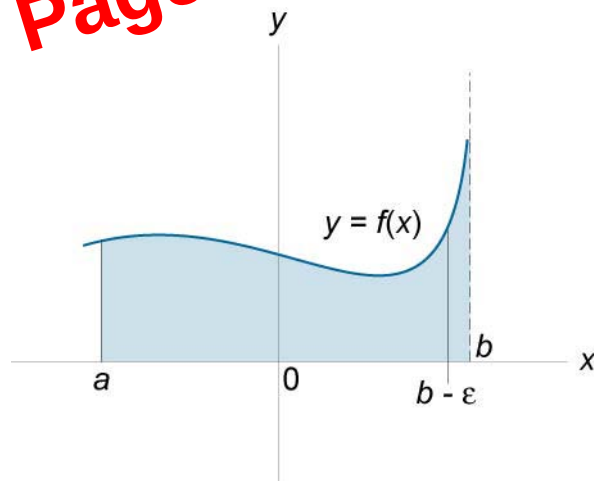


Figure 187.

$$\iint_S f(x, y) dA \leq \iint_R f(x, y) dA.$$

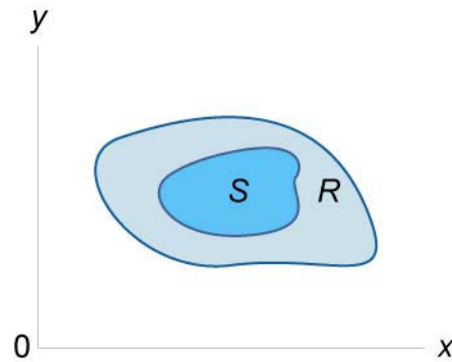


Figure 191.

1084. If $f(x, y) \geq 0$ on R and S are non-overlapping regions, then $\iint_{R \cup S} f(x, y) dA = \iint_R f(x, y) dA + \iint_S f(x, y) dA$.
Here $R \cup S$ is the union of the regions R and S .

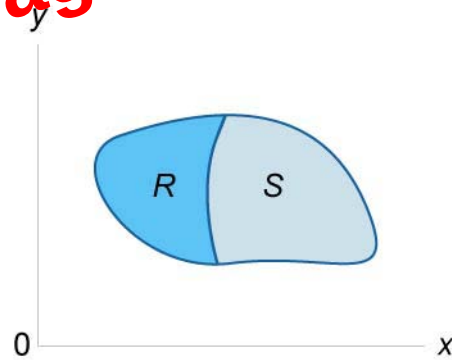


Figure 192.

1130. Length of a Curve

$$L = \int_C ds = \int_{\alpha}^{\beta} \left| \frac{d\vec{r}}{dt}(t) \right| dt = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2} dt,$$

where C is a piecewise smooth curve described by the position vector $\vec{r}(t)$, $\alpha \leq t \leq \beta$.

If the curve C is two-dimensional, then

$$L = \int_C ds = \int_{\alpha}^{\beta} \left| \frac{d\vec{r}}{dt}(t) \right| dt = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} dt.$$

If the curve C is the graph of a function $y = f(x)$ in the xy -plane ($a \leq x \leq b$), then

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx.$$

1131. Length of a Curve in Polar Coordinates

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta} \right)^2 + r^2} d\theta,$$

where the curve C is given by the equation $r = r(\theta)$, $\alpha \leq \theta \leq \beta$ in polar coordinates.

1132. Mass of a Wire

$$m = \int_C \rho(x, y, z) ds,$$

where $\rho(x, y, z)$ is the mass per unit length of the wire.

If C is a curve parametrized by the vector function $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$, then the mass can be computed by the formula

where $\vec{v}(\vec{r})$ is the fluid velocity.

1161. Mass Flux (across the surface S)

$$\Phi = \iiint_S \rho \vec{v}(\vec{r}) \cdot d\vec{S},$$

where $\vec{F} = \rho \vec{v}$ is the vector field, ρ is the fluid density.

1162. Surface Charge

$$Q = \iint_S \sigma(x, y) dS,$$

where $\sigma(x, y)$ is the surface charge density.

1163. Gauss' Law

The **electric flux** through any closed surface is proportional to the charge Q enclosed by the surface.

$$\Phi = \iiint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0},$$

where

Φ is the electric flux,

$\vec{E}$ is the magnitude of the electric field strength,

$\epsilon_0 = 8,85 \times 10^{-12} \frac{F}{m}$ is permittivity of free space.

1172. Population Dynamics (Logistic Model)

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{M} \right),$$

where $P(t)$ is population at time t , k is a positive constant, M is a limiting size for the population.

The solution of the differential equation is

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-kt}}, \text{ where } P_0 = P(0) \text{ is the initial population at time } t = 0.$$

10.2 Second Order Ordinary Differential Equations

1173. Homogeneous Linear Equations with Constant Coefficients

$$r^2 + pr + q = 0.$$

The characteristic equation is

$$\lambda^2 + p\lambda + q = 0.$$

If λ_1 and λ_2 are distinct real roots of the characteristic equation, then the general solution is

$$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}, \text{ where}$$

C_1 and C_2 are integration constants.

If $\lambda_1 = \lambda_2 = -\frac{p}{2}$, then the general solution is

$$y = (C_1 + C_2 x) e^{-\frac{p}{2}x}.$$

If λ_1 and λ_2 are complex numbers:

Chapter 11

Series

11.1 Arithmetic Series

Initial term: a_1

Nth term: a_n

Difference between successive terms: d

Number of terms in the series: n

Sum of the first n terms: S

$$1184. a_n = a_{n-1} + d = a_{n-2} + 2d = \dots = a_1 + (n-1)d$$

$$1185. a_1 + a_n = a_2 + a_{n-1} = \dots = a_i + a_{n+1-i}$$

$$1186. a_i = \frac{a_{i-1} + a_{i+1}}{2}$$

$$1187. S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

$$1248. f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$1249. a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$1250. b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

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Page 327 of 338

1260. Range of Probability Values

$$0 \leq P(A) \leq 1$$

1261. Certain Event

$$P(A) = 1$$

1262. Impossible Event

$$P(A) = 0$$

1263. Complement

$$P(\bar{A}) = 1 - P(A)$$

1264. Independent Events

$$P(A/B) = P(A),$$

$$P(B/A) = P(B)$$

1265. Addition Rule for Independent Events

$$P(A \cup B) = P(A) + P(B)$$

1266. Multiplication Rule for Independent Events

$$P(A \cap B) = P(A) \cdot P(B)$$

1267. General Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

where

$A \cup B$ is the union of events A and B,

$A \cap B$ is the intersection of events A and B.

1268. Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\mathbf{1269.} \quad P(A \cap B) = P(B) \cdot P(A/B) = P(A) \cdot P(B/A)$$