$$(x+h)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} h^k \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{\cancel{x^n} + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \binom{n}{3} x^{n-3} h^3 + \dots + h^n \cancel{-x^n}}{h}$$

$$\frac{n!}{2!(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{2\,\cancel{(n-2)!}} = \frac{n(n-1)}{2}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \frac{n(n-1)(n-2)}{6} x^{n-3} h^3 + \dots + h^n}{h}$$

$$= \lim_{h\to 0} n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \frac{n(n-1)(n-2)}{6} x^{n-3} h^2 + \dots + h^{n-1}$$

$$= n x^{n-1}$$

$$\frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{3\cdot 2\cdot 1\cdot \cancel{(n-3)!}} = \frac{n(n-1)(n-2)}{6}$$

khanacademy/binomial_theorem/

(More on the binomial theorem)