

Optimization Problems

→ How do you maximize a function?

* Extreme points: maximums, minimums, points of inflection.

* At extreme points, the derivative of the function must be equal to zero.

$$\rightarrow EU(Y) = (1-\pi)u(Y_1) + \pi u(Y_2)$$

$$Y_1 = W - pC$$

$$Y_2 = W - L + C - pC$$

$$\begin{aligned} \text{* 1st Derivative: } \frac{dEU(Y)}{dC} &= (1-\pi) \frac{du(Y_1)}{dY_1} \cdot \frac{dY_1}{dC} + \pi \frac{du(Y_2)}{dY_2} \cdot \frac{dY_2}{dC} \\ &= (1-\pi)u'(Y_1)(-p) + \pi u'(Y_2)(1-p) \end{aligned}$$

$$\begin{aligned} \frac{d[(1-\pi)u(Y_1)]}{dC} &= (1-\pi) \frac{du(Y_1)}{dC} \\ Y_1 &= W - pC \quad \left[\text{If } C \uparrow \frac{dY_1}{dC} = -p \right] \\ \frac{du(Y_1)}{dC} &= \frac{du(Y_1)}{d(Y_1)} \cdot \frac{d(Y_1)}{dC} \\ &= u'(Y_1)(-p) \\ Y_2 &= W - L + (1-p)C \\ \frac{d(Y_2)}{dC} &= (1-p) \\ \frac{du(Y_2)}{dC} &= \frac{du(Y_2)}{d(Y_2)} \cdot \frac{d(Y_2)}{dC} \\ &= u'(Y_2)(1-p) \end{aligned}$$