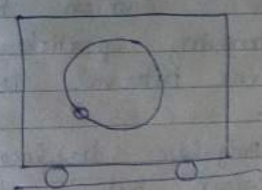
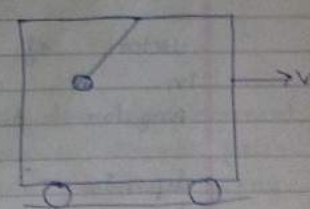
CIRCULAR MOTION

⇒ When a particle moves in [illegible] plane such that its distance from a fixed (or moving) point remains constant, then its motion [illegible] known as circular motion w.r.t [illegible] that fixed (or moving) point. The fixed point [illegible] called centre, and the distance [illegible] of particle from it is called radius.

We solve circular motion always in the frame of centre.

If the centre move, then it is stop & apply circular motion





KINEMATICS OF CIRCULAR MOTION:-

VARIABLES MOTION:-

(a) ANGULAR POSITION:- The angle made by the position vector w.r.t origin, with the reference line is called angular position. Clearly angular position depends on the choice of the origin

$\frac{d}{dx}(5x - 5)$ uniform motion = a is zero

$5 - 0$

(ii) INSTANTANEOUS ANGULAR ACCELERATION ↦

$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$ (like $a = \frac{d\vec{v}}{dt}$)

Since $\boxed{\vec{\omega} = \frac{d\theta}{dt}}$

like ($v = \frac{dx}{dt}$)

$\therefore \boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}}$; also $\boxed{\vec{\alpha} = \omega \frac{d\vec{\omega}}{d\theta}}$

⇒ If $\alpha = 0$, circular motion is said to be uniform.

⇒ Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rad/s^2

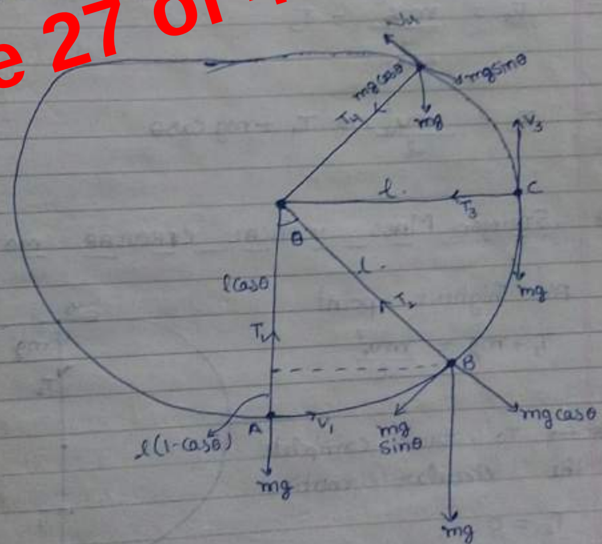
$e_t = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$. $|\vec{v}_2' - \vec{v}_1'| = 2v\sin\frac{\theta}{2}$

$e_N = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$. $|\vec{v}_2'| - |\vec{v}_1'| = 0$.

VERTICAL CIRCULAR MOTION :-

It is non- uniform circular motion.

Let a bob of mass m is moving in vertical circle with help of a string of length l as shown as :-

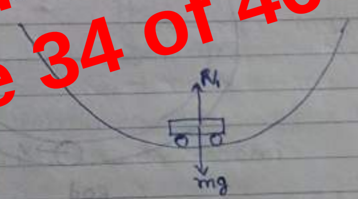


In vertical circular motion a bob with string (similar case) speed, Tension decreases as we move from lowest to highest point.

$$F_c = \frac{mv_1^2}{l} = T_1 - mg$$

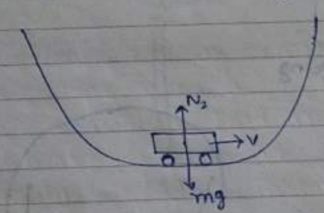
Q. Prove that a motor car moving over a concave bridge is heavier than the same car resting on the same bridge.

Ans:- if car is stationary



$N_1 = mg$

if car is moving



$N_2 - mg = \frac{mv^2}{R}$

$N_2 = \frac{mv^2}{R} + mg$

$N_2 > N_1$ (H.P)

Q. A car is moving with uniform speed over a circular bridge of radius R which subtends an angle 90° at its centre. Find the minimum possible speed so that the car can cross the bridge without losing the contact anywhere.

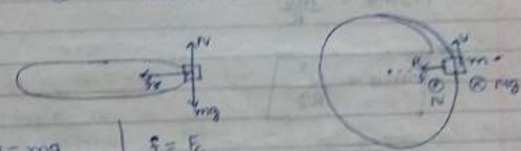
Ans:- $a_T = 0$: $v =$ constant

CIRCULAR TURNING ON ROADS :-

The necessary centripetal force is being provided to the vehicles by following three ways

1. By friction only
2. By banking of roads only
3. By friction and banking of road both

(i) BY FRICTION ONLY :- Let car is moving with constant speed on a Horizontal Circular Road.



$N = mg$	$f = F_c$
$f_{max} = \mu mg$	$f = \frac{mv^2}{R}$

⇒ Maximum value of v for which car will not slide

$$f = f_{max}$$

$$\frac{mv^2}{R} = \mu mg$$

$$\boxed{v_{max} = \sqrt{\mu g R} \qquad \mu_{min} = \frac{v^2}{gR}}$$

(i) V_{min} :- friction is acting upward (Along the circles).

$$N\cos\theta + \mu N\sin\theta = mg \quad \text{— (i)}$$

$$N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R} \quad \text{— (ii)}$$

$$\frac{(ii)}{(i)} \cdot \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{v^2}{Rg}$$

$$\frac{\tan\theta - \mu}{1 + \mu\tan\theta} = \frac{v^2}{Rg} \qquad \left\{ \begin{array}{l} \mu = \tan\phi \\ \phi = \text{Angle of Repose} \end{array} \right.$$

$$\frac{\tan\theta - \tan\phi}{1 + \tan\phi\tan\theta} = \frac{v^2}{Rg}$$

☆ $$\boxed{V_{min} = \sqrt{Rg\{\tan(\theta - \phi)}}$$

If $\theta < \phi$ then $V_{min} = 0$

(ii) V_{max} :→ friction is acting downward. (along the incline).

→ g' & N are maximum at pale and minimum at equitorial position.

→ fr on body due to rotation of Earth is zero at pale & equitorial position & at mid at height.