

$$(x+h)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} h^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \rightarrow 0} \frac{x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \binom{n}{3} x^{n-3} h^3 + \dots + h^n}{h}$$

$$\frac{n!}{2!(n-2)!} = \frac{n(n-1)(n-2)!}{2(n-2)!}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \frac{n(n-1)(n-2)}{6} x^{n-3} h^3 + \dots + h^n}{h}$$

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$$= \lim_{h \rightarrow 0} n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \frac{n(n-1)(n-2)}{6} x^{n-3} h^2 + \dots + h^{n-1}$$

$$\frac{n!}{3!(n-3)!} =$$

$$= n x^{n-1}$$

$$= \frac{n(n-1)(n-2)(n-3)!}{3 \cdot 2 \cdot 1 \cdot (n-3)!}$$

$$= \frac{n(n-1)(n-2)}{6}$$

[khanacademy/binomial\\_theorem/](http://khanacademy/binomial_theorem/)  
(More on the binomial theorem)